Qualification Accredited

Oxford Cambridge and RSA

A LEVEL

Examiners' report

FURTHER MATHEMATICS B (MEI)

H645

For first teaching in 2017

Y421/01 Summer 2023 series

Version 1

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate answers is also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from OCR.

Paper Y421/01 series overview

This is one of the two major examination components for the A Level examination for GCE Further Mathematics B. It is a two-hour fifteen-minute paper consisting of 120 marks. The paper consists of two sections, A and B. Section A will have between 25 and 35 marks and will comprise more straightforward questions. Section B will have between 85 and 95 marks and will comprise a mixture of more and less straightforward questions.

Inevitably, the report that follows will concentrate on aspects of the candidates' performance where improvement is possible to assist centres on preparing candidates for future series. However, this should not obscure the fact that a significant number of candidates who sat this paper produced solutions which were a pleasure for examiners to assess. Many candidates demonstrated a most impressive level of mathematical ability and insight which enabled them to meet the various challenges posed by this paper on all the associated mechanics content; precision, command of correct mathematical notation and excellent presentational skills were evident in many scripts.

The specification includes some guidance about the level of written evidence required in assessment questions; these were provided to reflect the increased functionality of the available calculators and the changes in assessment objectives, since there is a significant change from when the equivalent legacy qualifications were designed.

The word 'Determine' in a question does not simply imply that candidates should find the answer but, to quote the specification, 'this command word indicates that justification should be given for any results found, including working where appropriate.' This command word featured in Questions 1, 2, 3 (b), 4 (a), 4 (b), 5 (b), 6 (a), 6 (b), 7 (a), 8 (a), 9 (a), 9 (b), 10 (a), 10 (b), 11 (b), 12 (a) (ii) and 12 (b).

The phrase 'Show that' generally indicates that the answer has been given, and that candidates should provide an explanation that has enough detail to cover every step of their working. This command phrase features in Questions 8 (b), 10 (c), 11 (a) and 13 (d).

While there is no specific level of working needed to justify answers to questions which use the command word 'Find …', method marks may still be available for valid attempts that do not result in a correct answer, and standard advice (included in the specification) that candidates should state explicitly any expressions, integrals, parameters and variables that they use a calculator to evaluate (using correct mathematical notation rather than model specific calculator notation).

OCR support

Full details of 'command words' can be found in section 2b of the specification. OCR also publish a classroom poster and associated guidance on command words, which centres should make sure candidates are familiar with. These can be found on Teach Cambridge.

Regardless of the final required accuracy, candidates should be careful of not rounding prematurely, but also take care to avoid over specifying rounded answers where the context does not support that level of accuracy.

One general point with regards to the answering of certain mechanics questions should be made in this overview. This is that unless told otherwise the value that candidates should use for the acceleration due to gravity, g, is 9.8 and not 9.81 or 10 (and this value is stated explicitly on the front cover of the examination paper). However, when a different value is stated in the question (for example, in Question 9) then this value must be used.

Candidates who did well on this paper generally:	Candidates who did less well on this paper generally:
• used formal language and notation correctly • defined the letters used for speed, time, etc. in unstructured questions • understood the level of response required for the command words used in the questions • made efficient use of their calculator • read questions carefully and provided the answers that were requested.	• made careless mistake in algebraic manipulation • did not give sufficient evidence on 'Show that' and 'Determine' questions • used imprecise notation or language • did not read the question carefully and gave answers not to the required degree of accuracy (or did not use all the information given in the question).

Section A overview

This section consists of more straightforward questions to act as a gentle start to this long exam paper. Most candidates answered the questions in this section very well, although for some candidates there seemed to be some conceptual misunderstanding to the topics assessed in Questions 4 and 5.

Question 1

1 A car of mass 800 kg moves in a straight line along a horizontal road.

There is a constant resistance to the motion of the car of magnitude 600 N.

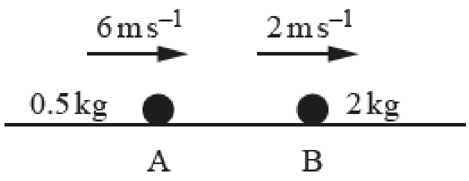
When the car is travelling at a speed of $15\,\text{m}\,\text{s}^{-1}$ the power developed by the car is 27 kW.

Determine the acceleration of the car when it is travelling at $15\,\text{m}\,\text{s}^{-1}$. **[4]**

This question was answered extremely well with almost all candidates obtaining the correct answer. When errors occurred, it was usually due to a sign error when applying Newton's second law or not using the correct value of 27000 for the power.

