Accredited

Oxford Cambridge and RSA

A Level Further Mathematics B (MEI)

Y421 Mechanics Major

Sample Question Paper Version 2

Date – Morning/Afternoon

Time allowed: 2 hours 15 minutes

You must have:
- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

You may use:
- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

- The total number of marks for this paper is **120**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **20** pages. The Question Paper consists of **12** pages.

Section A (26 marks)

Answer **all** the questions

1 A particle P has position vector $\mathbf{r}$ m at time t s given by $\mathbf{r} = (t^3 - 3t^2)\,\mathbf{i} - (4t^2 + 1)\,\mathbf{j}$ for $t \geq 0$.
Find the magnitude of the acceleration of P when $t = 2$. **[4]**

2 A particle of mass 5 kg is moving with velocity $2\mathbf{i} + 5\mathbf{j}$ m s^{-1}. It receives an impulse of magnitude 15 N s in the direction $\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$. Find the velocity of the particle immediately afterwards. **[3]**

3 The fixed points E and F are on the same horizontal level with EF = 1.6 m. A light string has natural length 0.7 m and modulus of elasticity 29.4 N. One end of the string is attached to E and the other end is attached to a particle of mass M kg. A second string, identical to the first, has one end attached to F and the other end attached to the particle. The system is in equilibrium in a vertical plane with each string stretched to a length of 1 m, as shown in Fig. 3.

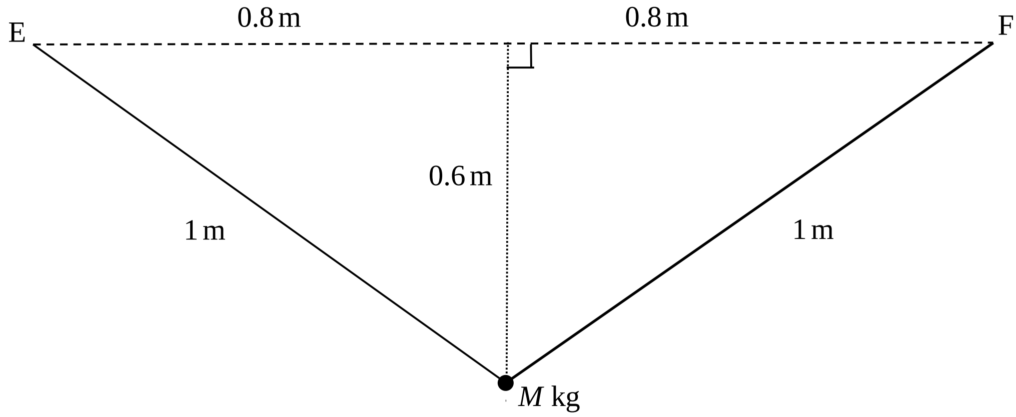


Fig. 3

(i) Find the tension in each string. **[2]**

(ii) Find M. **[3]**

4 A fixed smooth sphere has centre O and radius a. A particle P of mass m is placed at the highest point of the sphere and given an initial horizontal speed u.

For the first part of its motion, P remains in contact with the sphere and has speed v when OP makes an angle θ with the upward vertical. This is shown in Fig. 4.

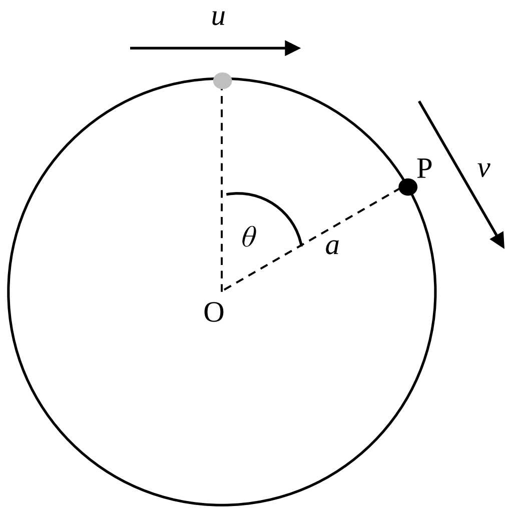


Fig. 4

(i) By considering the energy of P, show that $v^2 = u^2 + 2ga(1-\cos\theta)$. **[2]**

(ii) Show that the magnitude of the normal contact force between the sphere and particle P is $mg(3\cos\theta - 2) - \dfrac{mu^2}{a}$. **[2]**

The particle loses contact with the sphere when $\cos\theta = \dfrac{3}{4}$.

(iii) Find an expression for u in terms of a and g. **[2]**

5 Fig. 5 shows a light inextensible string of length 3.3 m passing through a small smooth ring R. The ends of the string are attached to fixed points A and B, where A is vertically above B. The ring R has mass 0.27 kg and is moving with constant speed in a horizontal circle of radius 1.2 m. The distances AR and BR are 2 m and 1.3 m respectively.

