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Examiners' report

FURTHER MATHEMATICS B (MEI)

H635

For first teaching in 2017

Y413/01 Summer 2023 series

Version 1

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Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate answers is also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from OCR.

Paper Y413/01 series overview

This was the fourth examination series for H635 AS Further Maths B (MEI) optional paper Y413 Modelling with Algorithms. Most candidates were well prepared for the examination, making good attempts on all questions. Candidates seemed to have enough time to answer the questions and nearly all seemed to make good use of the Printed Answer Booklet (that is very few answers appeared in the spare copies of graphs and tables or in the additional answer space).

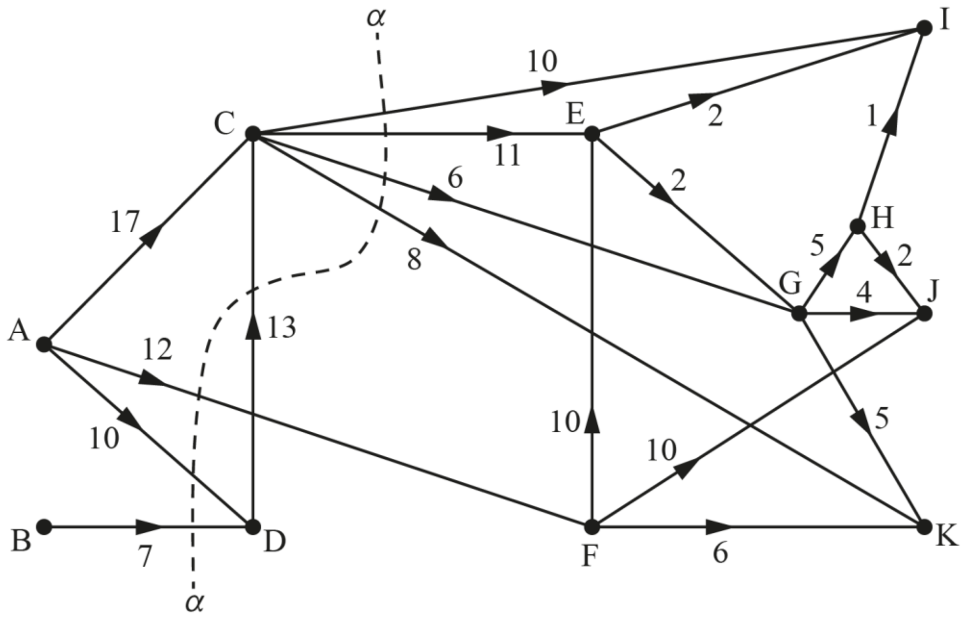
Based on the responses seen by examiners this series the following general points should be considered by centres in preparation of candidates for future sittings of this paper.

- Diagrams should be completed in pencil so that candidates can rub out and replace their answer rather than trying to correct in pen on what is likely to be a diagram, which does not have enough space for multiple attempts. This was especially true in Questions 2 (a), 2 (b) (Dijkstra's algorithm) and 4 (d) (i) (drawing a resource histogram) in which it was difficult at times for examiners to know what work candidates were genuinely trying to correct or what was in fact their final attempt.
- This is primarily a methods examination and so it is vital that candidates make their method and application of any corresponding algorithm clear; spotting and writing down the solution (for example in the flow chart question) will rarely gain any credit.
- Section 2b of the specification contains the meanings of several instructions that will be used in examination questions. Candidates need to be familiar with the requirements of these command words. For example, 'determine' in this mathematical context requires that justification should be given for any results found and not just the writing down or stating of the answer.
- Candidates are reminded to use the number of marks available as the main guide to how detailed their answer should be and not the space given in the Printed Answer Booklet. The Printed Answer Booklet is designed to provide more than enough space for the candidate's response, often with additional space so that candidates can correct any errors.

Candidates who did well on this paper generally:	Candidates who did less well on this paper generally:
• applied the corresponding algorithm correctly and showed detailed working/reasoning (and therefore did not rely on ad-hoc methods) • read questions carefully and provided the answers that were requested • made efficient use of their calculator.	• did not give sufficient detail in questions which required an explanation or in response to the command to 'determine' their answer • used imprecise mathematical notation or incorrect language.

