

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Friday 19 May 2023 – Afternoon

AS Level Further Mathematics B (MEI)

Y411/01 Mechanics a

**Time allowed: 1 hour 15 minutes
plus your additional time allowance**

YOU MUST HAVE:

**the Printed Answer Booklet or any suitable paper
supplied by the centre. The Printed Answer Booklet may
be enlarged by the centre.**

**the Formulae Booklet for Further Mathematics B (MEI)
a scientific or graphical calculator**

READ INSTRUCTIONS OVERLEAF



INSTRUCTIONS

Use black ink. You can use an HB pencil, but only for graphs and diagrams.

If you use the Printed Answer Booklet write your answer to each question in the space provided in the PRINTED ANSWER BOOKLET. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.

If you use the Printed Answer Booklet fill in the boxes on the front of the Printed Answer Booklet.

Answer ALL the questions.

Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.

Give your final answers to a degree of accuracy that is appropriate to the context.

The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.

Do NOT send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

The total mark for this paper is 60.

The marks for each question are shown in brackets [].

ADVICE

Read each question carefully before you start your answer.

- 1 Throughout ALL parts of this question, the resistance to the motion of a car has magnitude kv^2 N, where $v \text{ m s}^{-1}$ is the speed of the car and k is a constant.

At first, the car travels along a straight horizontal road with constant speed 20 m s^{-1} . The power developed by the car at this speed is 5000 W .

- (a) Show that $k = \frac{5}{8}$. [3]
- (b) Find the power the car must develop in order to maintain a constant speed of 28 m s^{-1} when travelling along the same horizontal road. [1]

The car climbs a hill which is inclined at an angle of 2° to the horizontal. The power developed by the car is 13000 W , and the car has a constant speed of 20 m s^{-1} .

- (c) Determine the mass of the car. [3]

- 2 A ball P of mass m kg is held at a height of 12.8 m above a horizontal floor. P is released from rest and rebounds from the floor. After the first bounce, P reaches a maximum height of 5 m above the floor.**

Two models, A and B, are suggested for the motion of P.

Model A assumes that air resistance may be neglected.

- (a) Determine, according to model A, the coefficient of restitution between P and the floor. [3]**

Model B assumes that the collision between P and the floor is perfectly elastic, but that work is done against air resistance at a constant rate of E joules per metre.

- (b) Show that, according to model B, $E = \frac{39}{89}mg$. [3]**
- (c) Show that both models predict that P will attain the same maximum height after the second bounce. [4]**

- 3 The time period T of a satellite in circular orbit around a planet satisfies the equation

$$GMT^2 = 4\pi^2 R^3,$$

where

G is the universal gravitational constant,

M is the mass of the planet,

R is the radius of the orbital circle.

- (a) Find the dimensions of G . [2]

A student suggests the following formula to model the approach speed between two orbiting bodies.

$$v = kG^\alpha c^\beta r^\gamma m_1 m_2 (m_1 + m_2),$$

where

v is the approach speed of the two bodies,

k is a dimensionless constant,

c is the speed of light,

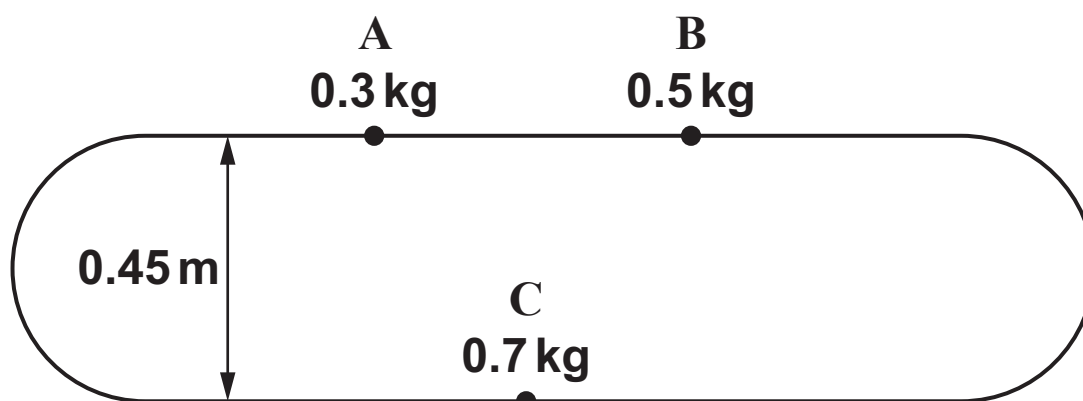
r is the distance between the two bodies,

m_1 and m_2 are the masses of the bodies.

