**Modified Enlarged 18pt**

**OXFORD CAMBRIDGE AND RSA EXAMINATIONS**

# Friday 23 June 2023 – Afternoon

## A Level Further Mathematics B (MEI)

## Y435/01 Extra Pure

**Time allowed: 1 hour 15 minutes**
**plus your additional time allowance**

**YOU MUST HAVE:**
**the Printed Answer Booklet or any suitable paper provided by the centre. The Printed Answer Booklet may be enlarged by the centre.**
**the Formulae Booklet for Further Mathematics B (MEI)**
**a scientific or graphical calculator**

**READ INSTRUCTIONS OVERLEAF**

## INSTRUCTIONS

Use black ink. You can use an HB pencil, but only for graphs and diagrams.

If you use the Printed Answer Booklet write your answer to each question in the space provided in the PRINTED ANSWER BOOKLET. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.

If you use the Printed Answer Booklet fill in the boxes on the front of the Printed Answer Booklet.

Answer ALL the questions.

Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.

Give your final answers to a degree of accuracy that is appropriate to the context.

Do NOT send this Question Paper for marking. Keep it in the centre or recycle it.

## INFORMATION

The total mark for this paper is 60.

The marks for each question are shown in brackets [ ].

## ADVICE

Read each question carefully before you start your answer.

1 A surface is defined in 3-D by $z = 3x^3 + 6xy + y^2$.

Determine the coordinates of any stationary points on the surface. [7]

2 A sequence is defined by the recurrence relation $4t_{n+1} - t_n = 15n + 17$ for $n \geqslant 1$, with $t_1 = 2$.

(a) Solve the recurrence relation to find the particular solution for $t_n$. [7]

Another sequence is defined by the recurrence relation $(n+1)u_{n+1} - u_n^2 = 2n - \frac{1}{n^2}$ for $n \geqslant 1$, with $u_1 = 2$.

(b) (i) Explain why the recurrence relation for $u_n$ CANNOT be solved using standard techniques for non-homogeneous first order recurrence relations. [1]

(ii) Verify that the particular solution to this recurrence relation is given by $u_n = an + \frac{b}{n}$ where $a$ and $b$ are constants whose values are to be determined. [5]

A third sequence is defined by $v_n = \frac{t_n}{u_n}$ for $n \geqslant 1$.

(c) Determine $\lim_{n \to \infty} v_n$. [2]

3 A surface, $S$, is defined by $g(x, y, z) = 0$ where $g(x, y, z) = 2x^3 - x^2y + 2xy^2 + 27z$. The normal to $S$ at the point $\left(1, 1, -\frac{1}{9}\right)$ and the tangent plane to $S$ at the point $(3, 3, -3)$ intersect at $P$.

Determine the position vector of $P$. [8]

4 The set $G$ is given by $G = \{\mathrm{M}: \mathrm{M} \text{ is a real } 2 \times 2 \text{ matrix and } \det \mathrm{M} = 1\}$.

(a) Show that $G$ forms a group under matrix multiplication, $\times$. You may assume that matrix multiplication is associative. [5]

(b) The matrix $\mathbf{A}_n$ is defined by $\mathbf{A}_n = \begin{pmatrix} 1 & 0 \\ n & 1 \end{pmatrix}$ for any integer $n$. The set $S$ is defined by

$S = \{\mathbf{A}_n : n \in \mathbb{Z}, n \geqslant 0\}$.

(i) Determine whether $S$ is closed under $\times$. [2]

(ii) Determine whether $S$ is a subgroup of $(G, \times)$. [2]

(c) (i) Find a subgroup of $(G, \times)$ of order 2. [2]

(ii) By considering the inverse of the non-identity element in any such subgroup, or otherwise, show that this is the only subgroup of $(G, \times)$ of order 2. [2]

The set of all real $2 \times 2$ matrices is denoted by $H$.

(d) With the help of an example, explain why $(H, \times)$ is NOT a group. [2]

5 The matrix $\mathbf{P}$ is given by $\mathbf{P} = \begin{pmatrix} a & 0 \\ 2 & 3 \end{pmatrix}$ where $a$ is a constant and $a \neq 3$.

(a) Given that the acute angle between the directions of the eigenvectors of $\mathbf{P}$ is $\frac{1}{4}\pi$ radians, determine the possible values of $a$. [8]

(b) You are given instead that $\mathbf{P}$ satisfies the matrix equation $\mathbf{I} = \mathbf{P}^2 + r\mathbf{P}$ for some rational number $r$.

(i) Use the Cayley-Hamilton theorem to determine the value of $a$ and the corresponding value of $r$. [4]

(ii) Hence show that $\mathbf{P}^4 = s\mathbf{I} + t\mathbf{P}$ where $s$ and $t$ are rational numbers to be determined.
You should NOT calculate $\mathbf{P}^4$. [3]

END OF QUESTION PAPER

BLANK PAGE

BLANK PAGE