Question 1 (a)

1 Fig. 1.1



The diagram in **Fig. 1.1** represents a system of pipes through which a fluid can flow from two sources to three sinks. It also shows a cut α.

The weights on the arcs show the capacities of the pipes in gallons per minute.

(a) Add a supersource S and a supersink T to the network in the Printed Answer Booklet, giving appropriate weightings and directions to the connecting arcs. **[2]**

Many candidates correctly added a supersource S and a supersink T, with appropriate weightings and directions to the connecting arcs. Some did not include relevant weights (of at least 39 on SA, 7 on SB, 13 on IT, 16 on JT and 19 on KT) or they did not include all arrows directed either away from the supersource or towards the supersink. Though the question said that the flow through the network went from two sources to three sinks, several candidates only had two of the three sink nodes attached to the supersink.

Question 1 (b)

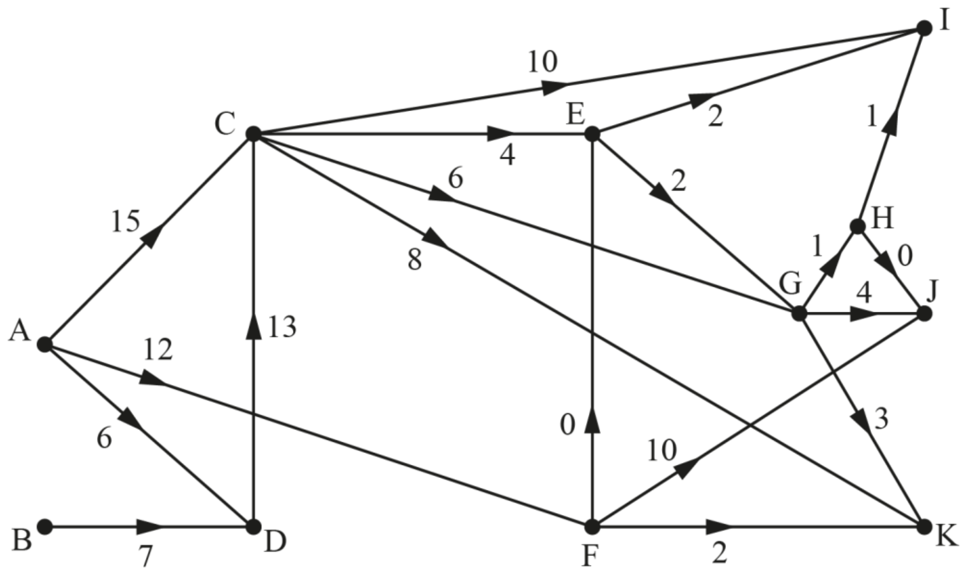
(b) Calculate the capacity of the cut α. **[1]**

Almost all candidates correctly stated the capacity of the cut as 64.

Question 1 (c)

The diagram in **Fig. 1.2** shows a feasible flow through the network.

Fig. 1.2



(c) Write down the amount of fluid, in gallons per minute, that **Fig. 1.2** shows flowing from the sources to the sinks. **[1]**

As in Question 1 (b), almost all candidates gave the correct flow of 40.

Question 1 (d)

(d) Prove that the flow shown in **Fig. 1.2** is the maximum possible flow through the network. **[2]**

It was extremely rare for candidates to either state a correct cut with capacity 40 or for them to give enough detail in proving that this flow was indeed maximal.

Assessment for learning

Proving a flow is maximal:

Step 1: state a cut, based on the capacity of saturated arcs directed from source to sink and arcs with zero flow directed from sink to source, as either a set of nodes or as a list of arcs which the cut passes through, and conclude that therefore the minimum cut is less than or equal to this value.

Step 2: state the value of the current flow through the network and conclude that therefore the maximum flow is greater than or equal to this value.

Step 3: if these two values are the same then by the maximum flow-minimum cut theorem a maximum flow for the network has been found.

Misconception

When calculating the capacity of a cut candidates must use the capacity of the arcs and not the current flow which is passing through those arcs (otherwise all cuts for the network would have the same capacity).

Question 2 (a)

2 The journey times (in minutes) between eight towns, A, B, …, H, for which there is a direct route are given in the table in **Fig. 2**.