Question 2

2



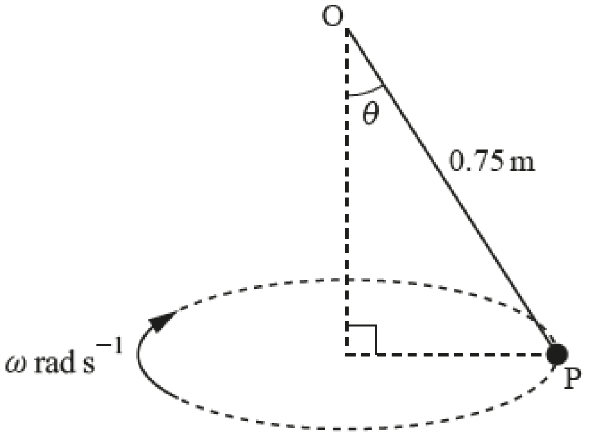
Two small uniform smooth spheres A and B have masses 0.5 kg and 2 kg respectively. The two spheres are travelling in the same direction in the same straight line on a smooth horizontal surface. Sphere A is moving towards B with speed $6\,\text{m}\,\text{s}^{-1}$ and B is moving away from A with speed $2\,\text{m}\,\text{s}^{-1}$ (see diagram). Spheres A and B collide. After this collision A moves with speed $0.2\,\text{m}\,\text{s}^{-1}$.

Determine the possible speeds with which B moves after the collision. **[4]**

Although some candidates needlessly spent time calculating the coefficients of restitution associated with the two possible cases of the motion of A after collision, most candidates correctly found the two possible speeds with which B moved after the collision (the one associated with A continuing to move in its original direction of its motion before collision, or the second case in which the direction of motion of A was reversed). When errors occurred, they were usually errors of sign or incorrectly applying the values given in the question.

Question 3 (a)

3



The diagram shows a particle P, of mass 0.2 kg, which is attached by a light inextensible string of length 0.75 m to a fixed point O.

Particle P moves with constant angular speed ω rad s^{-1} in a horizontal circle with centre vertically below O. The string is inclined at an angle θ to the vertical.

The greatest tension that the string can withstand without breaking is 15 N.

(a) Find the greatest possible value of θ, giving your answer to the nearest degree. **[2]**

Most candidates correctly resolved vertically to find the greatest possible value of θ with the most common errors being; using 0.2 as the weight, using sine instead of cosine, or not giving the final answer to the nearest degree as requested (although on this occasion any answer that was either an answer which rounded to 82 or 1.44 (radians) was accepted).

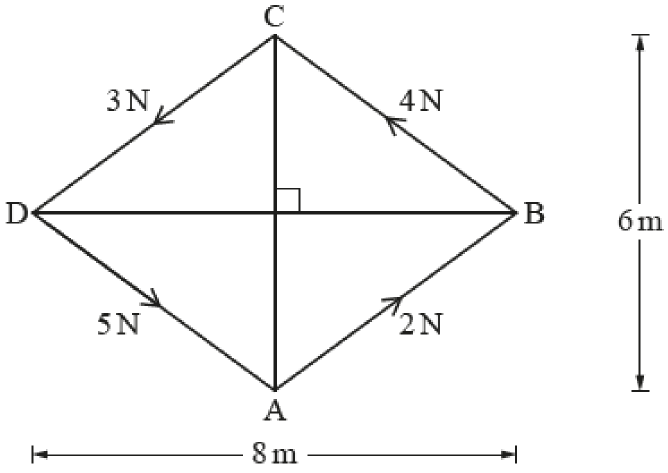
Question 3 (b)

(b) Determine the greatest possible value of ω. **[3]**

Part (b) was also answered well with almost all candidates correctly applying Newton's second law horizontally to find the greatest possible value of ω. The most common error was using 0.75 as the radius (rather than the correct expression 0.75sinθ) or using inexact values that lead to a value of ω that was not exactly 10 (although on this occasion any answer that was an answer that rounded to 10 was accepted – provide it came from correct working).

Question 4 (a)

4



A rigid lamina of negligible mass is in the form of a rhombus ABCD, where AC = 6 m and BD = 8 m. Forces of magnitude 2 N, 4 N, 3 N and 5 N act along its sides AB, BC, CD and DA, respectively, as shown in the diagram. A further force **F** N, acting at A, and a couple of magnitude G Nm are also applied to the lamina so that it is in equilibrium.

(a) Determine the magnitude and direction of **F**. **[4]**

Since the first examination in 2019 candidates have struggled to answer questions that involve forces in equilibrium that contain a couple, and this was no different this series. All that was required in this part was to an introduce a common angle (either to the horizontal or vertical) and then resolve horizontally and vertically. Those that did usually deduced the correct magnitude of **F** as 1.2 N and furthermore correctly stated that it acted in the direction AC.

Assessment for learning

When a question asks for a direction, it is advisable to give this in relation to information already given in the question. For example, in this part candidates who referenced AC were usually more successful than those who simply mentioned that the direction was 'vertical' (as it was not sufficiently clear if this was vertically upwards or downwards).

