Modified Enlarged 18pt

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Thursday 22 June 2023 – Afternoon

A Level Further Mathematics B (MEI)

Y434/01 Numerical Methods

Time allowed: 1 hour 15 minutes
plus your additional time allowance

YOU MUST HAVE:
the Printed Answer Booklet or any suitable paper provided by the centre. The Printed Answer Booklet may be enlarged by the centre.
the Formulae Booklet for Further Mathematics B (MEI)
a scientific or graphical calculator
Insert for questions 3(c) and 8(a) (with this document)

READ INSTRUCTIONS OVERLEAF

DC (ST/JG) 334551/1

INSTRUCTIONS

Use black ink. You can use an HB pencil, but only for graphs and diagrams.

If you use the Printed Answer booklet write your answer to each question in the space provided in the PRINTED ANSWER BOOKLET. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.

If you use the Printed Answer booklet fill in the boxes on the front of the Printed Answer Booklet.

Answer ALL the questions.

Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.

Give your final answers to a degree of accuracy that is appropriate to the context.

Do NOT send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

The total mark for this paper is 60.

The marks for each question are shown in brackets [ ].

ADVICE

Read each question carefully before you start your answer.

BLANK PAGE

1 You are given that $(x_1, y_1) = (0.9, 2.3)$ and $(x_2, y_2) = (1.1, 2.7)$.

The values of x_1 and x_2 have been ROUNDED to 1 decimal place.

(a) Determine the range of possible values of $x_2 - x_1$. [2]

The values of y_1 and y_2 have been CHOPPED to 1 decimal place.

(b) Determine the range of possible values of $y_2 - y_1$. [2]

You are given that $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.

(c) Determine the range of possible values of m. [2]

(d) Explain why your answer to part (c) is much larger than your answer to part (a) and your answer to part (b). [1]

2 A car tyre has a slow puncture. Initially the tyre is inflated to a pressure of 34.5 psi. The pressure is checked after 3 days and then again after 5 days. The time t in days and the pressure, P psi, are shown in the table below. You are given that the pressure in a car tyre is measured in pounds per square inch (psi).

t	0	3	5
P	34.5	29.4	27.0

The owner of the car believes the relationship between P and t may be modelled by a polynomial.

(a) Explain why it is not possible to use Newton's forward difference interpolation method for these data. [1]

(b) Use Lagrange's form of the interpolating polynomial to find an interpolating polynomial of degree 2 for these data. [4]

The car owner uses the polynomial found in part (b) to model the relationship between P and t.

Subsequently it is found that when $t = 6$, $P = 26.0$ and when $t = 10$, $P = 24.4$.

(c) Determine whether the owner's model is a good fit for these data. [2]

(d) Explain why the model would not be suitable in the long term. [1]

3 **The diagram opposite shows the graph of $y = f(x)$ for values of x from 1 to 3.5.**

The table shows some values of x and the associated values of y.

