Modified Enlarged 36pt

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Monday 15 May 2023 – Afternoon

AS Level Further Mathematics B (MEI)

Y410/01 Core Pure

Time allowed: 1 hour 15 minutes plus your additional time allowance

YOU MUST HAVE:
the Printed Answer Booklet or any suitable paper provided by the centre. The Printed Answer Booklet may be enlarged by the centre
the Formulae Booklet for Further Mathematics B (MEI)
a scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

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INSTRUCTIONS

Use black ink. You can use an HB pencil but only for graphs and diagrams.

If you use the Printed Answer Booklet write your answer to each question in the space provided in the PRINTED ANSWER BOOKLET. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.

If you use the Printed Answer Booklet fill in the boxes on the front of the Printed Answer Booklet.

Answer ALL the questions.

Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.

Give your final answers to a degree of accuracy that is appropriate to the context.

Do NOT send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

The total mark for this paper is 60.

The marks for each question are shown in brackets [ ].

ADVICE

Read each question carefully before you start your answer.

1 The transformation R of the plane is reflection in the line $x = 0$.

(a) Write down the matrix M associated with R. [1]

(b) Find M^2. [1]

(c) Interpret the result of part (b) in terms of the transformation R. [1]

2 In this question you must show detailed reasoning.

The equation $x^2 - kx + 2k = 0$, where k is a non-zero constant, has roots α and β.

Find $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ in terms of k, simplifying your answer. [4]

3 In this question you must show detailed reasoning.

The function $\mathrm{f}(z)$ is given by

$\mathrm{f}(z) = 2z^3 - 7z^2 + 16z - 15.$

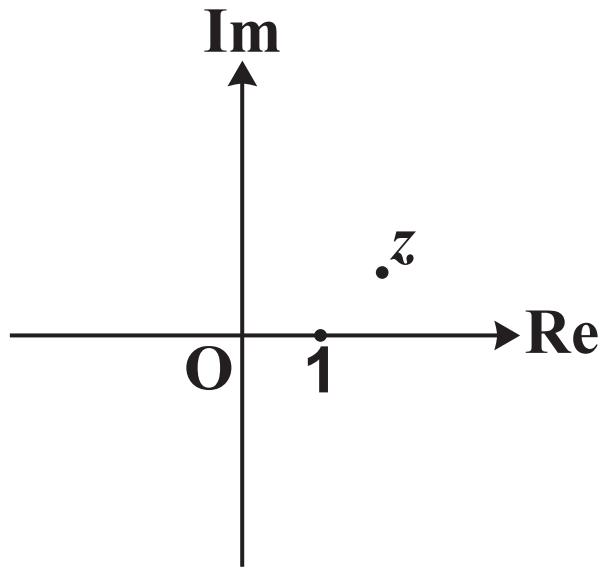
By first evaluating $\mathrm{f}\left(\frac{3}{2}\right)$, find the roots of

$\mathrm{f}(z) = 0.$ [6]

4 You are given that $\sum_{r=1}^{n} (ar + b) = n^2$ for all n, where a and b are constants.

By finding $\sum_{r=1}^{n} (ar + b)$ in terms of a, b and n, determine the values of a and b. [6]

5 The Argand diagram below shows the points representing 1 and z, where $|z| = 2$.



Mark the points representing the following complex numbers on the copy of the diagram in the Printed Answer Booklet or on the insert, labelling them clearly. [4]

z^*

$\frac{1}{z}$

$1 + z$

$\mathrm{i}z$

6 The matrix $\mathbf{M}$ is $\begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}$.

(a) Calculate $\mathbf{M}^2$, $\mathbf{M}^3$ and $\mathbf{M}^4$. [2]

(b) Hence make a conjecture about the matrix $\mathbf{M}^n$. [1]

(c) Prove your conjecture. [5]

7 **In this question you must show detailed reasoning.**

The complex number $\sqrt{3}+\mathrm{i}$ is denoted by z.

(a) By expanding $(\sqrt{3}+\mathrm{i})^5$, express z^5 in the form $a+b\mathrm{i}$ where a and b are real and exact. [3]

(b) (i) Express z in modulus-argument form. [3]

(ii) Hence find z^5 in modulus-argument form. [2]

(iii) Use this result to verify your answers to part (a). [2]

8 The equations of three planes are

$$\begin{aligned} 2x + y + 3z &= 3, \\ 3x - y - 2z &= 2, \\ -4x + 3y + 7z &= k, \end{aligned}$$

where k is a constant.

(a) By considering a suitable determinant, show that the planes do NOT meet at a single point. [2]

(b) Given that the planes form a sheaf, determine the value of k. [4]

9 A transformation T of the plane is represented by the matrix

$\mathbf{M} = \begin{pmatrix} k+1 & -1 \\ 1 & k \end{pmatrix}$, where k is a constant.

Show that, for all values of k, T has no invariant lines through the origin. [6]

10 The plane P has normal vector $2\mathbf{i} + a\mathbf{j} - \mathbf{k}$, where a is a positive constant, and the point (3, −1, 1) lies in P. The plane $x - z = 3$ makes an angle of 45° with P.

Find the cartesian equation of P. [7]

END OF QUESTION PAPER

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