Fig. 2

	A	B	C	D	E	F	G	H
A	–	26	11	43	–	–	–	–
B	26	–	12	–	34	17	6	–
C	11	12	–	–	–	–	–	–
D	43	–	–	–	14	7	15	–
E	–	34	–	14	–	–	–	9
F	–	17	–	7	–	–	5	38
G	–	6	–	15	–	5	–	45
H	–	–	–	–	9	38	45	–

(a) Using the vertices given in the Printed Answer Booklet, draw a network to represent the information shown in **Fig. 2**. **[2]**

This question was answered extremely well. Nearly all candidates added all the required arcs to the vertices and included the correct corresponding weights too.

Question 2 (b) (i)

(b) (i) Apply Dijkstra's algorithm to the network drawn in part **(a)**, to find the length of the quickest route from A to H. **[5]**

Many candidates scored full marks in this question. It is vital when applying Dijkstra's algorithm that candidates show all the working values at each node and that the order of labelling is correct. In this case, the correct working values of 26 and 23 at B had to be seen to indicate that the algorithm was initially being applied correctly. A small number of candidates labelled two different nodes with the same label. Several candidates correctly applied the algorithm but then did not state the length of the quickest route from A to H.

Assessment for learning

The Printed Answer Booklet provided the standard key for the completion of the boxes at each node together with a specific instruction to candidates that they should not cross out their temporary labels. Several candidates still did so. It is vital that all the values at each node can be seen to check that the algorithm has been applied correctly. Candidates should be encouraged to not cross out temporary labels in classroom practice.

Question 2 (b) (ii)

(ii) Write down the quickest route from A to H. **[1]**

Most candidates who completed Question 2 (b)(i) went on to state the quickest route from A to H correctly too.

Question 2 (c)

(c) Using your answers to part **(b)**, determine the total length of the arcs in a minimum spanning tree for the network given in **Fig. 2**.

You should not apply an algorithm to find this minimum spanning tree. **[2]**

Many candidates simply stated the total length of the arcs in the required minimum spanning tree and therefore scored only 1 mark. The command word 'determine' indicated that more information would be needed to justify the response. For both marks to be credited candidates had to explain that as the quickest route from A to H passed through all the nodes in the network this implied that the length of the shortest path from A to H was therefore equivalent to the total length of the MST (for the network).

Question 3 (a)

3 The list below shows the sizes of 10 items.

17 15 18 9 23 20 14 12 25 11

(a) Use the quick sort algorithm to sort the list of numbers above into **descending** order. You should use the first value as the pivot for each sublist. **[3]**

This question was answered extremely well but the most common error was not including a fifth pass.

Misconception

Candidates are reminded that, for many sorting algorithms, even when a list is in the correct order this does not necessarily mean that the algorithm is complete. In this application of quick sort, after the 4th pass the list read:

25 23 20 18 17 15 14 12 11 9

But a 5th pass was still required as this list still contained a sublist of two values that had yet to be sorted; the 12 and the 11.

Question 3 (b) (i)

The first fit decreasing algorithm is used to pack items with the sizes listed above into bins that have a capacity of m, where m is a positive integer.

The following packing is achieved.

Bin 1:	25	20	
Bin 2:	23	18	
Bin 3:	17	15	14
Bin 4:	12	11	9

(b) (i) By considering the placement of the three largest items, determine the possible values of m. **[1]**

Most candidates correctly implied that the placement of the three largest items meant that the value of *m* was either 45, 46 or 47.

Question 3 (b) (ii)

(ii) By considering the total in each bin, further refine the possible values of m. **[1]**

Those candidates who correctly answered Question 3 (b)(i) usually went on to realise that *m* was either 46 or 47.

Question 3 (c)

(c) Using the original list, show the result of applying the first fit algorithm to pack items with the sizes listed above into bins that have a capacity of m, where m takes any of the possible values found in part **(b)(ii)**. **[2]**

This question was answered successfully with nearly all candidates scoring at least 1 mark for correctly assigning the first five values to the first two bins. Occasionally errors occurred by introducing a fifth bin unnecessarily.