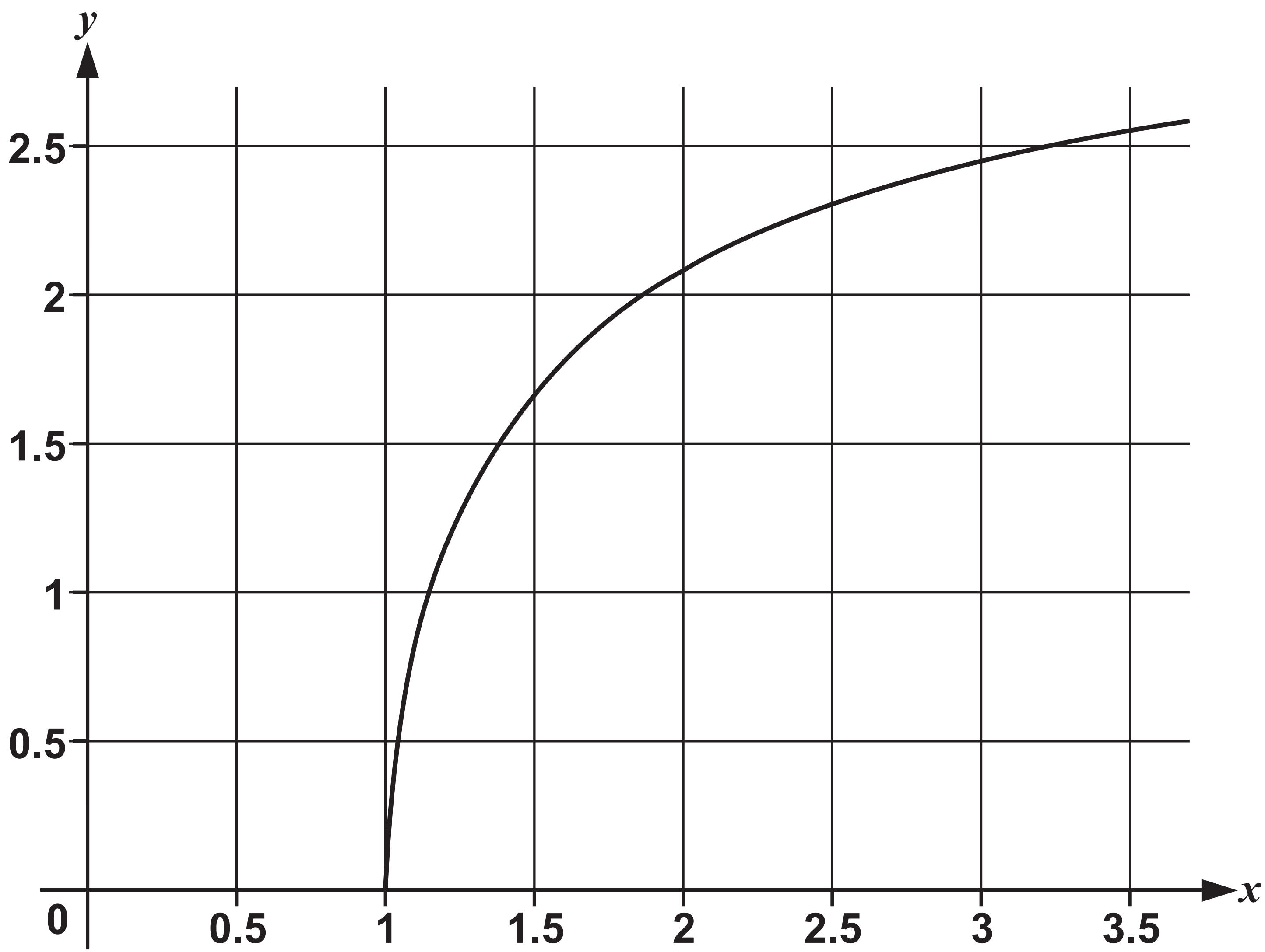
x	1.5	2	2.5
y	1.682137	2.094395	2.318559

(a) **Use the forward difference method to calculate an approximation to $\frac{dy}{dx}$ at $x = 2$. [2]**

(b) **Use the central difference method to calculate an approximation to $\frac{dy}{dx}$ at $x = 2$. [2]**

(c) **On the copy of the diagram in the Printed Answer Booklet or in the insert, show how the central difference method gives the approximation to $\frac{dy}{dx}$ at $x = 2$ which was found in part (b). [1]**

(d) **Explain whether your answer to part (a) or your answer to part (b) is likely to give a better approximation to $\frac{dy}{dx}$ at $x = 2$. [1]**



y
x
0
0.5
1
1.5
2
2.5
3
3.5
0.5
1
1.5
2
2.5

4 A spreadsheet is used to approximate $\int_a^b f(x)\,dx$ using the midpoint rule with 1 strip.

The output is shown in the table below.