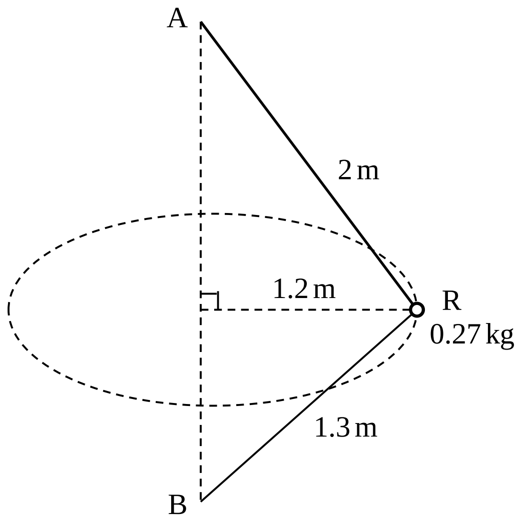


Fig. 5

(i) Show that the tension in the string is 6.37 N. **[4]**

(ii) Find the speed of R. **[4]**

Section B (94 marks)

Answer **all** the questions

6 Fig. 6 shows a pendulum which consists of a rod AB freely hinged at the end A with a weight at the end B. The pendulum is oscillating in a vertical plane. The total energy, E, of the pendulum is given by

$$E = \frac{1}{2} I\omega^2 - mgh\cos\theta \,,$$

where

- ω is its angular speed
- m is its mass
- h is the distance of its centre of mass from A
- θ is the angle the rod makes with the downward vertical
- g is the acceleration due to gravity
- I is a quantity known as the moment of inertia of the pendulum.

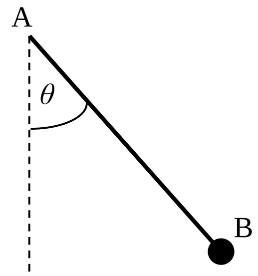


Fig. 6

(i) Use the expression for E to deduce the dimensions of I. **[4]**

It is suggested that the period of oscillation, T, of the pendulum is given by $T = kI^{\alpha}(mg)^{\beta}h^{\gamma}$, where k is a dimensionless constant.

(ii) Use dimensional analysis to find the values of α, β and γ. **[5]**

A class experiment finds that, when all other quantities are fixed, T is proportional to $\frac{1}{\sqrt{m}}$.

(iii) Determine whether this result is consistent with your answer to part **(ii)**. **[1]**

7 A uniform ladder of length 8 m and weight 180 N stands on a rough horizontal surface and rests against a smooth vertical wall. The ladder makes an angle of 20° with the wall. A woman of weight 720 N stands on the ladder. Fig. 7 shows this situation modelled with the woman's weight acting at a distance x m from the lower end of the ladder.

The system is in equilibrium.

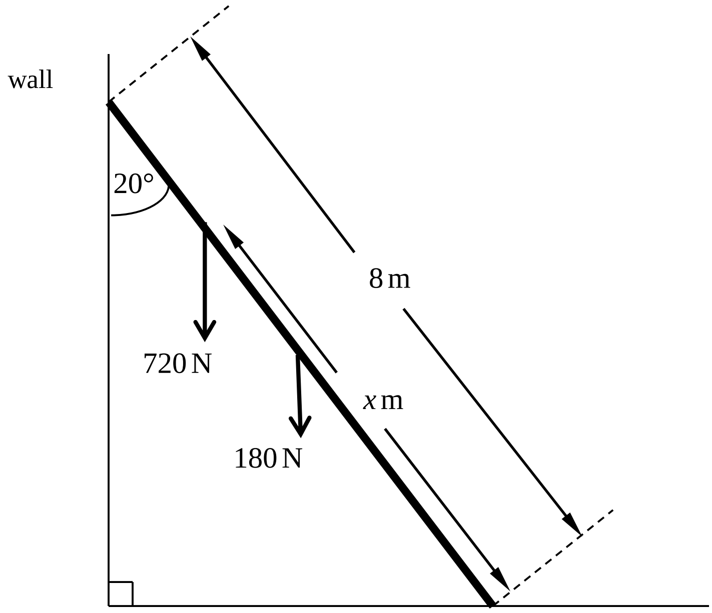


Fig. 7

(i) Show that the frictional force between the ladder and the horizontal surface is F N, where $F = 90(1 + x)\tan 20^\circ$. **[4]**

(ii) (*A*) State with a reason whether F increases, stays constant or decreases as x increases. **[1]**

(*B*) Hence determine the set of values of the coefficient of friction between the ladder and the surface for which the woman can stand anywhere on the ladder without it slipping. **[4]**

8 A tractor has a mass of 6000 kg. When developing a power of 5 kW, the tractor is travelling at a steady speed of $2.5\,\text{m s}^{-1}$ across a horizontal field.

(i) Calculate the magnitude of the resistance to the motion of the tractor. **[2]**

The tractor comes to horizontal ground where the resistance to motion is different. The power developed by the tractor during the next 10 s has an average value of 8 kW. During this time, the tractor accelerates uniformly from $2.5\,\text{m s}^{-1}$ to $3\,\text{m s}^{-1}$.

(ii) (*A*) Show that the work done against the resistance to motion during the 10 s is 71 750 J. **[4]**

(*B*) Assuming that the resistance to motion is constant, calculate its value. **[3]**

The tractor can usually travel up a straight track inclined at an angle α to the horizontal, where $\sin\alpha = \frac{1}{20}$, while accelerating uniformly from 3 m s^{-1} to 3.25 m s^{-1} over a distance of 100 m against a resistance to motion of constant magnitude of 2000 N.

The tractor develops a fault which limits its maximum power to 16kW.