- (b) Use dimensional analysis to determine the values of α , β and γ . [5]

- (c) Calculate, according to the student's model, how many times greater the approach speed is between a pair of stars which are 6.13 light-years apart and the same pair of stars if they were 8.64 light-years apart. (A light-year is a unit of distance.) [2]

- 4 The diagram shows three beads, A, B and C, of masses 0.3 kg, 0.5 kg and 0.7 kg respectively, threaded onto a smooth wire circuit consisting of two straight and two semi-circular sections. The circuit occupies a VERTICAL plane, with the two straight sections horizontal and the upper section 0.45 m directly above the lower section.



Initially, the beads are at rest. A and B are each given an impulse so that they move towards each other, A with a speed of 8 m s^{-1} and B with a speed of 1.6 m s^{-1} . In the subsequent collision between A and B, A is brought to rest.

- (a) Show that the coefficient of restitution between A and B is $\frac{1}{3}$. [3]

Bead B next collides with C.

- (b) Show that the speed of B before this collision is 4.37 m s^{-1} , correct to 3 significant figures. [2]

In this collision between B and C, B is brought to rest.

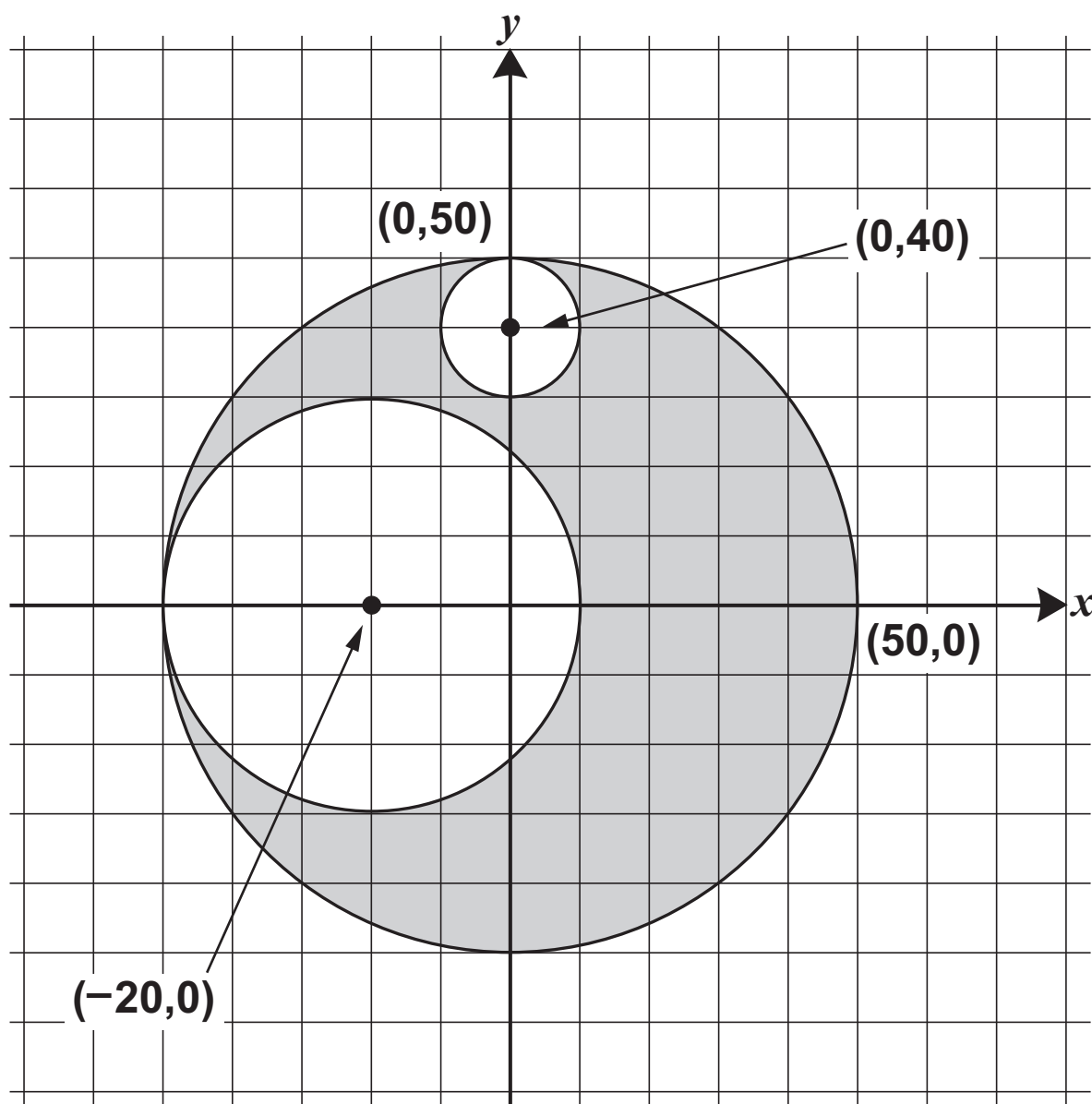
- (c) Determine whether C next collides with A or with B. [3]
- (d) Explain why, if B has a greater mass than C, B could NOT be brought to rest in their collision. [2]