	B	C	D
3	x	$f(x)$	M_1
4	1.5	1.3103707	0.65518535

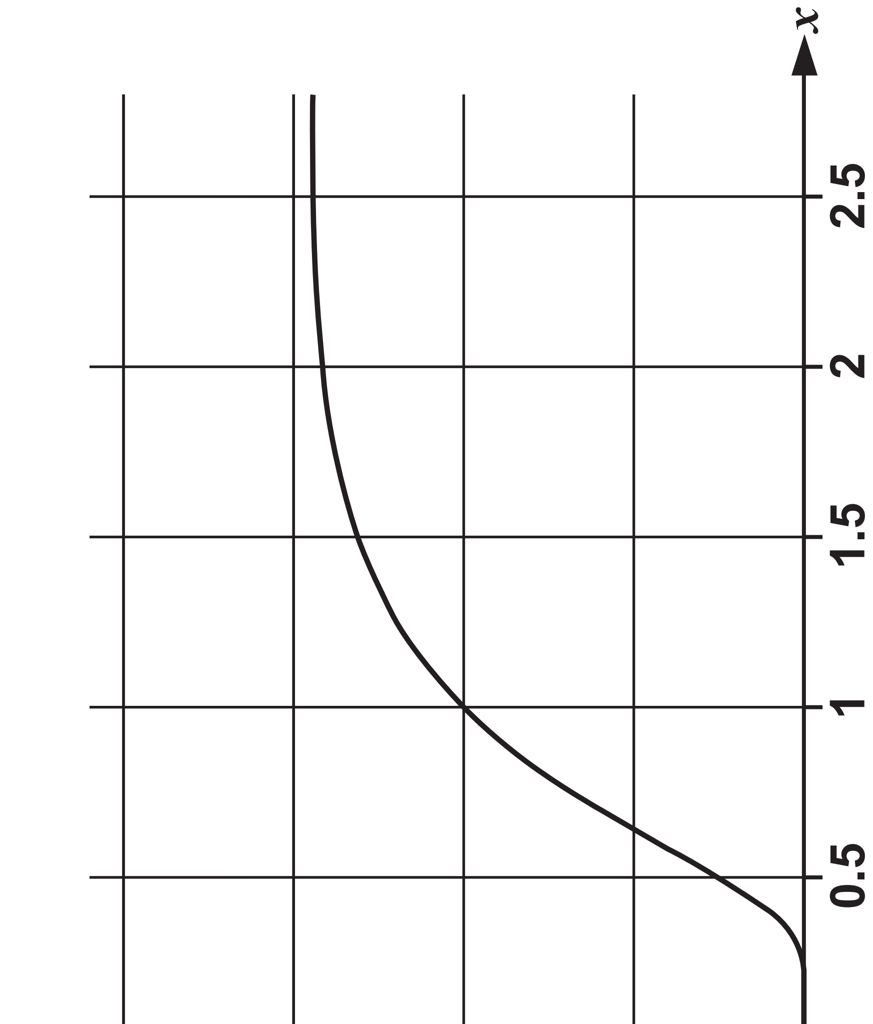
The formula in cell C4 is $\boxed{=B4\text{^}(1/B4)}$.

The formula in cell D4 is $\boxed{=0.5 * C4}$.

(a) Write the integral in standard mathematical notation. [3]

A graph of $y = f(x)$ is included in the diagram opposite.