(iii) Determine whether the tractor could now perform the same motion up the track.
[You should assume that the mass of the tractor and the resistance to motion remain the same.] **[7]**

9

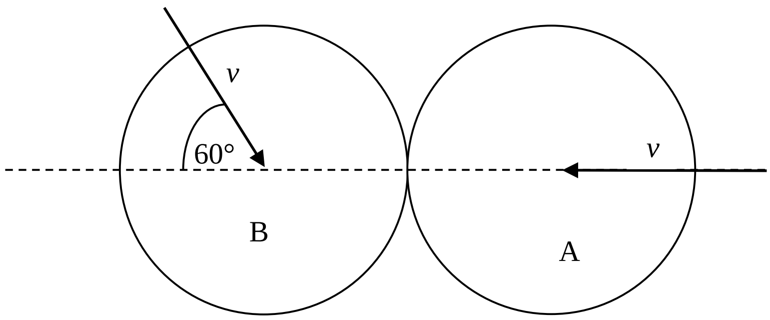


Fig. 9

Fig. 9 shows the instant of impact of two identical uniform smooth spheres, A and B, each with mass m. Immediately before they collide, the spheres are sliding towards each other on a smooth horizontal table in the directions shown in the diagram, each with speed v. The coefficient of restitution between the spheres is $\frac{1}{2}$.

(i) Show that, immediately after the collision, the speed of A is $\frac{1}{8}v$. Find its direction of motion. **[6]**

(ii) Find the percentage of the original kinetic energy that is lost in the collision. **[7]**

(iii) State where in your answer to part **(i)** you have used the assumption that the contact between the spheres is smooth. **[1]**

10 In this question take $g = 10$.

A smooth ball of mass 0.1 kg is projected from a point on smooth horizontal ground with speed $65\,\mathrm{m\,s^{-1}}$ at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$. While it is in the air the ball is modelled as a particle moving freely under gravity. The ball bounces on the ground repeatedly. The coefficient of restitution for the first bounce is 0.4.

(i) Show that the ball leaves the ground after the first bounce with a horizontal speed of $52\,\mathrm{m\,s^{-1}}$ and a vertical speed of $15.6\,\mathrm{m\,s^{-1}}$. Explain your reasoning carefully. **[4]**

(ii) Calculate the magnitude of the impulse exerted on the ball by the ground at the first bounce. **[2]**

Each subsequent bounce is modelled by assuming that the coefficient of restitution is 0.4 and that the bounce takes no time. The ball is in the air for T_1 seconds between projection and bouncing the first time, T_2 seconds between the first and second bounces, and T_n seconds between the $(n-1)$th and nth bounces.

(iii) (*A*) Show that $T_1 = \frac{39}{5}$. **[2]**

(*B*) Find an expression for T_n in terms of n. **[2]**

(iv) According to the model, how far does the ball travel horizontally while it is still bouncing? **[3]**

(v) According to the model, what is the motion of the ball after it has stopped bouncing? **[1]**

11 The region bounded by the x-axis and the curve $y=\frac{1}{2}k\left(1-x^2\right)$ for $-1\le x\le 1$ is occupied by a uniform lamina, as shown in Fig. 11.1.

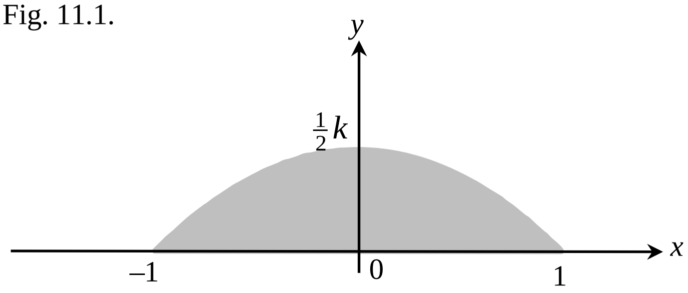


Fig. 11.1

(i) In this question you must show detailed reasoning.

Show that the centre of mass of the lamina is at $\left(0, \frac{1}{5}k\right)$. **[7]**

A shop sign is modelled as a uniform lamina in the form of the lamina in part **(i)** attached to a rectangle ABCD, where AB = 2 and BC = 1. The sign is suspended by two vertical wires attached at A and D, as shown in Fig. 11.2.

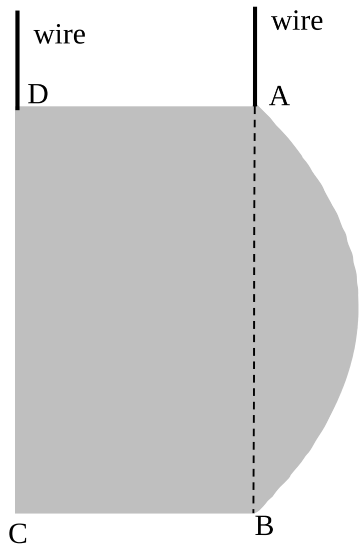


Fig. 11.2

(ii) Show that the centre of mass of the sign is at a distance

$$\frac{2k^2+10k+15}{10k+30}$$

from the midpoint of CD. **[4]**

The tension in the wire at A is twice the tension in the wire at D.

(iii) Find the value of k. **[5]**

12 Fig. 12 shows x- and y- coordinate axes with origin O and the trajectory of a particle projected from O with speed $28\,\mathrm{m\,s^{-1}}$ at an angle α to the horizontal. After t seconds, the particle has horizontal and vertical displacements x m and y m.

