

Monday 15 May 2023 – Afternoon

AS Level Further Mathematics B (MEI)

Y410/01 Core Pure

Time allowed: 1 hour 15 minutes



You must have:

- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator



INSTRUCTIONS

- Use black ink. You can use an HB pencil but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is **60**.
- The marks for each question are shown in brackets [].
- This document has **4** pages.

ADVICE

- Read each question carefully before you start your answer.

1 The transformation R of the plane is reflection in the line $x = 0$.

(a) Write down the matrix \mathbf{M} associated with R . [1]

(b) Find \mathbf{M}^2 . [1]

(c) Interpret the result of part (b) in terms of the transformation R . [1]

2 **In this question you must show detailed reasoning.**

The equation $x^2 - kx + 2k = 0$, where k is a non-zero constant, has roots α and β .

Find $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ in terms of k , simplifying your answer. [4]

3 **In this question you must show detailed reasoning.**

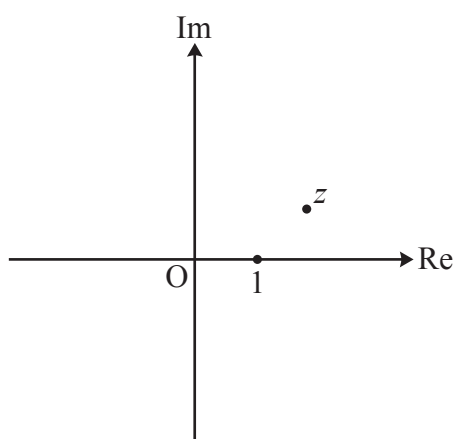
The function $f(z)$ is given by $f(z) = 2z^3 - 7z^2 + 16z - 15$.

By first evaluating $f\left(\frac{3}{2}\right)$, find the roots of $f(z) = 0$. [6]

4 You are given that $\sum_{r=1}^n (ar + b) = n^2$ for all n , where a and b are constants.

By finding $\sum_{r=1}^n (ar + b)$ in terms of a , b and n , determine the values of a and b . [6]

- 5 The Argand diagram below shows the points representing 1 and z , where $|z| = 2$.



Mark the points representing the following complex numbers on the copy of the diagram in the Printed Answer Booklet, labelling them clearly.

- z^*
- $\frac{1}{z}$
- $1 + z$
- iz [4]

- 6 The matrix \mathbf{M} is $\begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}$.

- (a) Calculate \mathbf{M}^2 , \mathbf{M}^3 and \mathbf{M}^4 . [2]
- (b) Hence make a conjecture about the matrix \mathbf{M}^n . [1]
- (c) Prove your conjecture. [5]

- 7 **In this question you must show detailed reasoning.**

The complex number $\sqrt{3} + i$ is denoted by z .

- (a) By expanding $(\sqrt{3} + i)^5$, express z^5 in the form $a + bi$ where a and b are real and exact. [3]
- (b) (i) Express z in modulus-argument form. [3]
- (ii) Hence find z^5 in modulus-argument form. [2]
- (iii) Use this result to verify your answers to part (a). [2]

8 The equations of three planes are

$$2x + y + 3z = 3,$$

$$3x - y - 2z = 2,$$

$$-4x + 3y + 7z = k,$$

where k is a constant.

(a) By considering a suitable determinant, show that the planes do **not** meet at a single point. [2]

(b) Given that the planes form a sheaf, determine the value of k . [4]

9 A transformation T of the plane is represented by the matrix $\mathbf{M} = \begin{pmatrix} k+1 & -1 \\ 1 & k \end{pmatrix}$, where k is a constant.

Show that, for all values of k , T has no invariant lines through the origin. [6]

10 The plane P has normal vector $2\mathbf{i} + a\mathbf{j} - \mathbf{k}$, where a is a positive constant, and the point $(3, -1, 1)$ lies in P . The plane $x - z = 3$ makes an angle of 45° with P .

Find the cartesian equation of P . [7]

END OF QUESTION PAPER

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