- 5 FIG. 5.1 shows the uniform cross-section of a solid S which is formed from a cylinder by boring two cylindrical tunnels the entire way through the cylinder. The radius of S is 50 cm, and the two tunnels have radii 10 cm and 30 cm.

The material making up S has uniform density.

Coordinates refer to the axes shown in FIG. 5.1 and the units are centimetres.

FIG. 5.1

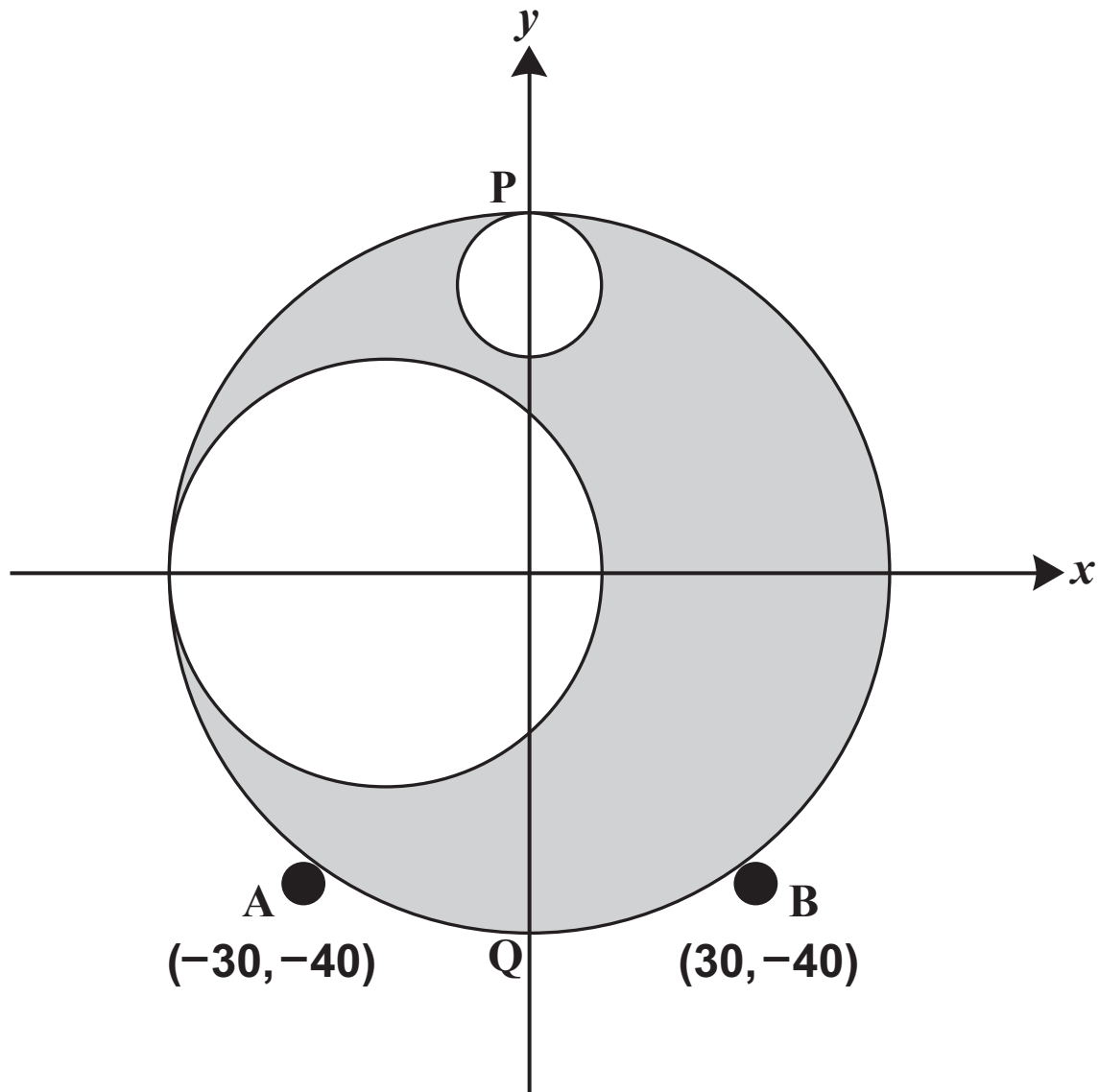


The centre of mass of S is (\bar{x}, \bar{y}) .

