Modified Enlarged 18pt

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Monday 5 June 2023 – Afternoon

A Level Further Mathematics B (MEI)

Y433/01 Modelling with Algorithms

Time allowed: 1 hour 15 minutes
plus your additional time allowance

YOU MUST HAVE:
the Printed Answer Booklet
the Formulae Booklet for Further Mathematics B (MEI)
a scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

DC (DE/JG) 333045

INSTRUCTIONS

Use black ink. You can use an HB pencil, but only for graphs and diagrams.

Write your answer to each question in the space provided in the PRINTED ANSWER BOOKLET. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.

Fill in the boxes on the front of the Printed Answer Booklet.

Answer ALL the questions.

Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.

Give your final answers to a degree of accuracy that is appropriate to the context.

Do NOT send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

The total mark for this paper is 60.

The marks for each question are shown in brackets [ ].

ADVICE

Read each question carefully before you start your answer.

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1 Ten suitcases are to be transported in containers. Each container can hold a maximum of n kg, where n is a positive integer.

The total weight of the ten suitcases is 216 kg.

You are given that at least 4 containers are needed to transport all ten suitcases.

(a) Determine the maximum value of n for which ANY set of 10 suitcases with total weight 216 kg needs at least 4 containers. [2]

The exact weights, in kg, of the suitcases are:

17	23	18	14	26
21	24	15	31	27

(b) Apply the quick sort algorithm to sort the list of numbers into DESCENDING order. You should use the first value as the pivot for each sublist. [3]

The first fit decreasing algorithm is applied to the sorted list of numbers. The following allocation of suitcases to containers is obtained.

Container 1:	31	27	
Container 2:	26	24	18
Container 3:	23	21	17
Container 4:	15	14	

(c) Determine the possible values of n that are consistent with this result from applying the first fit decreasing algorithm to the sorted list of these weights. [2]

2 The diagram opposite shows an activity network for a project. The arc weights show activity durations in hours. The numbers in circles are event numbers.

(a) Explain the significance of the dummy activity from event 3 to event 4. [1]

(b) Using the diagram in the Printed Answer Booklet, carry out a forward pass and a backward pass through the entire network to find the following.

The minimum completion time for the project.

The critical activities. [5]

The duration of activity J changes to x hours.

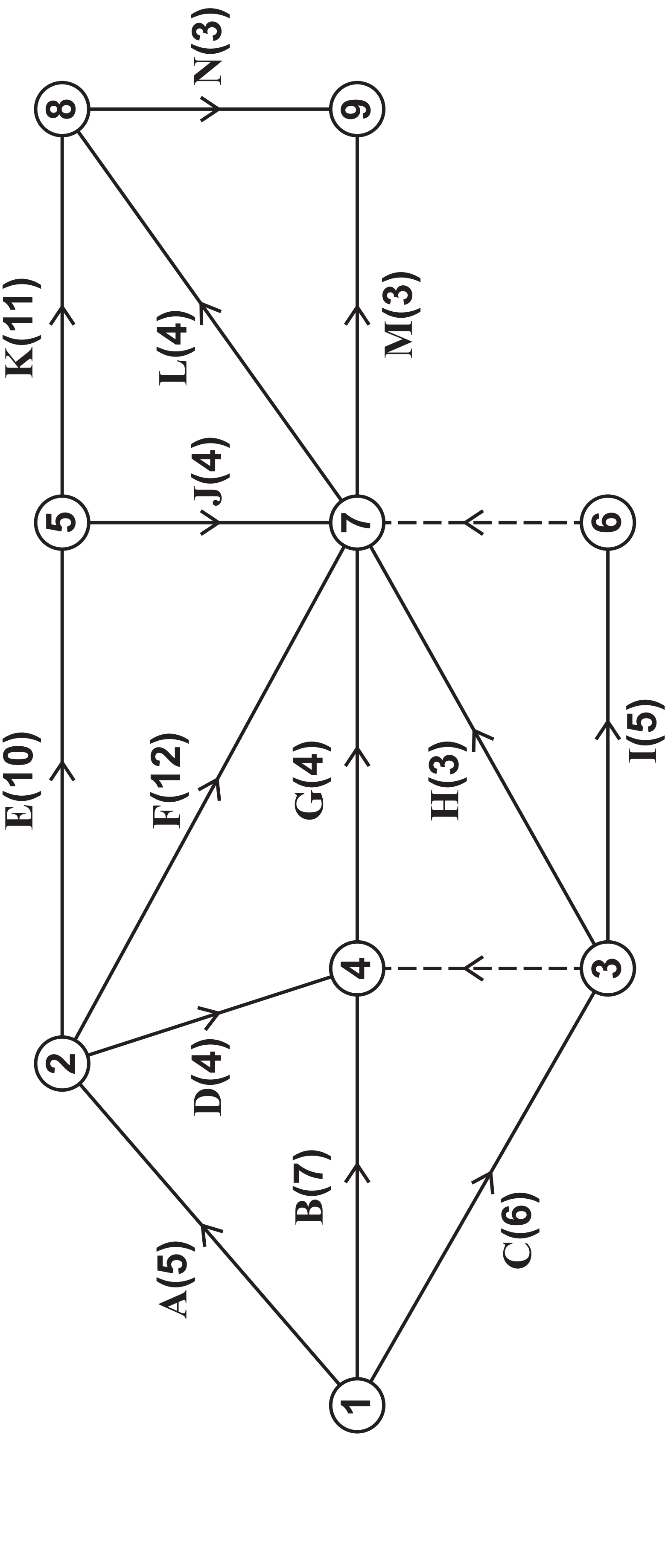
(c) Determine the following, in terms of x where necessary.