Question 4 (b)

(b) Determine the value of G. **[2]**

This part was not answered particularly well with many candidates leaving this part blank. Of those that did attempt this part, many did not choose the most ideal point to take moment about (namely A as about this point the force found in part (a) could be ignored). Those candidates who did take moments about A (and derived an equation of the form 6(3sinθ) + 6(4sinθ) where θ was the angle between AB and the upward vertical) usually wrote the most successful responses.

Question 5 (a)

5 A particle P of mass m kg is projected with speed $u\,\mathrm{m\,s^{-1}}$ along a rough horizontal surface. During the motion of P, a constant frictional force of magnitude F N acts on P. When the velocity of P is $v\,\mathrm{m\,s^{-1}}$, it experiences a force of magnitude kv N due to air resistance, where k is a constant.

(a) Determine the dimensions of k. **[3]**

This part was answered extremely well with most candidates using the correct dimensions of force and velocity together with $[F] = [k] \times [v]$ to correctly obtain the dimensions of *k*.

Question 5 (b)

At time T s after projection P comes to rest. A formula approximating the value of T is

$$T = \frac{mu}{F} - \frac{kmu^2}{2F^2} + \frac{1}{3}k^2 m^\alpha u^\beta F^\gamma.$$

(b) Use dimensional analysis to find α, β and γ. **[4]**

It was clear that several candidates had a fundamental misunderstanding of dimensional analysis. Most candidates started off by writing $\mathrm{T} = \frac{\mathrm{MLT^{-1}}}{\mathrm{MLT^{-2}}} - \frac{\mathrm{MT^{-1}M(LT^{-1})^2}}{(\mathrm{MLT^{-2}})^2} + (\mathrm{MT^{-1}})^2\mathrm{M}^\alpha(\mathrm{LT^{-1}})^\beta(\mathrm{MLT^{-2}})^\gamma$ and while this is not strictly incorrect many did not realise that as each term of the equation had to have the same dimensions that it was therefore the case that $\mathrm{T} = (\mathrm{MT^{-1}})^2\mathrm{M}^\alpha(\mathrm{LT^{-1}})^\beta(\mathrm{MLT^{-2}})^\gamma$. A number of candidates who started with the statement that

$\mathrm{T} = \frac{\mathrm{MLT^{-1}}}{\mathrm{MLT^{-2}}} - \frac{\mathrm{MT^{-1}M(LT^{-1})^2}}{(\mathrm{MLT^{-2}})^2} + (\mathrm{MT^{-1}})^2\mathrm{M}^\alpha(\mathrm{LT^{-1}})^\beta(\mathrm{MLT^{-2}})^\gamma$ did not make any significant progress (or incorrectly treated it as an equation and therefore started to combine like terms). The most successful candidates were those that realised that $\mathrm{T} = (\mathrm{MT^{-1}})^2\mathrm{M}^\alpha(\mathrm{LT^{-1}})^\beta(\mathrm{MLT^{-2}})^\gamma$ and then formed and solved three equations in α, β and γ.

Misconception

The most common misconception in this part was assuming that *k* was dimensionless even though most candidates had correctly found the dimensions of *k* in the first part.

Section B overview

The questions in Section B were less straightforward in nature than those in Section A and also provided candidates an opportunity to apply their understanding of the mechanics content in a less structured setting.

Question 6 (a)

6 At time t seconds, where $t \geqslant 0$, a particle P has position vector $\mathbf{r}$ metres, where

$$\mathbf{r} = (2t^2 - 12t + 6)\mathbf{i} + (t^3 + 3t^2 - 8t)\mathbf{j}.$$

The velocity of P at time t seconds is $\mathbf{v}\,\mathrm{m\,s^{-1}}$.

(a) Find $\mathbf{v}$ in terms of t. **[1]**

Almost all candidates answered this part correctly by differentiating the given displacement vector to find **v** in terms of *t*.

Question 6 (b)

(b) Determine the speed of P at the instant when it is moving parallel to the vector $\mathbf{i} - 4\mathbf{j}$. **[5]**

Candidates struggled in this part to set up a correct equation to find the time when P was moving parallel to the given vector with many surprisingly setting a scalar product equal to 1 or considered the displacement rather than the velocity vector. Those candidates who realised that $\begin{pmatrix} 4t - 12 \\ 3t^2 + 6t - 8 \end{pmatrix} = k\begin{pmatrix} 1 \\ -4 \end{pmatrix}$ and solved for k were mostly successful. Of those candidates who correctly found that $t = 2$ (when P was moving in the given direction) most correctly went on to find the required speed (rather than leaving their answer as a velocity vector).

Misconception

Candidates are reminded that the direction of a moving body is governed by its velocity and not its displacement.