(a) Show that $\bar{x} = 12$ and find the value of \bar{y} . [4]

Solid S is placed onto two rails, A and B, whose point of contacts with S are at $(-30, -40)$ and $(30, -40)$ as shown in FIG. 5.2. Two points, P $(0, 50)$ and Q $(0, -50)$, are marked on FIG. 5.2.

FIG. 5.2

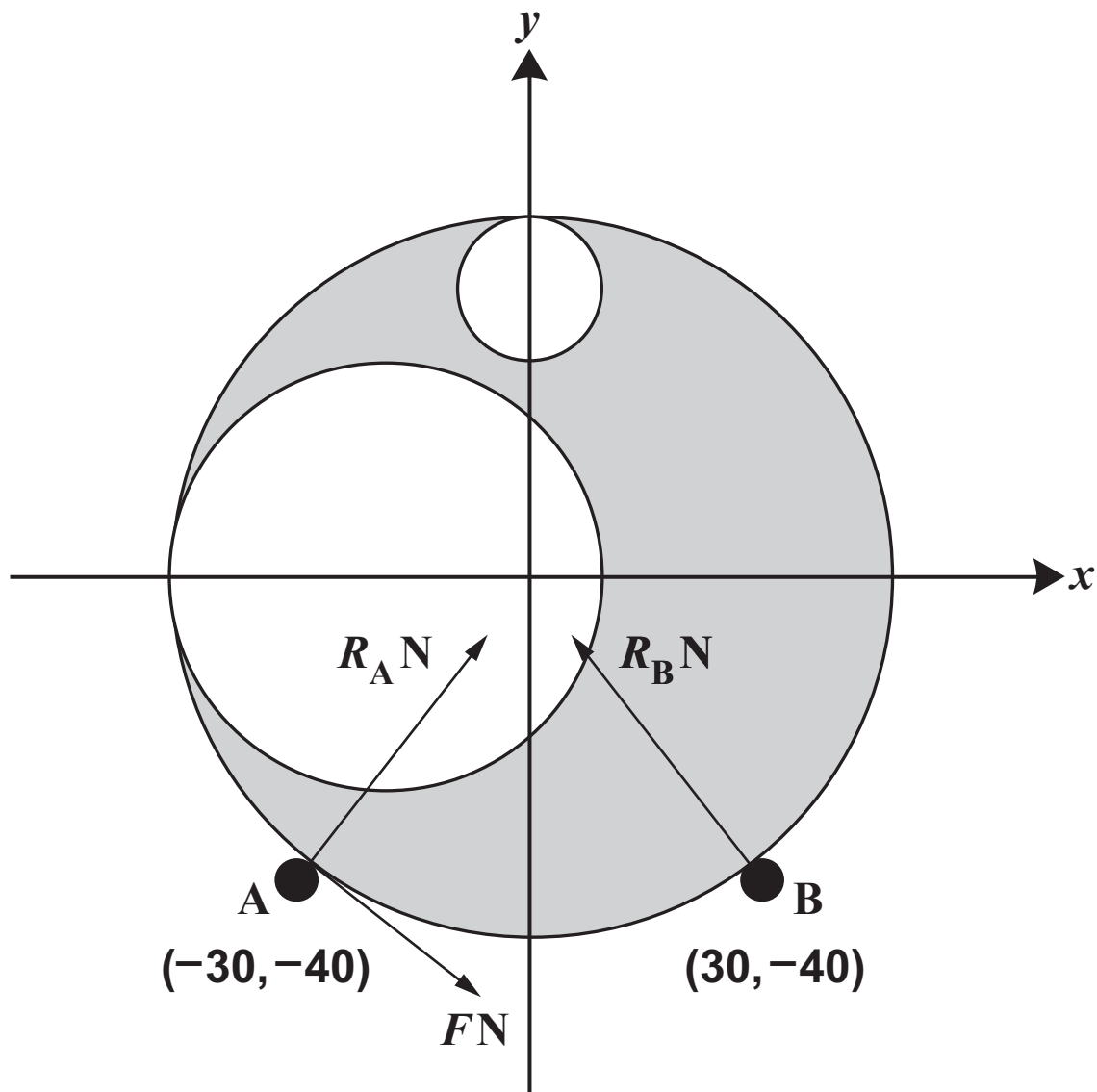


At first, you should assume that the contact between S and the two rails is SMOOTH.