The new early event time for event 7.

The new late event time for event 7. [3]

It is given that the total float for activity K is now 5 hours.

(d) Find the value of x. [1]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3 A DIRECTED network consists of eight nodes, A–H, and fifteen arcs.

A shortest path from A to H needs to be found for this network.

The objective for an LP formulation for finding the shortest path from A to H is to minimise the objective function given by

$$19AB + 37AD + 27AE + 41BC + 15BD + 23EF + 51AF + 20DC + 14DF + 32DG + 19CH + 10CG + 42DH + 27FH + 6GH$$

(a) Draw the network in the Printed Answer Booklet. [3]

(b) Complete the LP formulation associated with the shortest path problem by listing the constraints. [3]

(c) Apply Dijkstra’s algorithm to the completed network from part (a) to find the shortest path from A to H. [5]

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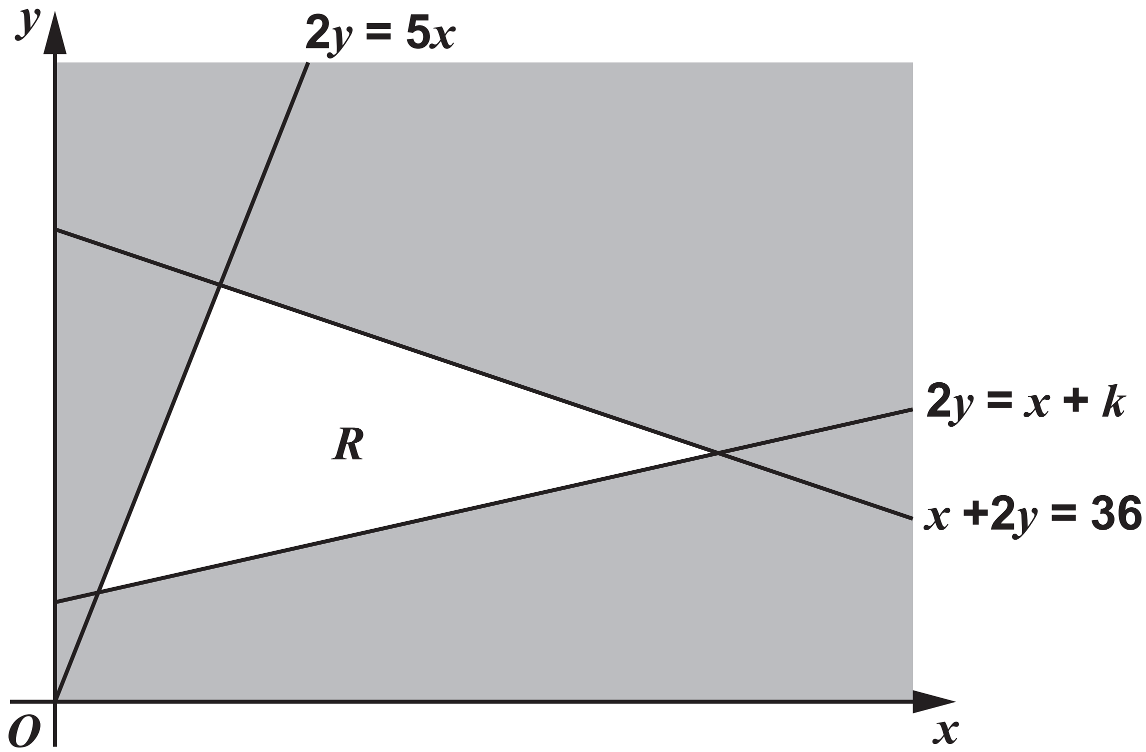