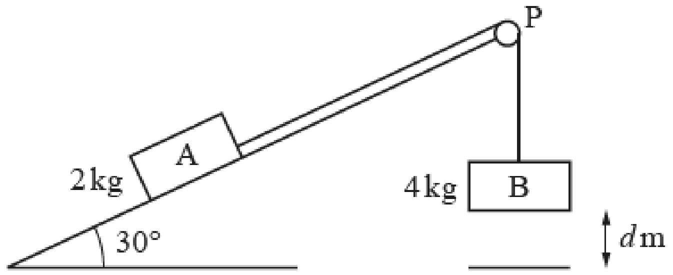
Question 6 (c)

(c) Determine the value of t when the magnitude of the acceleration of P is $20.2\,\mathrm{m\,s^{-2}}$. **[3]**

In comparison to part (b) this part was answered extremely well with almost all candidates correctly differentiating to find the acceleration vector and then using Pythagoras to find the required value of *t*.

Question 7 (a)

7 One end of a rope is attached to a block A of mass 2 kg. The other end of the rope is attached to a second block B of mass 4 kg. Block A is held at rest on a fixed rough ramp inclined at 30° to the horizontal. The rope is taut and passes over a small smooth pulley P which is fixed at the top of the ramp. The part of the rope from A to P is parallel to a line of greatest slope of the ramp. Block B hangs vertically below P, at a distance d m above the ground, as shown in the diagram.



Block A is more than d m from P. The blocks are released from rest and A moves up the ramp.

The coefficient of friction between A and the ramp is $\frac{1}{2\sqrt{3}}$.

The blocks are modelled as particles, the rope is modelled as light and inextensible, and air resistance can be ignored.

(a) Determine, in terms of g and d, the work done against friction as A moves d m up the ramp. **[3]**

This part was answered extremely well with most candidates correctly resolving perpendicular to the plane for A, then using both $F = \mu R$ and Work done = $F \times d$ correctly to find the work done against friction as A moved up the ramp.

Question 7 (b)

(b) Given that the speed of B immediately before it hits the ground is $1.75\,\mathrm{m\,s^{-1}}$, use the work–energy principle to determine the value of d. **[5]**

Several candidates did not read the question carefully and instead gave a solution using Newton's second law and constant acceleration formulae (therefore scoring no marks). The most common errors when applying the work-energy principle were either having the gravitational potential energy for A as 2*gd* rather than 2*gd*sin30, not including the result from part (a) or not including the KE gained by both A and B (most candidates forgot to include A in their calculation).

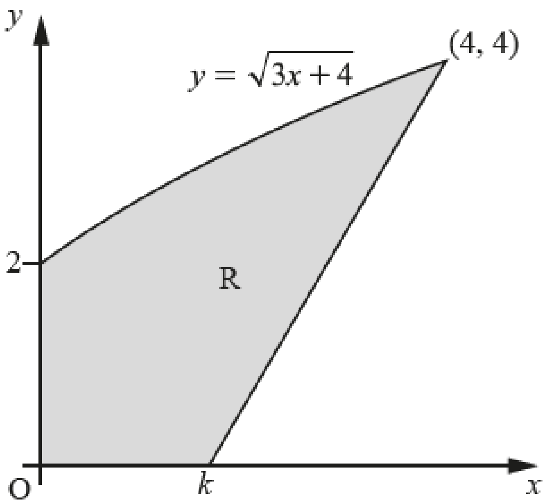
Question 7 (c)

(c) Suggest one improvement, apart from including air resistance, that could be made to the model to make it more realistic. **[1]**

Many candidates did correctly state one improvement that would make the model more realistic (for example, include the dimensions of the pulley, model the rope as elastic, or model the friction at the pulley) but some incorrectly gave a modelling assumption that already existed for the problem. Many candidates did not make it clear what the improvement was, for example, just stating 'do not model the blocks as particles' does not make it clear what form the improvement is going to take.

Question 8 (a)

8



The diagram shows the shaded region R bounded by the curve $y = \sqrt{3x+4}$, the x-axis, the y-axis, and the straight line that passes through the points $(k, 0)$ and $(4, 4)$, where $0 < k < 4$.

Region R is occupied by a uniform lamina.

(a) Determine, in terms of k, an expression for the y-coordinate of the centre of mass of the lamina. Give your answer in the form $\frac{a+bk}{c+dk}$, where a, b, c and d are integers to be determined. **[6]**

This question was not answered well with many candidates not finding the required expression for the *y*- coordinate of the centre of mass of the lamina. Many candidates used the extremely inefficient method of applying calculus in their attempt to calculate the centre of mass of the triangle below the line segment from (k, 0) to (4, 4), when all that was required was realising that the centre of mass of this triangle is at a distance of $\frac{4}{3}$ from the *x*-axis. The most logical way of tackling this problem was to calculate the area below the curve as $\frac{112}{9}$ (which could be done directly on a calculator), then work out $A\bar{y} = \frac{1}{2}\int_0^4 3x + 4\,\mathrm{d}x$ as 20 (again from a calculator) and combining this with the triangle below the line to give an equation of the form

$$20 - \frac{1}{2}(4)(4-k)\left(\frac{4}{3}\right) = \left(\frac{112}{9} - \frac{1}{2}(4)(4-k)\right)\bar{y}$$

and from this equation the required answer of $\bar{y} = \frac{42+12k}{20+9k}$ drops out relatively easily.