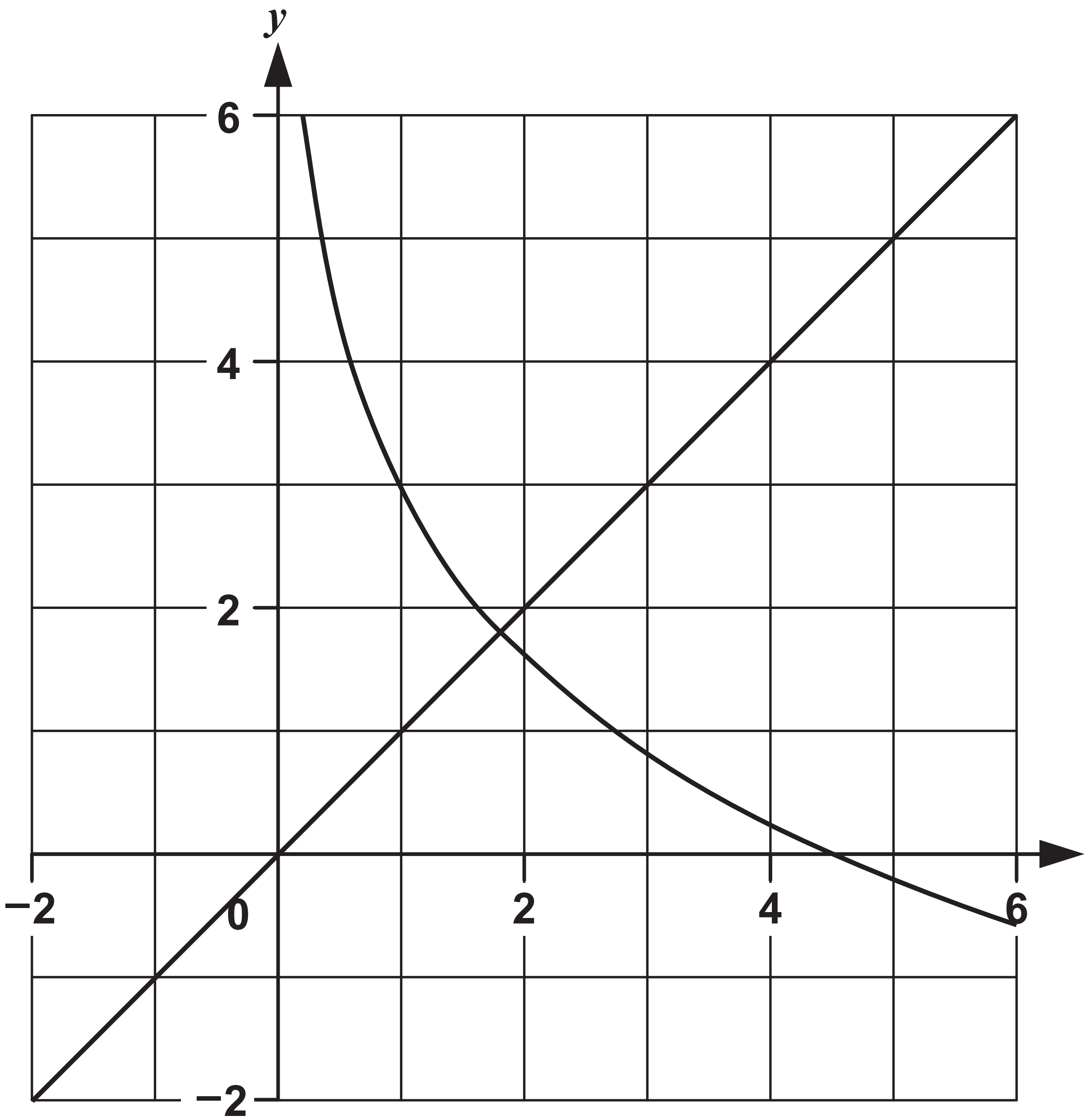
(b) Explain whether 0.65518535 is an over-estimate or an under-estimate of $\int_a^b f(x)\,dx$. [1]


y
x
0
0.5
1
1.5
2
2.5

5 **The equation $3 - 2\ln x - x = 0$ has a root near $x = 1.8$.**

A student proposes to use the iterative formula $x_{n+1} = g(x_n) = 3 - 2\ln x_n$ to find this root.

The diagram shows the graphs of $y = x$ and $y = g(x)$ for values of x from -2 to 6.



(a) With reference to the graph, explain why it might not be possible to use the student's iterative formula to find the root near $x = 1.8$. [1]

(b) Use the relaxed iteration $x_{n+1} = \lambda g(x_n) + (1 - \lambda)x_n$, with $\lambda = 0.475$ and $x_0 = 2$, to determine the root correct to 6 decimal places. [3]

A student uses the same relaxed iteration with the same starting value. Some analysis of the iterates is carried out using a spreadsheet, which is shown in the table below.

r	difference	ratio
0		
1	–0.1834898	
2	–0.0049137	0.02678
3	–6.44E-06	0.00131
4	–3.862E-09	0.0006
5	–2.313E-12	0.0006

(c) Explain what the analysis tells you about the order of convergence of this sequence of approximations. [2]

6 (a) (i) Calculate the relative error when π is CHOPPED to 2 decimal places in approximating $\pi^2 + 2$. [2]

(ii) WITHOUT doing any calculation, explain whether the relative error would be the same when π is CHOPPED to 2 decimal places when approximating $(\pi + 2)^2$. [1]

The table shows some spreadsheet output. The values of x in column A are exact.

	A	B	C
1	x	10^x	$\log_{10} 10^x$
2	1E-12	1	1.00001E-12
3	1E-11	1	9.99998E-12

The formula in cell B2 is =10^A2 .

This has been copied down to cell B3.

The formula in cell C2 is =LOG(B2) .

This formula has been copied down to cell C3.