Question 3 (d) (i)

A list of n numbers is to be sorted into descending order using the quick sort algorithm. The number of comparisons made is used as a measure of complexity of the quick sort algorithm.

(d) (i) By considering a worst case, determine an expression in terms of n, for the total number of comparisons required to sort the list using the quick sort algorithm in the worst case. **[2]**

This was probably the most demanding question of the paper for candidates and very few were able to access both marks. Even though the question directed candidates to 'consider a worst case', many did not give an example of such a case and for those that did, very few could describe the comparisons that would happen in the first couple of passes. The question specifically said a 'list of *n* numbers' but many candidates attempted to consider the worst case on the original list of numbers given at the start of the question. Many candidates knew what the expression in terms of *n* was in this worse case and so scored 1 mark as a Special Case, but very few could determine, with sound reasoning, this result.

Question 3 (d) (ii)

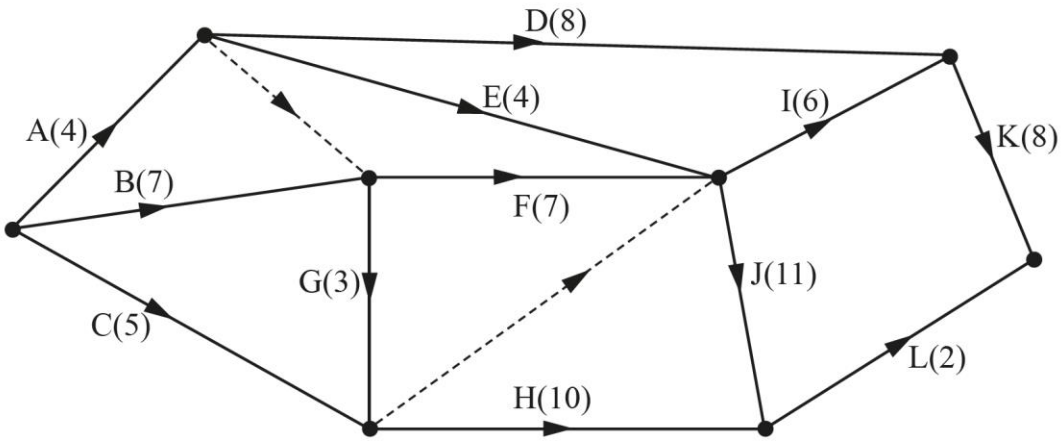
(ii) Hence state, in the worst case, the complexity of the quick sort algorithm. **[1]**

Almost all candidates knew that in the worst case the complexity of the quick sort algorithm was quadratic.

Question 4 (a)

4 **Fig. 4.1** shows an activity network for a project. The arc weights show activity durations in days.

Fig. 4.1



(a) Complete the table in the Printed Answer Booklet to show the immediate predecessors for each activity. **[2]**

Most candidates had some success with this question with nearly all candidates scoring at least 1 mark for correctly completing five rows of the table. The most common errors were due to the dummy activities and for not including C and/or G in the immediate predecessors for either I or J.

Question 4 (b)

(b) Carry out a forward pass and a backward pass through the entire network to find

- The minimum completion time for the project
- The critical activities **[5]**

Nearly all candidates managed to score the method marks for attempting both a forward and backward pass through the network. It was uncommon to give the corresponding accuracy mark as many candidates did not consider the dummy activities when completing either one or both passes (most notably the backward pass). Most candidates correctly stated the minimum completion time and critical activities for the project though.

Misconception

The start event node should be labelled with a double zero and neither of these two boxes should be left blank.

Question 4 (c)

(c) Determine the total float for activity H. **[2]**

In this 'determine' question almost all candidates showed the correct calculation of the total float for activity H and didn't just state the answer of 6.

Question 4 (d) (i)

The table in **Fig. 4.2** shows the number of workers required for each activity.

It is given that when an activity is started it must be completed without interruption.

Fig. 4.2

Activity	Number of workers
A	1
B	1
C	2
D	1
E	2
F	1
G	1
H	1
I	2
J	1
K	1
L	3

(d) **(i)** Draw a resource histogram to show the number of workers required each day when each activity begins at its earliest possible start time. **[2]**

While several candidates did not attempt this question, a good number did draw the histogram correctly. The most common errors were not realising that 6 workers were needed in the 4 – 5 time interval. A few candidates had the project finishing either earlier or later than the completion time found in Question 4 (b).