Air resistance should be neglected.

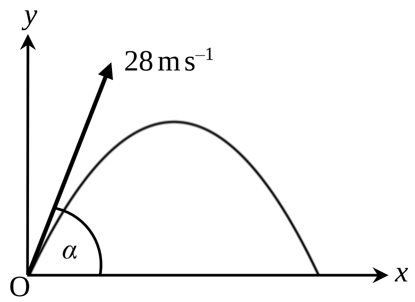


Fig. 12

(i) Show that the equation of the trajectory is given by

$$\tan^2\alpha - \frac{160}{x}\tan\alpha + \frac{160y}{x^2} + 1 = 0 . \qquad (*)$$ **[5]**

(ii) (A) Show that if (*) is treated as an equation with $\tan\alpha$ as a variable and with x and y as constants, then (*) has two distinct real roots for $\tan\alpha$ when $y < 40 - \frac{x^2}{160}$. **[3]**

(B) Show the inequality in part **(ii)** (A) as a locus on the graph of $y = 40 - \frac{x^2}{160}$ in the Printed Answer Booklet and label it R. **[1]**

S is the locus of points (x, y) where (*) has **one** real root for $\tan\alpha$.
T is the locus of points (x, y) where (*) has **no** real roots for $\tan\alpha$.

(iii) Indicate S and T on the graph in the Printed Answer Booklet. **[2]**

(iv) State the significance of R, S and T for the possible trajectories of the particle. **[3]**

A machine can fire a tennis ball from ground level with a maximum speed of $28\,\mathrm{m\,s^{-1}}$.

(v) State, with a reason, whether a tennis ball fired from the machine can achieve a range of 80 m. **[1]**

END OF QUESTION PAPER

…day June 20XX – Morning/Afternoon

A Level Further Mathematics B (MEI)

Y421 Mechanics Major

SAMPLE MARK SCHEME

Duration: 2 hours 15 minutes

MAXIMUM MARK 120

This document consists of 20 pages

Text Instructions

1. Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✘	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

2. Subject-specific Marking Instructions for A Level Further Mathematics B (MEI)

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some papers. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

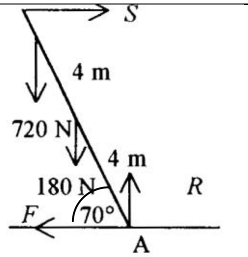
j If in any case the scheme operates with considerable unfairness consult your Team Leader.

k Anything in the mark scheme which is in square brackets […] is not required for the mark to be earned on this occasion, but shows what a complete solution might look like

Question			Answer	Marks	AOs	Guidance	
1			$\mathbf{v} = (3t^2 - 6t)\,\mathbf{i} - 8t\,\mathbf{j}$	M1	1.1a	Differentiate	
			$\mathbf{a} = (6t - 6)\,\mathbf{i} - 8\,\mathbf{j}$	M1	1.1	Differentiate	
			When $t = 2$, $\mathbf{a} = 6\mathbf{i} - 8\mathbf{j}$	M1	1.1	Substitute $t = 2$	
			Magnitude of acceleration = 10 m s^{-2}	A1	1.1		
				[4]			
2			Impulse is $5(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$	B1	1.1		
			$5(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}) = 5(\mathbf{v} - 2\mathbf{i} - 5\mathbf{j})$	M1	3.4	Use Impulse = change in momentum	
			$\mathbf{v} = 3\mathbf{i} + 7\mathbf{j} - 2\mathbf{k}$	A1	1.1		
				[3]			
3	(i)		Use Hooke's law: $T = \frac{\lambda e}{l} = \frac{29.4 \times 0.3}{0.7} = 12.6$	M1	3.4	Use of Hooke's law	
			Tension is 12.6 N	A1	1.1	Correct answer	
				[2]			
3	(ii)		Let θ be angle between each string and vertical, then $2T\cos\theta = Mg$	M1	3.4	Resolve vertically	
			$M = \frac{2 \times 12.6 \times 0.6}{9.8}$	M1	1.1a	Substitution	
			$= 1.54$	A1	1.1		
				[3]			

Question			Answer	Marks	AOs	Guidance	
4	(i)		Conservation of energy: $\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mga(1-\cos\theta)$ $v^2 = u^2 + 2ga(1-\cos\theta)$ AG	M1 A1 [2]	3.4 1.1	Using conservation of energy or work-energy equation Correct use of c.o.e. leading to given answer	
4	(ii)		N2L: $R = mg\cos\theta - \frac{mv^2}{a}$ $= mg\cos\theta - \frac{m}{a}\left(u^2 + 2ga\left(1-\cos\theta\right)\right)$ $R = mg\left(3\cos\theta - 2\right) - \frac{mu^2}{a}$. AG	B1 A1 [2]	3.4 1.1	Clearly shown	N2L is Newton's second law
4	(iii)		Use $R = 0$, $\cos\theta = \frac{3}{4}$ $u = \frac{1}{2}\sqrt{ga}$	M1 A1 [2]	3.1b 1.1	Both used	

