Modified Enlarged 36pt

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Friday 16 June 2023 – Afternoon

AS Level Further Mathematics B (MEI)

Y414/01 Numerical Methods

Time allowed: 1 hour 15 minutes
plus your additional time allowance

YOU MUST HAVE:
the Printed Answer Booklet or any suitable paper provided by the centre. The Printed Answer Booklet may be enlarged by the centre.
the Formulae Booklet for Further Mathematics B (MEI)
a scientific or graphical calculator
Insert for Question 6(a) (with this document)

READ INSTRUCTIONS OVERLEAF

DC (ST) 334296/2

INSTRUCTIONS

Use black ink. You can use an HB pencil, but only for graphs and diagrams.

If you use the Printed Answer Booklet write your answer to each question in the space provided in the PRINTED ANSWER BOOKLET. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.

If you use the Printed Answer Booklet fill in the boxes on the front of the Printed Answer Booklet.

Answer ALL the questions.

Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.

Give your final answers to a degree of accuracy that is appropriate to the context.

Do NOT send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

The total number of marks for this paper is 60.

The marks for each question are shown in brackets [ ].

ADVICE

Read each question carefully before you start your answer.

1 **The table opposite shows some spreadsheet output. The values displayed in cells K21 and K22 are exact, but the values displayed in rows 21 and 22 in columns L, M and N have been found using appropriate cell formulae.**

(a) (i) Write the value displayed in cell N21 in standard mathematical notation. [1]

The formula in cell N22 is

$\boxed{= \text{L22} - \text{M22}}$.

(ii) Explain why the value displayed in cell N22 is not zero. [2]

(b) (i) Determine the absolute error when $\sqrt{e^{0.1}}$ is CHOPPED to 3 decimal places. [2]

(ii) WITHOUT doing any further calculation, explain whether the absolute error found in part (b)(i) will be the same as the absolute error when $e^{\sqrt{0.1}}$ is CHOPPED to 3 decimal places. [1]

	K	L	M	N
20	x	e^x	10^x	$e^x - 10^x$
21	0.000000001	1.000000001	1.000000002	1.30258E-09
22	1E-10	1	1	1.30258E-10

2 **The table gives three values of x and the associated values of y.**

x	3	5	8
y	9.26	19.3	37.96

Use Lagrange's method to construct the interpolating polynomial of degree 2 for the values in the table, giving your answer in the form

$$y = ax^2 + bx + c,$$

where a, b and c are constants to be determined. [4]

3 You are given that

$$P = \frac{43.2}{x+y} \text{ and } Q = \frac{43.2}{x-y}$$

and that $x = 2.17$ and $y = 2.14$.

The values of x and y have been ROUNDED to 2 decimal places.

(a) Calculate the range of possible values of P. [2]

(b) Calculate the range of possible values of Q. [2]

(c) Explain why the answer to part (b) is much larger than the answer to part (a). [1]

4 It is given that the function $g(x)$ is continuous and differentiable and that $g'(1.8) = 0.576$, correct to 3 decimal places.

(a) Use this result to determine the error when $g(1.8)$ is used to approximate $g(1.802)$, giving your answer correct to 3 significant figures. [2]

The largest root, α, of the equation $x^3 - x^2 - 2x + 1 = 0$ is approximately 1.8. The iterative formula $x_{n+1} = g(x_n)$ where $g(x_n) = \sqrt[3]{x_n^2 + 2x_n - 1}$ is to be used to find this root.

(b) Use the iterative formula with $x_0 = 1.8$ to find α correct to 5 decimal places. [3]

The other positive root of the equation, β, is approximately 0.445. It is given that $\mathrm{g}'(0.445) = 4.87$, correct to 2 decimal places.

(c) Explain why the iterative formula may not be used to find β. [1]

(d) Use the relaxed iteration $x_{n+1} = (1 - \lambda)x_n + \lambda \mathrm{g}(x_n)$, with $\lambda = -0.258$ and $x_0 = 0.445$, to determine β correct to 8 decimal places. [3]

5 The table shows some approximations to $\int_0^1 \sqrt[5]{x^2 + 3x + 2}\,\mathrm{d}x$, correct to 8 decimal places, which have been found using the midpoint rule and the trapezium rule. The number of strips, n, used in each application of the two rules is also given.

n	M_n	T_n
1	1.30258554	1.28983372
2	1.29948881	

(a) Determine the value of T_2, giving your answer correct to 8 decimal places. [2]