(b) (i) Write the value displayed in cell C2 in standard mathematical notation. [1]

(ii) Explain why the values in cells C2 and C3 are neither zero nor the same as the values in cells A2 and A3 respectively. [2]

BLANK PAGE

7 The value of a function, $y = \mathrm{f}(x)$, and its gradient function, $\frac{\mathrm{d}y}{\mathrm{d}x}$, when $x = 2$, is given in TABLE 7.1.

TABLE 7.1

x	$\mathrm{f}(x)$	$\frac{\mathrm{d}y}{\mathrm{d}x}$
2	6	−2.8

(a) Determine the approximate value of the error when $\mathrm{f}(2)$ is used to estimate $\mathrm{f}(2.03)$. [2]

The Newton-Raphson method is used to find a sequence of approximations to a root, α, of the equation $\mathrm{f}(x) = 0$. The spreadsheet output showing the iterates, together with some further analysis, is shown in TABLE 7.2.

TABLE 7.2

	A	B	C	D
1	r	x_r	difference	ratio
2	0	12		
3	1	−13.1165572	−25.1165572	
4	2	1.76283279	14.87939004	−0.5924136
5	3	2.18052157	0.41768878	0.02807163
6	4	2.182419024	0.001897454	0.00454275
7	5	2.182419066	4.13985E-08	2.1818E-05

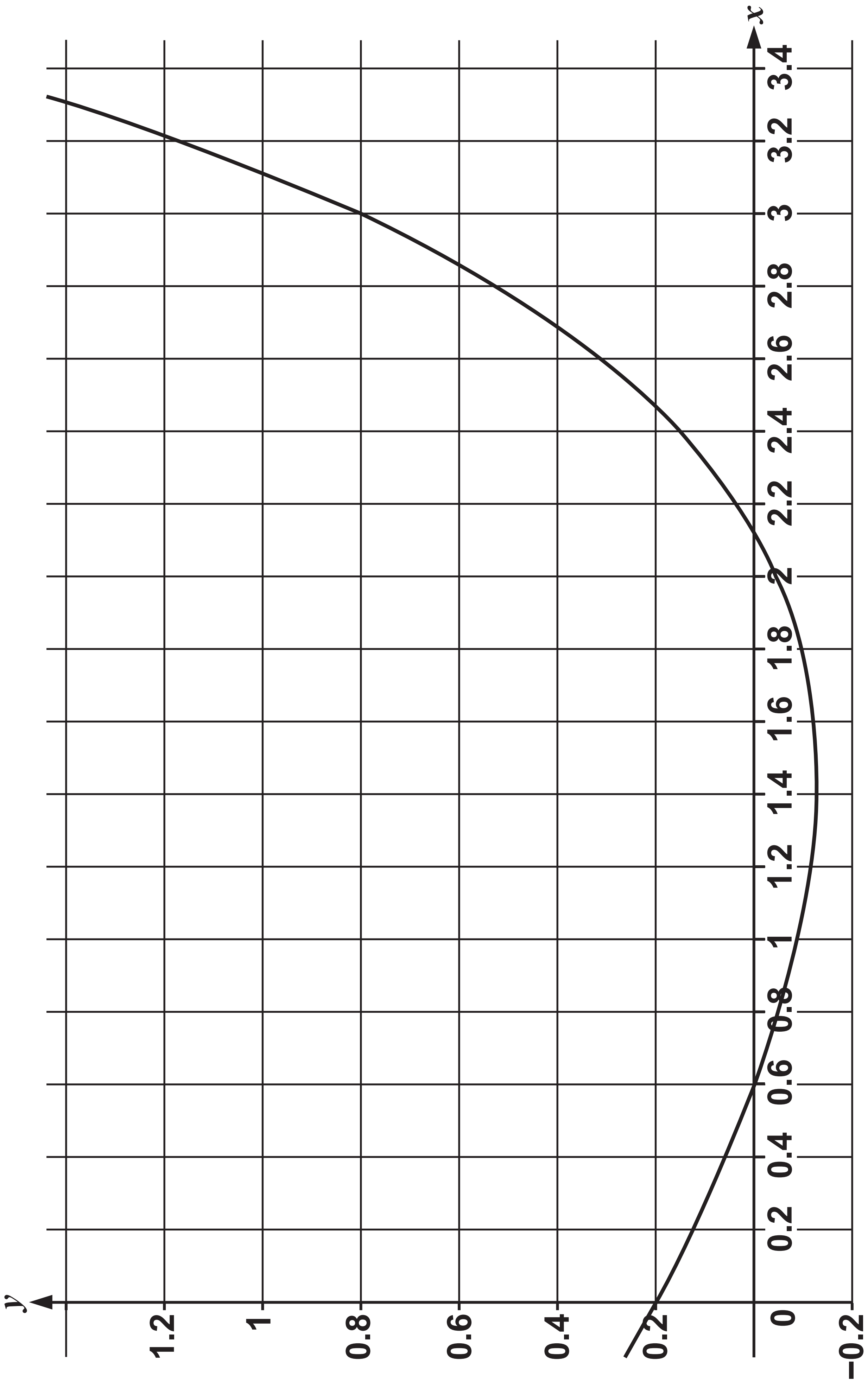
(b) (i) Explain what the values in column D tell you about the order of convergence of this sequence of approximations. [2]