Question 8 (b)

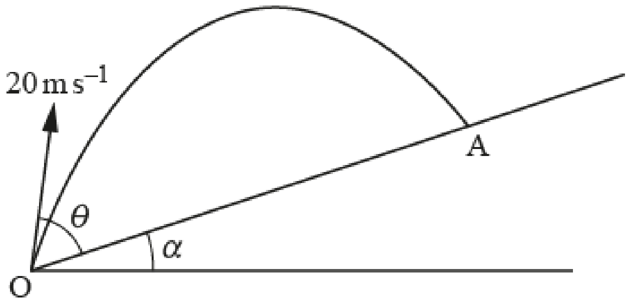
(b) Show that the y-coordinate of the centre of mass of the lamina cannot be $\frac{3}{2}$. **[2]**

Most candidates who attempted this part scored at least 1 mark for setting their answer to part (a) equal to 1.5 and solving for *k*. Some candidates who correctly derived that in this case *k* would equal 8 did not explicitly reference why this value was not possible (and hence the *y*-coordinate could not be 1.5).

Question 9 (a)

9 In this question take $g = 10$.

A small ball P is projected with speed $20\,\text{m s}^{-1}$ at an angle of elevation of $(\alpha+\theta)$ from a point O at the bottom of a smooth plane inclined at an angle α to the horizontal, where $\tan\alpha = \frac{5}{12}$ and $\tan\theta = \frac{3}{4}$. The ball subsequently hits the plane at a point A, where OA is a line of greatest slope of the plane, as shown in the diagram.



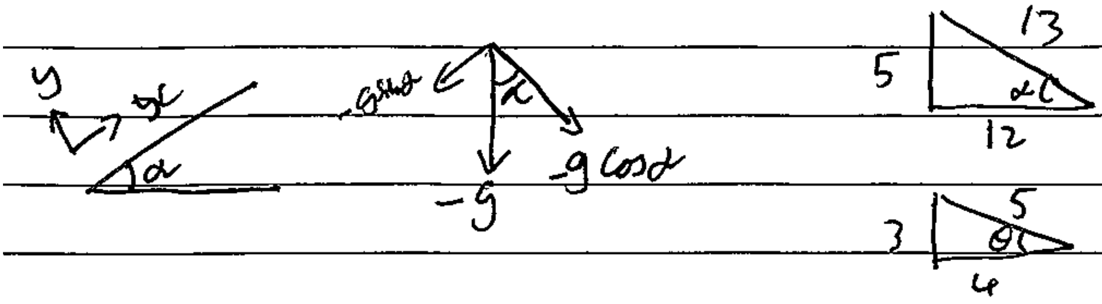
(a) Determine the following, in either order.

- The components of the velocity of P, parallel and perpendicular to the plane, immediately before P hits the plane at A.
- The distance OA. **[9]**

The responses to this unstructured part were very mixed with some candidates scoring full marks and others making very little progress. In the most successful responses candidates correctly applied

$s = ut + 0.5at^2$ perpendicular to the plane with $a = -g\cos\alpha$ and $u = 20\sin\theta$ to find the time of flight as 2.6. Those that did start by finding the time of flight usually went on to correctly apply $v = u + at$ parallel and perpendicular to the plane (to find the components of the velocity of P before impact) and then finish the problem by applying $s = ut + 0.5at^2$, this time parallel to the plane, to find the distance OA as 28.6. Although the question explicitly stated that the value of *g* should be taken to be 10 many candidates used the value of 9.8 and so lost 2 of the 9 marks available.

Exemplar 1



$$S_y = 20\sin\theta\, t - \frac{g}{2}\cos\alpha\, t^2 = 0$$

$$t \neq 0 \Rightarrow 20\sin\theta = \frac{g}{2}\cos\alpha\, t$$

$$t = \frac{20\sin\theta}{\frac{g}{2}\cos\alpha} = \frac{40\sin\theta}{g\cos\alpha} \text{ at } A \quad (= 2.6)$$

$$S_x = 20\cos\theta\, t - \frac{g\sin\alpha}{2}t^2$$

$$= 20\cos\theta\left(\frac{40\sin\theta}{g\cos\alpha}\right) - \frac{g\sin\alpha}{2}\left(\frac{40\sin\theta}{g\cos\theta}\right)^2$$

$$= 20\times\frac{4}{5}\left(\frac{26}{g}\right) - \frac{5g}{26}\left(\frac{26}{g}\right)^2$$

$$= \frac{416}{g} - \frac{130}{g} = \frac{286}{g} = 28.6\text{ m} = OA$$

y compt: $V = 20\sin\theta - g\cos\alpha\, t$

$= 12 - \frac{120}{13}\times 2.6 = -12\text{ ms}^{-1}$

x compt: $V = 20\cos\theta - g\sin\alpha\, t$

$= 16 - \frac{50}{13}\times 2.6 = 6\text{ ms}^{-1}$

So V parallel to plane $= 6\text{ ms}^{-1}$

V perpendicular to plane $= -12\text{ ms}^{-1}$

This response was fully correct. Each line of working was extremely clear as was the level of detail in the algebraic working to show the required results for this part.