(b) Determine the angle PQ makes with the vertical, after S settles into equilibrium. [2]

For the remainder of the question, you should assume that the contact between S and A is **ROUGH**, that the contact between S and B is **SMOOTH**, and that S does not move when placed on the rails. FIG. 5.3 shows **ONLY** the forces exerted on S by the rails. The normal contact forces exerted by A and B on S have magnitude R_A N and R_B N respectively. The frictional force exerted by A on S has magnitude F N.

FIG. 5.3

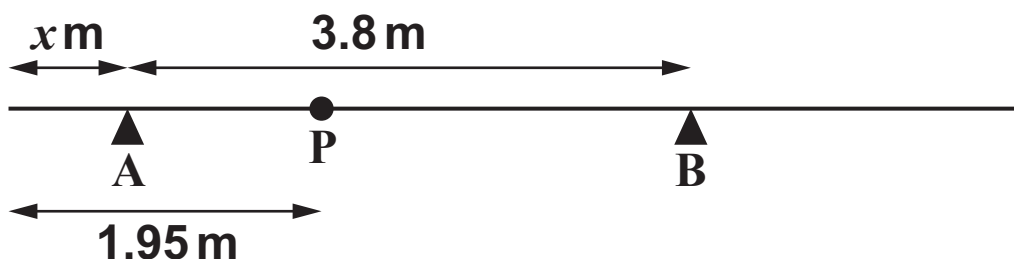


The weight of S is W N.

- (c) By taking moments about the origin, express F in the form λW , where λ is a constant to be determined. [2]
- (d) Given that S is in limiting equilibrium, find the coefficient of friction between A and S. [5]

- 6 A uniform beam of length 6m and mass 10kg rests horizontally on two supports A and B, which are 3.8m apart. A particle P of mass 4kg is attached 1.95m from one end of the beam (see FIG. 6.1).

FIG. 6.1

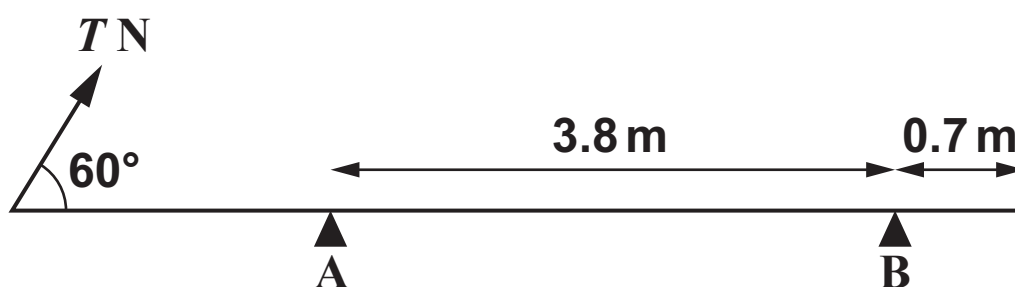


When A is x m from the end of the beam, the supports exert forces of equal magnitude on the beam.

(a) Determine the value of x . [4]

P is now removed. The same beam is placed on the supports so that B is 0.7 m from the end of the beam. The supports remain 3.8 m apart (see FIG. 6.2).

FIG. 6.2



The contact between A and the beam is smooth. The contact between B and the beam is rough, with coefficient of friction 0.4.

A small force of magnitude T N is applied to one end of the beam. The force acts in the same vertical plane as the beam and the angle the force makes with the beam is 60° .

As T is increased, forces T_L and T_S are defined in the following way.

T_L is the value of T at which the beam would start lifting, assuming that it is not already sliding.

T_S is the value of T at which the beam would start sliding, assuming that it has not already lifted.

(b) Show that $T_L = 49.1$, correct to 3 significant figures. [2]

(c) Determine whether the beam will first slide or lift. [5]

END OF QUESTION PAPER

BLANK PAGE

BLANK PAGE



Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact The OCR Copyright Team, The Triangle Building, Shaftesbury Road, Cambridge CB2 8EA.

OCR is part of Cambridge University Press & Assessment, which is itself a department of the University of Cambridge.