Question 4 (d) (ii)

(ii) Hence state the minimum number of workers required to complete the project (in the minimum time), according to the histogram drawn in part **(d)(i)**. **[1]**

Most candidates who had attempted Question 4 (d)(i) also correctly stated the minimum number of workers as 6.

Question 5 (a)

5 Four students, Jamal (J), Kai (K), Layla (L), and Mia (M) are planning to participate in a 200 m swimming relay race.

Each of the four students must swim exactly one 50 m length. Furthermore, the students must each swim a different type of swimming stroke. The four types of swimming stroke are backstroke (A), front crawl (C), breaststroke (R) and butterfly (U).

The table in **Fig. 5.1** shows the average time, in seconds, each of the four students took to complete each of the four types of swimming stroke during practice.

Fig. 5.1

	A	C	R	U
J	39	32	37	41
K	39	34	38	42
L	40	36	41	40
M	42	33	42	43

The four students need to know, based on the times in **Fig. 5.1**, who should be allocated to swim each stroke to give them the best chance of winning the relay race.

The constraints for an LP formulation for this problem are as follows.

$$JA + JC + JR + JU = 1$$
$$KA + KC + KR + KU = 1$$
$$LA + LC + LR + LU = 1$$
$$MA + MC + MR + MU = 1$$
$$JA + KA + LA + MA = 1$$
$$JC + KC + LC + MC = 1$$
$$JR + KR + LR + MR = 1$$
$$JU + KU + LU + MU = 1$$

(a) Explain the purpose of the line JA + JC + JR + JU = 1 in the LP formulation. **[1]**

Almost all candidates correctly stated that the purpose of this line in the LP formulation was to ensure that Jamal could only be allocated one of the four swimming strokes.

Question 5 (b)

(b) Write down the objective function of the LP formulation. **[2]**

This question was answered extremely well although some candidates did not realise the subtle difference in an LP formulation between the correct answer of 'minimise 39JA + 32JC +…' and an incorrect answer of 'minimise 39AJ + 32CJ +…'.

Misconception

In any LP formulation it is vital that the order in which the letters are used is correct and consistent with any information provided in the question. Many candidates used AJ even though in this context it had to be stated as JA; as dictated by the constraints set up in the question.

Question 5 (c)

The complete LP was run in an LP solver and the output is shown in the table in **Fig. 5.2**.

Fig. 5.2

Variable	Value
JA	0.000000
JC	0.000000
JR	1.000000
JU	0.000000
KA	1.000000
KC	0.000000
KR	0.000000
KU	0.000000
LA	0.000000
LC	0.000000
LR	0.000000
LU	1.000000
MA	0.000000
MC	1.000000
MR	0.000000
MU	0.000000

(c) State the predicted total time for the team to complete the relay race, according to the output given in **Fig. 5.2**. **[1]**

Almost all candidates correctly stated the total time as 149.

Question 5 (d)

A fifth student, Nina (N), is now available to swim one of the four 50 m lengths. Her times, on average, in seconds, to complete each of the four types of swimming stroke are shown in the table in **Fig. 5.3**.

Fig. 5.3

	A	C	R	U
N	38	43	37	42

(d) Explain how to adapt the LP formulation so that the LP solver can determine which of the five students should be allocated to swim each of the four 50 m lengths. **[3]**

The responses to this question were mixed with several candidates leaving it blank. Of those that did attempt this question, many did not give detailed enough descriptions of exactly how the LP formulation should be adapted with the addition of the fifth student Nina. For the 3 marks to be given, evidence of the following was required:

- additional terms needed to be added to the objective function (namely, adding 38NA + 43NC + 37NR + 42NU),
- a correct explanation of how the final four constraints needed to be modified so that only one person could be allocated to one of the strokes (so explicitly seeing mention of NA, NC, NR and NU),
- the need to change the other four constraints from = 1 to ≤ 1 to allow an allocation of 5 people to only 4 activities, and the addition of a constraint regarding Nina doing one of the four strokes (namely, NA + NC + NR + NU ≤ 1).