Question			Answer	Marks	AOs	Guidance	
5	(i)		Let θ = angle between AR and vertical and α = angle between BR and vertical $\cos\theta = \frac{4}{5}$ $\cos\alpha = \frac{5}{13}$ or equivalent $T\cos\theta = T\cos\alpha + 0.27g$ Substitute to give $T = 6.37$ AG	 **B1** **M1** **A1** **A1** **[4]**	 **1.1** **3.3** **2.1** **1.1**	 Resolve vertically	
	(ii)		$T\sin\theta + T\sin\alpha = \frac{0.27v^2}{1.2}$ Solve: $v = 6.57$ The speed is 6.57 m s^{-1}	**M1** **A1** **M1** **A1** **[4]**	**3.3** **1.1** **3.4** **1.1**	N2L in radial direction Eliminate	
6	(i)		Dimensions of $mgh\cos\theta$ = ML^2T^{-2} Dimensions of ω = T^{-1} Use dim I = dim E/dim (ω^2) Dim I = ML^2	**B1** **B1** **M1** **A1** **[4]**	**1.1a** **1.1** **1.1** **2.1**	 Equate dimensions	
6	(ii)		$T = \left(ML^2\right)^{\alpha}\left(MLT^{-2}\right)^{\beta}L^{\gamma}$ Equate powers: $\alpha + \beta = 0, \quad 2\alpha + \beta + \gamma = 0, \quad -2\beta = 1$ Solve: $\alpha = \frac{1}{2}, \quad \beta = -\frac{1}{2}, \quad \gamma = -\frac{1}{2}$	**M1** **M1** **A1** **A1** **A1** **[5]**	**3.3** **1.1** **1.1** **1.1** **1.1**	Write equation in terms of dimensions Apply standard method For one correct All correct	

Question			Answer	Marks	AOs	Guidance	
6	**(iii)**		$kI^{\alpha}g^{\beta}h^{\gamma}$ is constant $T\ \alpha\ m^{\gamma} = m^{-\frac{1}{2}}$: T is proportional to $\frac{1}{\sqrt{m}}$ result compatible with experiment	**B1** **[1]**	**2.2b**	FT their power of m	
7	**(i)**		 $F = S$ $180 \times 4\cos 70^{\circ} + 720 \times x\cos 70^{\circ} = 8S\sin 70^{\circ}$ $(720 + 720x)\sin 20° = 8F\cos 20°$ $\Rightarrow F = 90(1 + x)\tan 20^{\circ}$	**B1** **M1** **A1** **E1** **[4]**	**2.2a** **3.3** **1.1** **2.1**	 Moments equation with at least two terms Two terms correct Correctly shown	

Question			Answer	Marks	AOs	Guidance	
7	(ii)	(*A*)	All other terms constant so F increases as x increases.	B1 [1]	2.4	Clearly explained	
7	(ii)	(*B*)	So worst case is $x = 8$ giving $F = 810\tan 20^\circ$ with $R = 900$ Since $F \le F_{\max} = \mu R$ $\mu \ge \frac{810\tan 20^\circ}{900} = \frac{9\tan 20^\circ}{10} = 0.3275...$	B1 B1 M1 A1 [4]	3.1b 3.4 1.1 2.2a	 Inequality clearly established	 Set notation not required
8	(i)		$5000 = F \times 2.5$ $F = 2000$. Now $R = F$ so R is 2000 N	M1 A1 [2]	3.4 1.1	Use of $P = Fv$	
8	(ii)	(*A*)	$\frac{1}{2}\times 6000\times 3^2 - \frac{1}{2}\times 6000\times 2.5^2$ $= 8000\times 10 - W$ $W = 80000 - 27000 + 18750 = 71\,750$, so 71 750 J AG	M1 B1 B1 E1 [4]	3.3 1.1 3.4 1.1	Use of work-energy equation with all terms One KE term correct Work done by driving force correct	 KE is kinetic energy
8	(ii)	(*B*)	Distance travelled = $\frac{2.5+3}{2}\times 10 = 27.5\text{ m}$ Hence $F \times 27.5 = 71\,750$ $F = 2609.09...$ so resistance is 2610 N (3 s.f.)	B1 M1 A1 [3]	3.1b 2.2a 1.1	 Work done is Fd	

Question			Answer	Marks	AOs	Guidance	
8	(iii)		$3.25^2 = 3^2 + 200a$	M1	3.1b	Use *suvat* to find acceleration	
			Acceleration = $\dfrac{3.25^2 - 3^2}{200}$ = 0.0078125	A1	1.1		
			N2L up slope: $D - 2000 - 6000g\sin\alpha = 6000a$	M1	3.4	N2L used	
			D = 4986.875	A1	1.1		
			Max power required = 4986.875 × 3.25	M1	3.2a	Use of max speed	
			= 16.2 kW	A1	1.1		
			Max power available is now 16 kW so could not achieve16.2 kW	E1	3.2a		
				[7]			
9	(i)		Perpendicular to line of centres, momentum of A unchanged, so A has no component of velocity, so A moves parallel to BA	B1	3.4		
			Taking L to R as positive NEL along line of centres: $V_A - V_B = e(v + v\cos 60^\circ)$	M1	3.3	Use of NEL. Allow sign errors. Must be $\dfrac{\text{speed of separation}}{\text{speed of approach}}$	NEL is Newton's experimental law
			$V_A - V_B = \dfrac{3v}{4}$	A1	1.1		
			PCLM $V_A + V_B = -v + v\cos 60^\circ = -\dfrac{v}{2}$	M1 A1	3.4 1.1	Use of PCLM. Allow sign errors	PCLM is Principle of conservation of linear momentum
			Solve: $V_A = \dfrac{1}{8}v$ $(V_B = -\dfrac{5}{8}v)$ so speed of A is $\dfrac{1}{8}v$ towards right	A1	2.1		
				[6]			