Question 9 (b)

After P hits the plane at A it continues to move away from O. Immediately after hitting the plane at A the direction of motion of P makes an angle β with the horizontal.

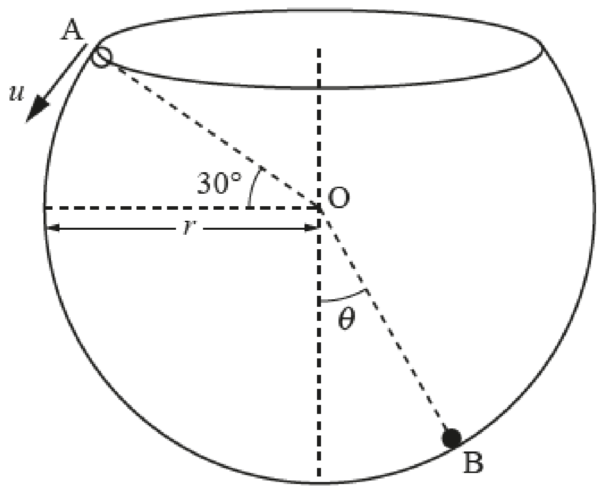
(b) Determine the maximum possible value of β, giving your answer to the nearest degree. **[3]**

The responses to this part were mixed, however, many candidates correctly realised that the greatest angle would occur when the collision with the plane was elastic. Only the highest scoring candidates realised though that the required angle was given by the expression,

$\arctan\left(\frac{[\dot{y}]}{\dot{x}}\right) + \arctan\left(\frac{5}{12}\right)$ with $\dot{x}, \dot{y}$ being the velocity components from part (a).

Question 10 (a)

10



A hollow sphere has centre O and internal radius r. A bowl is formed by removing part of the sphere. The bowl is fixed to a horizontal floor, with its circular rim horizontal and the centre of the rim vertically above O.

The point A lies on the rim of the bowl such that AO makes an angle of 30° with the horizontal (see diagram).

A particle P of mass m is projected from A, with speed u, where $u > \sqrt{\frac{gr}{2}}$, in a direction perpendicular to AO and moves on the smooth inner surface of the bowl.

The motion of P takes place in the vertical plane containing O and A. The particle P passes through a point B on the inner surface, where OB makes an acute angle θ with the vertical.

(a) Determine, in terms of m, g, u, r and θ, the magnitude of the force exerted on P by the bowl when P is at B. **[7]**

The responses to this question were mixed and while some candidates set their working out in a logical manner, examiners did report that at times it was difficult to follow some candidates' working. Surprisingly, a few candidates used $u^2 = \frac{gr}{2}$ for the initial speed of A.

Question 10 (b)

The difference between the magnitudes of the force exerted on P by the bowl when P is at points A and B is $4mg$.

(b) Determine, in terms of r, the vertical distance of B above the floor. **[4]**

This part was answered relatively well with many candidates correctly stating the magnitude of the force exerted on P at A as $\frac{mu^2}{r} - mg\cos 60$ and then setting up an equation in θ only (using the given information regarding the differences in magnitudes and different parts of the bowl) to then derive the required vertical distance of B above the floor.

Question 10 (c)

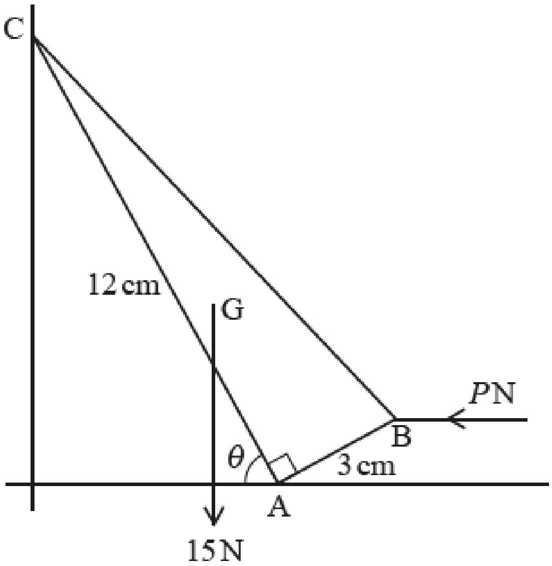
It is given that when P leaves the inner surface of the bowl it does not fall back into the bowl.