Exemplar 1

You should add a constraint such that Nina can only do one or less event i.e $NA + NC + NR + NU \leq 1$.

You need to add a ≤ 1 replacing all the equals as one student will be left out.

The objective function should include Nina's events.

This response scored 0 marks due to the lack of specific detail on how the LP formulation should be adapted. While it was clear that the candidate understood the steps required to re-formulate the problem with the addition of the 5th student, they did not give the required mathematical detail that such a question demands.

Question 5 (e)

When the adapted LP formulation is run through the LP solver it determines that Jamal should be allocated the front crawl (C) and Kai should be allocated the breaststroke (R).

(e) Determine the difference in the predicted times for the team to complete the relay race when Nina is in the team compared with when she is not in the team. **[2]**

Many candidates gave the correct time difference of 1 second but did not say where this difference came from. This only scored 1 mark. As this was a 'determine' question, explicit reference to L being assigned to U and N being assigned to A was needed for both marks to be given.

Question 6 (a)

6 Fig. 6.1

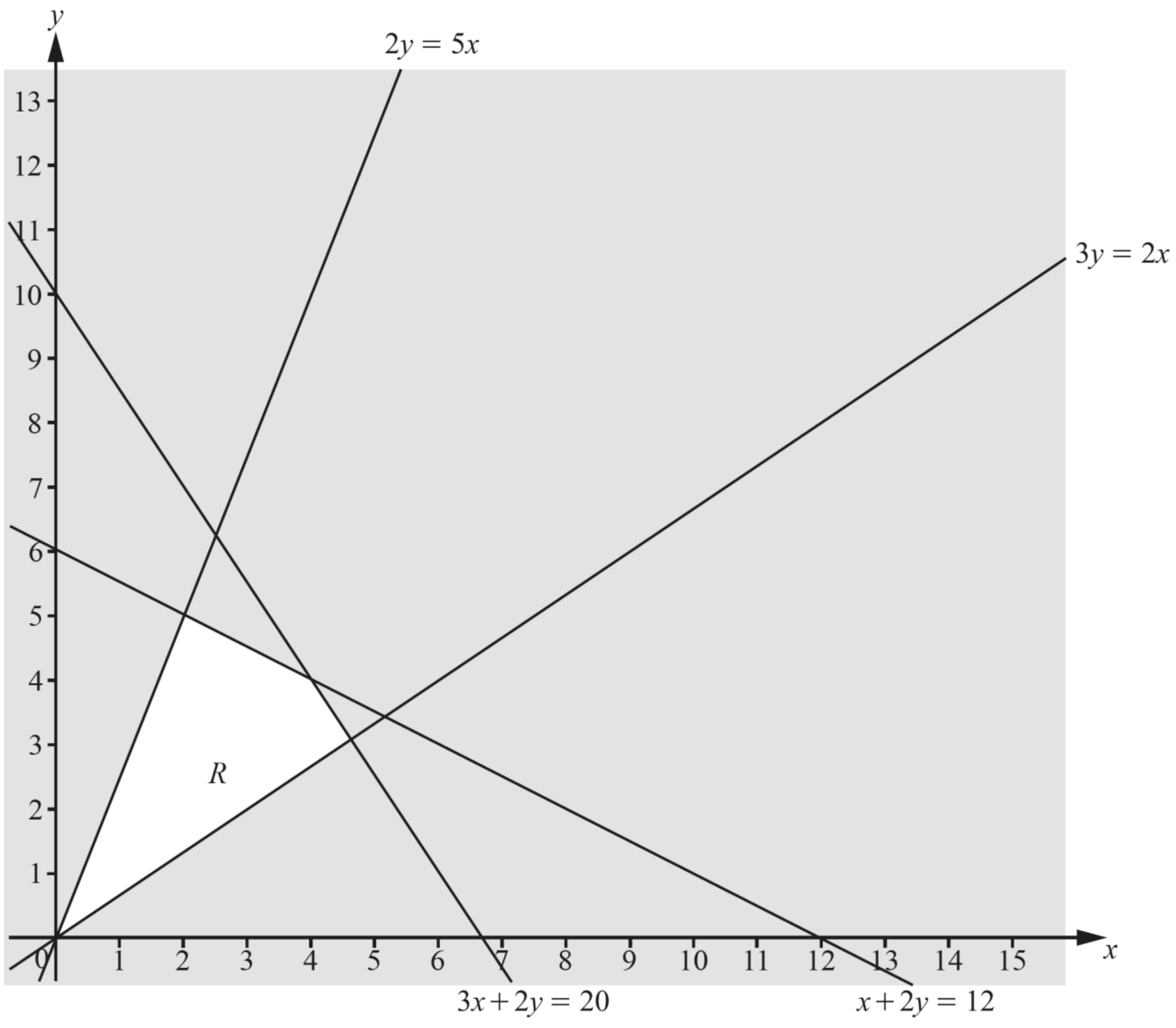


Fig. 6.1 shows the constraints of a linear programming problem, in which the objective is to maximise $P = x + ky$, where k is a positive constant.

The feasible region, R, is the unshaded region together with its boundaries.

(a) The simplex method could be used to solve this problem.

- Show how the constraints for the problem can be made into equations using slack variables.
- Show how the row for the objective function can be formed.
- Complete the initial tableau in the Printed Answer Booklet so that the simplex method may be used to solve this problem. **[6]**