Question			Answer	Marks	AOs	Guidance	
9	(ii)		Perpendicular to line of centres, momentum and so velocity of B unchanged: $v\sin 60^\circ = \frac{\sqrt{3}}{2}v$ Final KE of B = $\frac{1}{2}mV_B{}^2 + \frac{1}{2}m\left(\frac{\sqrt{3}v}{2}\right)^2 =$ $\frac{73}{128}mv^2$ Loss in KE = $2\times\frac{1}{2}\times mv^2 - \frac{1}{2}m\left(\frac{v}{8}\right)^2 - \frac{73}{128}mv^2$ = $\frac{27}{64}mv^2$ Percentage loss = $\frac{27}{64}\times 100 = 42\%$	M1 A1 M1 A1 M1 A1 A1 **[7]**	3.4 1.1 3.1b 1.1 1.1 1.1 2.1	Use of conservation of linear momentum for B Use of KE using both velocity components All KE terms present	
9	(iii)		The component of linear momentum of A perpendicular to line of centres does not change	B1 **[1]**	3.5b		
10	(i)		At projection $\uparrow 65\sin\alpha = 39$ and $\rightarrow 65\cos\alpha = 52$ After bounce, LM conserved horizontally, so $\rightarrow 52$ The vertical speed when it bounces is the same as at projection NEL$\uparrow$ so $0.4\times 39 = 15.6\uparrow$	 B1 E1 B1 E1 **[4]**	 3.3 3.4 3.4 1.1	 Both correct Accept "horizontal speed not affected by impact" Use of NEL	

Question			Answer	Marks	AOs	Guidance	
10	(ii)		$\uparrow 0.1(15.6-(-39))=5.46$	M1 A1 [2]	1.2 1.1	Attempt at change in momentum. Accept sign error All signs correct	
10	(iii)	(*A*)	$0=39T_1-5T_1^2$ so $T_1=\dfrac{39}{5}$ AG	M1 A1 [2]	3.4 1.1	Use of appropriate *suvat*	
10	(iii)	(*B*)	$T_n=(0.4)^{n-1}\times\dfrac{39}{5}$	M1 A1 [2]	3.1b 2.2a	Recognise geometric progression Correct answer	
10	(iv)		GP with $a=\dfrac{39}{5}$ and $r=0.4$: $S_\infty=\dfrac{a}{1-r}$ Total time = 13 s Total distance travelled while bouncing = 676 m	M1 A1 A1 [3]	3.1b 1.1 3.2a	Use of formula	
10	(v)		Carries on after bouncing phase, with horizontal velocity of 52 m s^{-1}	B1 [1]	3.5b		
11	(i)		DR $\bar{x}=0$ by symmetry $\bar{y}\sigma\int_{-1}^{1}\frac{1}{2}k(1-x^2)\mathrm{d}x=\sigma\int_{-1}^{1}\frac{1}{8}k^2(1-2x^2+x^4)\mathrm{d}x$ $\bar{y}\left[\frac{1}{2}k\left(x-\frac{1}{3}x^3\right)\right]_{-1}^{1}=\left[\frac{1}{8}k^2\left(x-\frac{2}{3}x^3+\frac{1}{5}x^5\right)\right]_{-1}^{1}$ $\frac{2}{3}k\bar{y}=\frac{2}{15}k^2$: $\bar{y}=\frac{1}{5}k$ AG	 B1 M1 M1 A1 A1 M1 A1 [7]	 2.4 2.5 1.1 2.1 1.1 1.1 1.1	 Symmetry needs to be stated Use of correct formula Reasonable attempt at integration LHS RHS Substitute limits	

Question			Answer	Marks	AOs	Guidance	
11	(ii)		Let X (= XG) be distance of com from CD				
			$X = \dfrac{2\left(\frac{1}{2}\right)+\frac{2}{3}k\left(1+\frac{1}{5}k\right)}{2+\frac{2}{3}k}$	M1 A1 A1	1.1 1.1 1.1	Moments Numerator Denominator	
			$= \dfrac{1+\frac{1}{15}(10k+2k^2)}{\frac{1}{3}(6+2k)} = \dfrac{2k^2+10k+15}{10k+30}$ AG	A1	2.1		
				[4]			

Question			Answer	Marks	AOs	Guidance	
11	(iii)		Consider forces vertically: $3T = \left(2 + \frac{2k}{3}\right)g$	**B1**	**3.4**		
			Take moments about D: $2T = \left(2 + \frac{2k}{3}\right)g \times \frac{\left(2k^2 + 10k + 15\right)}{(10k + 30)}$	**M1**	**3.1b**	Take moments: all terms present	
				M1	**1.1**	Solving 3-term quadratic, dependent on previous M1	
			Comment on taking positive root	**B1**	**2.3**		
			$k = \frac{1}{6}\left(-5 + \sqrt{115}\right) = 0.9539...$	**A1**	**1.1**	BC	
			Alternative Methods				
			$T_D : T_A = 1:2 \Rightarrow DG : GA = 2:1$	**M1**		Use of ratio of moments	
			$\Rightarrow DG = \frac{2}{3}$	**A1**			
			$\frac{2k^2 + 10k + 15}{10k + 30} = \frac{2}{3}$ $\Rightarrow k = \frac{1}{6}\left(-5 + \sqrt{115}\right) = 0.9539...$	**M1**		Solving 3-term quadratic, dependent on previous M1	
			Comment on taking positive root	**B1**			
				A1			
				[5]			