The diagram shows the constraints of a linear programming problem in which the objective is to maximise $P = x + ky$, where k is a positive constant.

The feasible region, R, is the unshaded region together with its boundaries.

You are given that the optimal value of P is 24.96.

Determine the following, explaining your reasoning.

The possible value(s) of k.

The corresponding coordinates of the optimal vertex. [7]

5 The diagram opposite represents a system of pipes through which a fluid flows continuously from a source to a sink. The weights on the arcs show the capacities of the pipes in litres per minute. All flows in pipes take integer values.

(a) By considering the SINK node, explain why the maximum flow through the network cannot be greater than 65 litres per minute. [1]

(b) (i) The cut α partitions the vertices into the sets {S, A, B}, {C, D, E, F, G, H, I, T}. Calculate the capacity of cut α. [1]

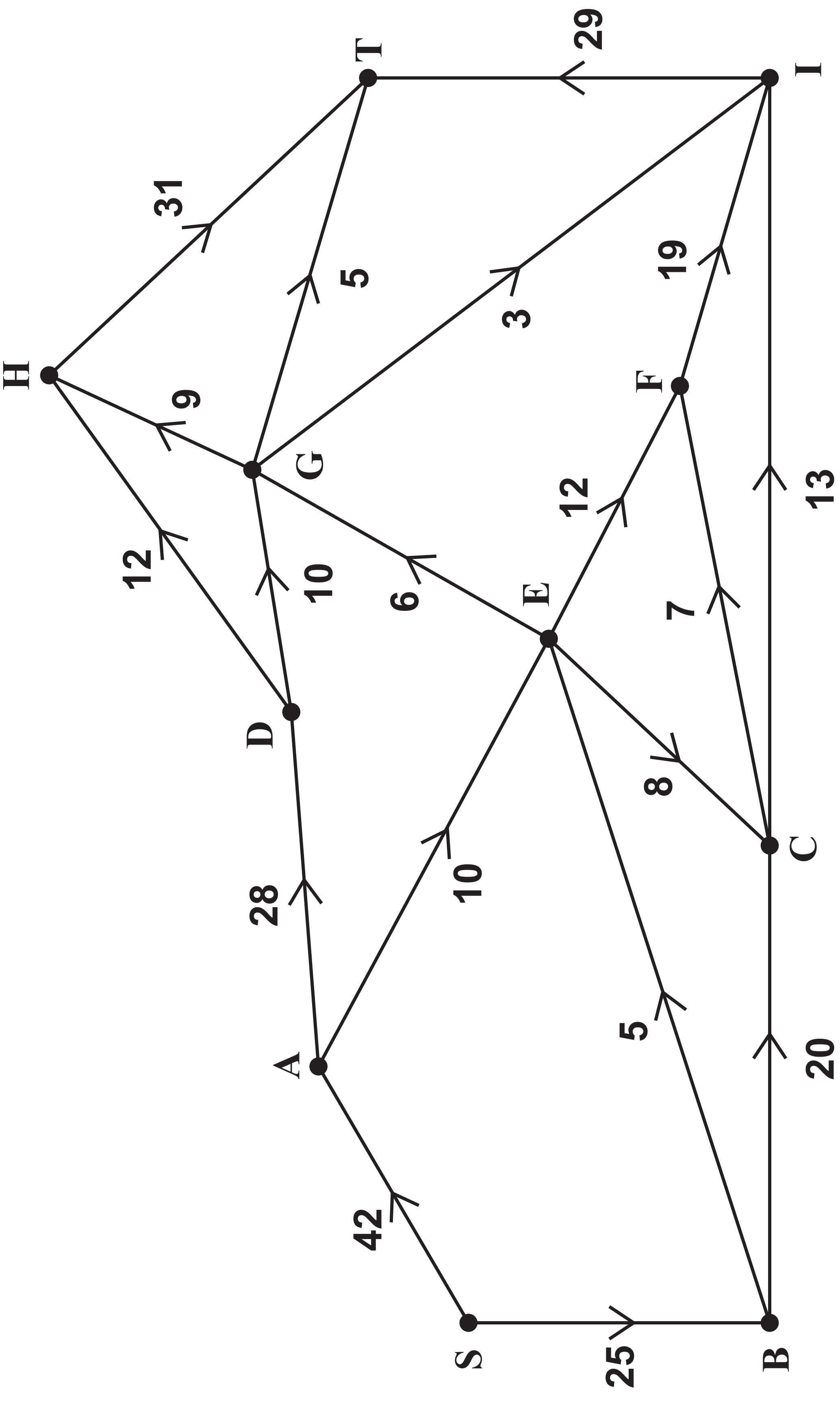
(ii) The cut β partitions the vertices into the sets {S, A, B, C, D}, {E, F, G, H, I, T}. Calculate the capacity of cut β. [1]