Several candidates only completed the initial tableau and did not show how the constraints for the problem were made into equations using slack variables. Most candidates could correctly deal with the two lines with negative gradients but struggled to form the correct inequalities for the two lines that passed through the origin.

Assessment for learning

Candidates should be reminded that when filling in an initial tableau that all cells must be completed, especially zeros.

Exemplar 2

$2y \leq 5x$

$2y - 5x \leq 0$

$2y - 5x + s_1 = 0$

$3y - 2x \geq 0$

$2x - 3y + s_2 = 0$

$x + 2y + s_3 = 12$

$3x + 2y + s_4 = 20$

Maximise

$P = x + ky \Rightarrow P - x - ky$

Subject to

$2y - 5x + s_1 = 0$

$2x - 3y + s_2 = 0$

$x + 2y + s_3 = 12$

$3x + 2y + s_4 = 20$

P	x	y	s_1	s_2	s_3	s_4	RHS
1	−1	−k	0	0	0	0	0
0	−5	2	1	0	0	0	0
0	2	−3	0	1	0	0	0
0	1	2	0	0	1	0	12
0	3	2	0	0	0	1	20

This response scored 4 of the 6 marks available even though the initial tableau was completed correctly. The candidate did not state the inequalities for the lines $x + 2y = 12$ and $3x + 2y = 20$ before re-writing as equations with slack variables. They also did not correctly state the equation of the objective function, as they did not state it equal to zero.

Question 6 (b)

After two iterations of the simplex method a computer produces the tableau in **Fig. 6.2**. It is given that the tableau in **Fig. 6.2** does not give an optimal solution to the LP problem.

Fig. 6.2

P	x	y	s_1	s_2	s_3	s_4	RHS
1	0	0	$-\frac{1}{6}+\frac{1}{12}k$	0	0	$\frac{1}{6}+\frac{5}{12}k$	$2+5k$
0	0	1	$\frac{1}{12}$	0	0	$\frac{5}{12}$	5
0	0	0	$\frac{1}{3}$	1	0	$-\frac{4}{3}$	4
0	0	0	$\frac{7}{12}$	0	1	$\frac{11}{12}$	11
0	1	0	$-\frac{1}{6}$	0	0	$\frac{1}{6}$	2