Question			Answer	Marks	AOs	Guidance	
12	(i)		$x = 28\cos\alpha t \quad y = 28\sin\alpha t - 4.9t^2$	B1	3.4	Both	
			Use $t = \dfrac{x}{28\cos\alpha}$ to eliminate t from y giving				
			$y = 28\sin\alpha \times \dfrac{x}{28\cos\alpha} - 4.9 \times \left(\dfrac{x}{28\cos\alpha}\right)^2$	M1	1.1	Process must be completed	
			$y = \tan\alpha x - \dfrac{x^2}{160}\sec^2\alpha$	A1	1.1	Simplification. May leave $\dfrac{1}{\cos^2\alpha}$	
			$y = \tan\alpha x - \dfrac{x^2}{160}\left(1 + \tan^2\alpha\right)$	A1	2.1	Using $\sec^2\alpha = 1 + \tan^2\alpha$	
			$\Rightarrow \dfrac{x^2}{160}\tan^2\alpha - x\tan\alpha + y + \dfrac{x^2}{160} = 0$				
			$\Rightarrow \tan^2\alpha - \dfrac{160}{x}\tan\alpha + \dfrac{160y}{x^2} + 1 = 0$ AG	A1	1.1		
				[5]			
12	(ii)	(*A*)	We require the discriminant to be positive so				
			$\left(-\dfrac{160}{x}\right)^2 > 4\left(\dfrac{160y}{x^2} + 1\right)$	M1	2.2a	Finding the discriminant	
			so $\dfrac{160^2}{4} > 160y + x^2$	A1	1.1	Obtain a ky term	
			and $y < 40 - \dfrac{x^2}{160}$ AG	E1	1.1	Completely shown	
				[3]			
12	(ii)	(*B*)	Clearly indicates region below curve with R	A1 [1]	1.1		

Question			Answer	Marks	AOs	Guidance	
12	(iii)		Labels curve with S Labels region above curve with T	**B1** **B1** **[2]**	**1.1** **1.1**		
12	(iv)		Two real distinct roots means two distinct values of tanα (and so α) and so R: two distinct trajectories through points in this region T: no real roots means no trajectories go through those points S: equal roots means there is a single trajectory through a point on the curve	**B1** **B1** **B1** **[3]**	**2.3** **2.2a** **2.2a**		
12	(v)		Graph gives x = 80 when y = 0, so No, because model does not take air resistance into account which would slow it down; or Yes, according to the model	**B1** **[1]**	**3.5a**	Reason must be seen	

Question	AO1	AO2	AO3(PS)	AO3(M)	Totals
1	4	0	0	0	4
2	2	0	0	1	3
3i	1	0	0	1	2
3ii	2	0	0	1	3
4i	1	0	0	1	2
4ii	1	0	0	1	2
4iii	1	0	1	0	2
5i	2	1	0	1	4
5ii	2	0	0	2	4
6i	3	1	0	0	4
6ii	4	0	0	1	5
6iii	0	1	0	0	1
7i	1	2	0	1	4
7iiA	0	1	0	0	1
7iiB	1	1	1	1	4
8i	1	0	0	1	2
8ii	3	1	1	2	7
8iii	3	0	3	1	7
9i	2	1	0	3	6
9ii	4	1	1	1	7
9iii	0	0	0	1	1
10i	1	0	0	3	4
10ii	2	0	0	0	2
10iiiA	1	0	0	1	2
10iiiB	0	1	1	0	2
10iv	1	0	2	0	3
10v	0	0	0	1	1
11i	4	3	0	0	7
11ii	3	1	0	0	4
11iii	2	1	1	1	5
12i	3	1	0	1	5
12iiA	2	1	0	0	3
12iiB	1	0	0	0	1
12iii	2	0	0	0	2
12iv	0	3	0	0	3
12v	0	0	0	1	1
Totals	60	21	11	28	120

Summary of Updates

Date	Version	Change
October 2019	2	Amendments to the front cover rubric instructions to candidates

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A Level Further Mathematics B (MEI)

Y421 Mechanics Major

Printed Answer Booklet   Version 2

Date – Morning/Afternoon

Time allowed: 2 hours 15 minutes

You must have:
- Question Paper Y421 (inserted)
- Formulae Further Mathematics B (MEI)

You may use:
- a scientific or graphical calculator

First name											
Last name											
Centre number						**Candidate number**					

INSTRUCTIONS
- The Question Paper will be found inside the Printed Answer Booklet.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **20** pages. The Question Paper consists of **12** pages.