(c) Using ONLY the capacities of cuts α and β explain what can be deduced about the maximum possible flow through the system. [1]

An LP formulation is set up to find the maximum flow through the network.

(d) Explain why a possible objective function for the LP formulation is AD + EG + EF + CF + CI which is to be maximised. [1]

The complete LP was run in an LP solver, and it was found that only arcs AE, BE, CF, DH, DG, GH, GT, and IT were saturated and that there was zero flow through arc EC.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(e) **By completing the diagram in the Printed Answer Booklet, determine the maximum value of the flow through the network. [3]**

(f) **Use a suitable cut to prove that this is the maximum flow. [2]**

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6 The initial simplex tableau for a maximisation LP problem is shown in FIG. 6.1.

FIG. 6.1

P	x	y	z	s_1	s_2	s_3	RHS
1	−2	−1	−3	0	0	0	0
0	1	2	−1	1	0	0	b
0	−3	0	2	0	1	0	50
0	1	−1	2	0	0	1	55

It is given that b is a POSITIVE constant.

(a) Formulate the information given in FIG. 6.1 as an LP problem by completing the following.

State the objective function.

List the constraints as simplified inequalities with integer coefficients. [2]

After TWO iterations of the simplex method the tableau shown in FIG. 6.2 is produced.

FIG. 6.2

P	x	y	z	s_1	s_2	s_3	RHS
1	0	$-\frac{21}{8}$	0	0	$-\frac{1}{8}$	$\frac{13}{8}$	$\frac{665}{8}$
0	0	$\frac{15}{8}$	0	1	$\frac{3}{8}$	$\frac{1}{8}$	$b+\frac{205}{8}$
0	0	$-\frac{3}{8}$	1	0	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{215}{8}$
0	1	$-\frac{1}{4}$	0	0	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{5}{4}$

(b) Write down the value of each variable after the second iteration. [1]

(c) Explain how the tableau in FIG. 6.2 shows that the solution obtained after the second iteration is not optimal. [1]

After a third iteration of the simplex method the resulting tableau does give an optimal solution to the problem. Furthermore, it is given that in this optimal solution the value of y is three times the value of x.

(d) By performing the third iteration of the simplex method, using the tableau in the Printed Answer Booklet, determine the optimal value of the objective function for this LP problem. [6]

The LP problem is modified so that b is now a NEGATIVE constant.

(e) Explain why the simplex method cannot be used to solve this modified problem. [1]

(f) The two-stage simplex method is to be used to solve this modified problem. Complete the initial tableau in the Printed Answer Booklet so that the two-stage simplex method may be used to solve this modified problem. [4]

END OF QUESTION PAPER