(ii) WITHOUT doing any further calculation, state the value of α as accurately as you can, justifying the precision quoted. [2]

8 **The graph of $y = 0.2\cosh x - 0.4x$ for values of x from 0 to 3.32 is shown on the graph opposite.**

The equation $0.2\cosh x - 0.4x = 0$ has two roots, α and β where $\alpha < \beta$, in the interval $0 < x < 3$. The secant method with $x_0 = 1$ and $x_1 = 2$ is to be used to find β.

(a) On the copy of the graph in the Printed Answer Booklet or in the insert, show how the secant method works with these two values of x to obtain an improved approximation to β. [1]


x
y
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6
2.8
3
3.2
3.4
−0.2
0
0.2
0.4
0.6
0.8
1
1.2

The spreadsheet output in the table below shows the result of applying the secant method with $x_0 = 1$ and $x_1 = 2$.

	I	J	K	L	M
2	r	x_r	$f(x_r)$	x_{r+1}	$f(x_{r+1})$
3	0	1	−0.0914	2	−0.0476
4	1	2	−0.0476	3.08529	0.95784
5	2	3.08529	0.95784	2.05134	−0.0298
6	3	2.05134	−0.0298	2.08259	−0.0181
7	4	2.08259	−0.0181	2.13042	0.00155
8	5	2.13042	0.00155	2.12664	−7E-05

(b) Write down a suitable cell formula for cell J4. [1]

(c) Write down a suitable cell formula for cell L4. [2]

(d) Write down the most accurate approximation to β which is displayed in the table. [1]

(e) Determine whether your answer to part (d) is correct to 5 decimal places. You should NOT calculate any more iterates. [2]

(f) It is decided to use the secant method with starting values $x_0 = 1$ and $x_1 = a$, where $a > 1$, to find α. State a suitable value for a. [1]

9 The trapezium rule is used to calculate 3 approximations to $\int_0^1 \sqrt[3]{\sinh(x)}\,dx$ with 1, 2 and 4 strips respectively. The results are shown in TABLE 9.1.

TABLE 9.1

n	T_n
1	0.52764369
2	0.66617652
4	0.72534275

(a) USE THESE RESULTS to determine TWO approximations to $\int_0^1 \sqrt[3]{\sinh(x)}\,dx$ using Simpson's rule. [2]

(b) Use your answers to part (a) to state the value of $\int_0^1 \sqrt[3]{\sinh(x)}\,dx$ as accurately as you can, justifying the precision quoted. [1]

TABLE 9.2 shows some further approximations found using the trapezium rule, together with some analysis of these approximations.

TABLE 9.2

n	T_n	difference	ratio
1	0.5276437		
2	0.6661765	0.138533	
4	0.7253427	0.059166	0.42709
8	0.7498821	0.024539	0.41475
16	0.7598858	0.010004	0.40766
32	0.7639221	0.004036	0.40348
64	0.7655404	0.001618	0.40095

(c) Explain what can be deduced about the order of the method in this case. [2]

(d) Use extrapolation to obtain the value of $\int_0^1 \sqrt[3]{\sinh(x)}\,\mathrm{d}x$ as accurately as you can, justifying the precision quoted. [4]

END OF QUESTION PAPER

BLANK PAGE

BLANK PAGE

BLANK PAGE