(c) Show that $u^2 > 2gr$. **[5]**

A common error in this part was to derive the result that $u > \sqrt{\frac{gr}{2}}$ which had already been given to the candidates in the opening stem of the question. This given result guarantees that the initial speed of P is sufficient for it not to leave the surface of the bowl before it reaches the top of the bowl again. A second common error was to consider the diameter of the rim of the bowl as being either *r* or 2*r* rather than the correct 2*r* cos30. In the most successful responses in this part candidates first found the time of flight, after it left the inner surface, for when P would be at the same horizontal level as the rim of the bowl and then compared the horizontal distance that would be travelled in this time with 2*r* cos30 (which leads nicely to the given result).

Question 11 (a)

11



The diagram shows the cross-section through the centre of mass of a uniform solid prism. The cross-section is a right-angled triangle ABC, with AB perpendicular to AC, which lies in a vertical plane. The length of AB is 3 cm, and the length of AC is 12 cm.

The prism is resting in equilibrium on a horizontal surface and against a vertical wall. The side AC of the prism makes an angle θ with the horizontal.

A horizontal force of magnitude P N is now applied to the prism at B. This force acts towards the wall in the vertical plane which passes through the centre of mass G of the prism and is perpendicular to the wall.

The weight of the prism is 15 N and the coefficients of friction between the prism and the surface, and between the prism and the wall, are each $\frac{1}{2}$.

(a) Show that the least value of P needed to move the prism is given by

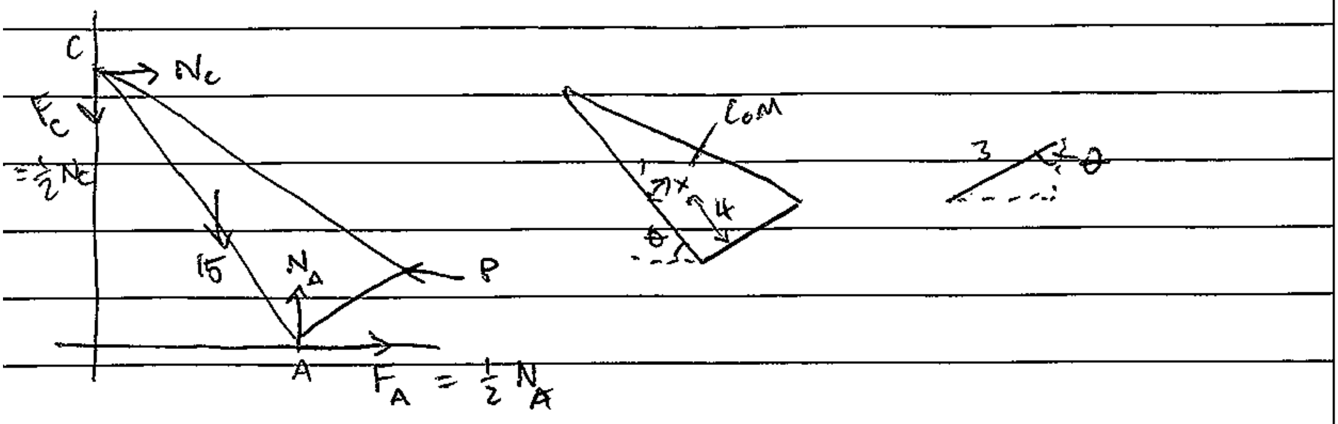
$$P = \frac{40\cos\theta + 95\sin\theta}{16\sin\theta - 13\cos\theta}.$$ **[8]**

Candidates found this the most demanding question on the paper with very few being able to derive the given result. While most candidates could resolve vertically and horizontally and apply $F = \mu R$ correctly, few were successful with taking moments. Those that took moments about either point A or B were far more successful than those that took moments about either point C or G. While the majority of candidates did take moments about A, very few had both required terms for the weight component (as if considering the weight having components parallel and perpendicular to AC then the moment equation consists of both a 4×(15cos θ) and a 1× (15sin θ) term). Even when a candidate had a correct moment equation very few could then eliminate all other unknown terms correctly to obtain an equation in P and θ only.

Assessment for learning

When a question involves multiple stages of working that include the need to introduce a certain number of variables that have not been defined in the question it is the responsibility of the candidate to define the variables they use. In this question many candidates did not make it explicitly clear which letters were being used for the forces acting on the prism. In this type of question candidates are advised to draw a diagram in the printed answer booklet with all forces clearly labelled to assist both theirs (and the examiners) understanding.