(b) Use your answer to part (a) and the values in the table to determine TWO Simpson's rule approximations to $\int_0^1 \sqrt[5]{x^2 + 3x + 2}\,\mathrm{d}x$. [3]

(c) Hence state the value of $\int_0^1 \sqrt[5]{x^2 + 3x + 2}\,dx$ as accurately as you can, justifying the precision quoted. [1]

(d) Use the fact that Simpson's rule is generally a fourth order method to obtain the value of $\int_0^1 \sqrt[5]{x^2 + 3x + 2}\,dx$ as accurately as you can, justifying the precision quoted. [3]

6 After taking his pet dog to the vet for a check-up, a dog owner is advised that the dog is too heavy and must therefore go on a special diet. The owner records the dog's mass, W, in kg, at the end of each week. The time, t weeks, from the appointment and the corresponding mass of the dog are shown in the difference table below.

t	W	ΔW	$\Delta^2 W$	$\Delta^3 W$
0	35.90			
1	33.91			
2	32.38			
3	31.25			
4	30.46			

(a) Complete the copy of the difference table in the Printed Answer Booklet or in the insert. [2]

The dog owner believes that the relationship between W and t may be modelled by a cubic polynomial.

(b) Explain how the values in the difference table support this belief. [1]

(c) Use Newton's forward difference interpolation method to construct a cubic polynomial to model the data, giving your answer in the form

$$W = at^3 + bt^2 + ct + d,$$

where a, b, c and d are constants to be determined. [4]

When $t = 6$ it is found that $W = 29.8$.

(d) Determine whether the model is a good fit for this value of t. [1]

(e) Explain why the model will not be suitable in the long term. [1]

7 The equation $5.39x^3 - 2.17x^2 - 1.11x + 0.45 = 0$ has a root, α, close to -0.5.

(a) Use the Newton-Raphson method with $x_0 = -0.5$ to determine the value of α correct to 7 decimal places. [4]

The equation has another root, β, which is close to 0.5. The Newton-Raphson method is implemented using a spreadsheet to find a sequence of approximations to β. These approximations, together with some further analysis, are shown in the output below and opposite.

A	B	C	D
r	x_r	difference	ratio
0	0.5		
1	0.46557	–0.03443	
2	0.44744	–0.01814	0.52682
3	0.4381	–0.00934	0.51472
4	0.43336	–0.00474	0.50775
5	0.43097	–0.00239	0.50398
6	0.42977	–0.0012	0.50202
7	0.42917	–0.0006	0.50102

A	B	C	D
r	x_r	difference	ratio
8	0.42887	−0.0003	0.50051
9	0.42872	−0.00015	0.50026
10	0.42865	−7.5E-05	0.50013
11	0.42861	−3.8E-05	0.50006
12	0.42859	−1.9E-05	0.50003
13	0.42858	−9.4E-06	0.50002
14	0.42858	−4.7E-06	0.50001

(b) Write down a suitable cell formula for

(i) cell C10, [1]

(ii) cell D12. [1]

(c) With reference to the values in column D, explain

what may be inferred about the order of convergence of this sequence of approximations,

whether this order of convergence is unusual. [3]

8 TABLE 8.1 shows some values of x and the associated values of a function $\mathrm{f}(x)$.

TABLE 8.1

x	1.2	2	2.8
$\mathrm{f}(x)$	0.9638087	1.3258177	1.4439304

(a) Use the forward difference method to determine an approximation to $\mathrm{f}'(2)$. [2]

(b) Use the central difference method to determine an approximation to $\mathrm{f}'(2)$. [2]

(c) Explain whether your answer to part (a) or your answer to part (b) is likely to be closest to the true value of $\mathrm{f}'(2)$. [1]

Some additional approximations to $\mathrm{f}'(2)$, found using the central difference method, together with some further analysis, are shown in TABLE 8.2 opposite. The additional values of $\mathrm{f}(x)$ have NOT been given.

TABLE 8.2

h	approximation	difference	ratio
0.4	0.2506259		
0.2	0.2390539	–0.011572	
0.1	0.2362293	–0.0028246	0.2440877
0.05	0.2355276	–0.0007017	0.2484204

(d) Use extrapolation to determine the value of $f'(2)$ as accurately as you can, justifying the precision quoted. [4]

END OF QUESTION PAPER

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