(b) Perform a third iteration of the simplex method. **[3]**

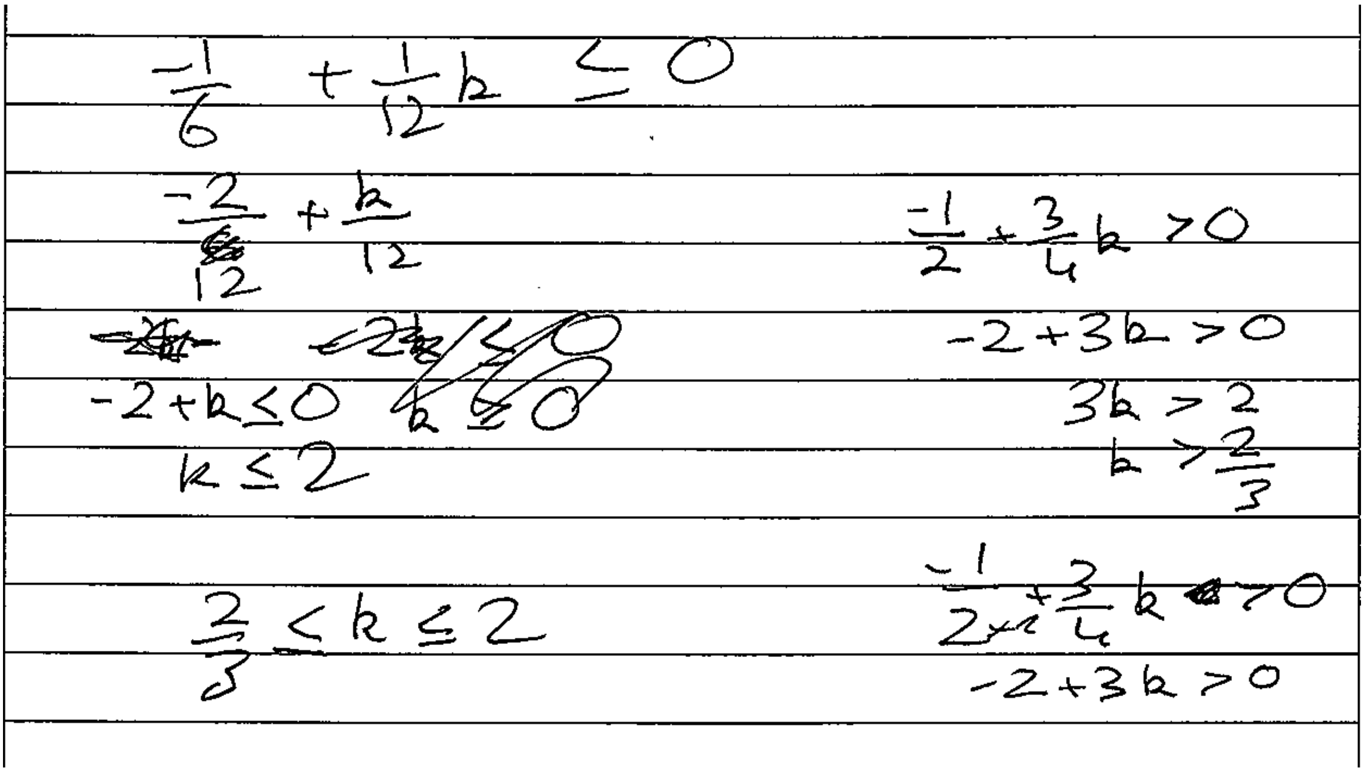
Candidates were generally very proficient in carrying out the simplex method. Many identified the correct pivot value (the ⅓ in the third row) and used this to carry out the third iteration, as requested. The most common errors were sign errors on certain values or difficulty in dealing with fractions. There were a significant minority who pivoted on an incorrect value and some who used a negative value as a pivot.

Question 6 (c)

(c) Given that an optimal solution to the LP problem is found after a third iteration of the simplex method, determine the range of possible values of P. **[4]**

Candidates found this question very demanding with very few realising that if after the 3rd iteration an optimal solution had been found then this implied that both $0.5 - 0.25k \geq 0$ and $-0.5 + 0.75k \geq 0$, leading to $^{2}/_{3} \leq k \leq 2$. However, even fewer realised that if the 3rd iteration provided an optimal solution, then the 2nd iteration did not. This meant that $-^{1}/_{6} + ^{k}/_{12} < 0$, which led to the further constraint on k that $k < 2$. Only the most successful candidates could correctly determine the range of values for P as $^{20}/_{3} \leq P < 12$.

Exemplar 3



This response scored 1 of the 4 marks available for considering –0.5 + 0.75k > 0. Although the inequality symbol used is incorrect, as an optimal solution occurs when there are no negative values in the objective row, a method mark could be given. Likewise, they considered $-^{2}/_{12} + {}^{k}/_{12} \leq 0$ when it should have contained the strict inequality < 0 because the second iteration was not optimal.

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