Section A (26 marks)

1	
2	

3 (i)	
3 (ii)	

4 (i)	
4 (ii)	
4 (iii)	

5 (i)	
5 (ii)	

Section B (94 marks)

6 (i)	
6 (ii)	

6 (iii)	
7 (i)	

7 (ii) (*A*)	
7 (ii) (*B*)	

PLEASE DO NOT WRITE IN THIS SPACE

8 (i)	
8 (ii) (*A*)	
8 (ii) (*B*)	

8 (iii)	

9 (i)	

9 (ii)	
9 (iii)	

10 (i)

10 (ii)

10 (iii) (*A*)	
10 (iii) (*B*)	
10 (iv)	
10 (v)	

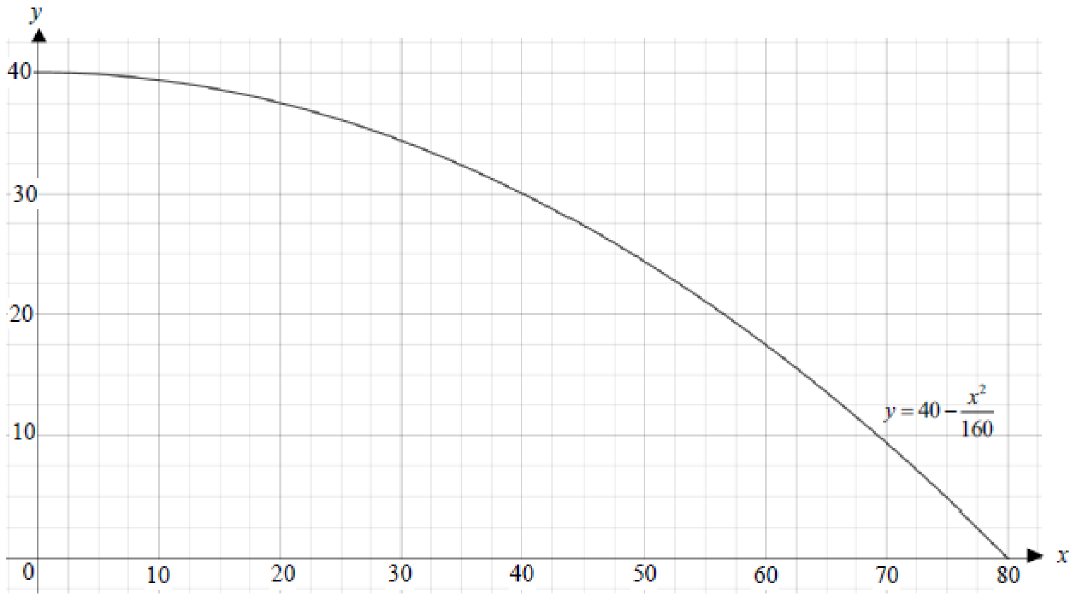
11 (i)	

11 (ii)	

11 (iii)

12 (i)	
12 (ii) (*A*) **&** **12 (iii)**	
	(answer space continued on next page)

12 (ii) (*B*) **&** **12 (iii)**	**(continued)**

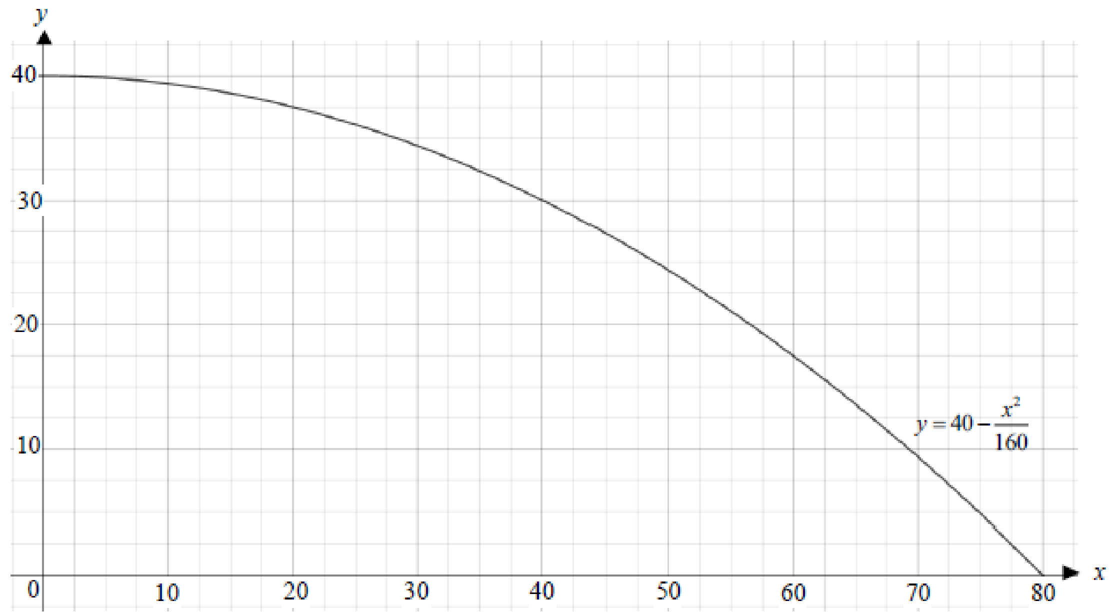


A spare copy of the graph for question 12 (ii) and 12 (iii) can be found on page 20

12 (iv)

12 (v)

Spare copy of graph for question 12 (ii) and 12 (iii)



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