Exemplar 2



$\rightarrow$: $N_C + \frac{1}{2}N_A = P$

$\uparrow$: $N_A = \frac{1}{2}N_C + 15$

$N_C = 2N_A - 30$

$2.5N_A - 30 = P$, $N_A = \frac{2}{5}P + 12$

$\circlearrowleft A$: $4\cos\theta \times 15 + 12\cos\theta \times \frac{1}{2}N_C + 3\cos\theta P = 12\sin\theta N_C$

$\cancel{60\cos\theta + 6\cos\theta N_C + 3\cos\theta P = 12\cos\theta N_C}$

$60\cos\theta + 6\cos\theta N_C + 3\cos\theta P = 12\sin\theta N_C$

$3\cos\theta P = 12\sin\theta N_C - 6\cos\theta N_C - 60\cos\theta$

$\cancel{3\cos\theta P = 12\sin\theta}$

$\cos\theta P = 4\sin\theta N_C - 2\cos\theta N_C - 20\cos\theta$

$\cos\theta P = (2N_A - 30)(4\sin\theta - 2\cos\theta) - 20\cos\theta$

$\cos\theta P = \left(\frac{4}{5}P + 24 - 30\right)(4\sin\theta - 2\cos\theta) - 20\cos\theta$

$\cos\theta P = \frac{4}{5}P(4\sin\theta - 2\cos\theta) - 24\sin\theta + 12\cos\theta - 20\cos\theta$

This response scored 6 of the 8 marks available. The candidate drew a clear diagram at the top of the response which showed the correct forces acting on the prism. The candidate correctly resolved vertically and horizontally (and correctly stated that $F_A = 0.5N_A$ and $F_C = 0.5N_C$). When it came to taking moments about A the candidate made the common mistake of only considering the anticlockwise moment of the weight component (and not realising that there would be a clockwise moment too). They did, however, eliminate all other variables to arrive at an equation in terms of P and θ only.

Question 11 (b)

(b) Determine the range in which the value of θ must lie. **[4]**

Many candidates left this part blank or incorrectly considered $40cos\theta + 95\sin\theta$ (the numerator of the expression from part (a)). While some did correctly realise that $P > 0 \Rightarrow 16sin\theta - 13cos\theta > 0$ and hence $\theta > 39.09$ ... very few also realised that for equilibrium $tan\theta \leq 4$ and hence $\theta \leq 75.96$ ...

Question 12 (a) (i)

12 Two small uniform smooth spheres A and B are of equal radius and have masses m and λm respectively. The spheres are on a smooth horizontal surface.

Sphere A is moving on the surface with velocity $u_1\mathbf{i} + u_2\mathbf{j}$ towards B, which is at rest.
The spheres collide obliquely. When the spheres collide, the line joining their centres is parallel to $\mathbf{i}$.

The coefficient of restitution between A and B is e.

(a) **(i)** Explain why, when the spheres collide, the impulse of A on B is in the direction of $\mathbf{i}$. **[1]**

It was rare for candidates to give the correct explanation that the impulse acts in a direction parallel to the line of centres because the sphere were smooth.

Question 12 (a) (ii)

(ii) Determine this impulse in terms of λ, m, e and u_1. **[6]**

This part was answered extremely well with most candidates correctly applying the conservation of linear momentum and Newton's experimental law consistently to derive the correct equations for the velocity of A and B after collision. Most then solved these equations correctly and obtained a correct expression for the velocity of either A and/or B after impact. Sadly, when it came to working out the impulse several candidates used either an incorrect mass (so used m instead of λm together with the velocity of B) or gave only the magnitude of the impulse.

Question 12 (b)

The loss in kinetic energy due to the collision between A and B is $\frac{1}{8}mu_1^{\,2}$.

(b) Determine the range of possible values of λ. **[6]**

Candidates found the algebraic demands of this part very demanding with many stopping after setting up a correct equation. Only the highest scoring could obtain the equation,

$\frac{\lambda(1-e^2)}{1+\lambda} = \frac{1}{4}$ (or similar) and then consider the fact that $0 \leq e \leq 1$ (which would then lead to the required range of possible values of λ).

Question 13 (a)

13 A particle P of mass m is fixed to one end of a light spring of natural length a and modulus of elasticity man^2, where $n > 0$. The other end of the spring is attached to the ceiling of a lift. The lift is at rest and P is hanging vertically in equilibrium.

(a) Find, in terms of g and n, the extension in the spring. **[3]**

This part was answered extremely well with almost all candidates applying Hooke's law correctly to find the required extension in the spring.

Question 13 (b)

At time $t = 0$ the lift begins to accelerate upwards from rest. At time t, the upward displacement of the lift from its initial position is y and the extension of the spring is x.

(b) Express, in terms of g, n, x and y, the upward displacement of P from its initial position at time t. **[2]**

The responses to this part were mixed with only the highest scoring expressing the upward displacement of P correctly as $y + gn^{-2} - x$.

Question 13 (c)

(c) Given that $\ddot{y} = kt$, where k is a positive constant, express the upward acceleration of P in terms of $\ddot{x}$, k and t. **[1]**

Although several candidates left this (and part (b)) blank, a number did give the correct answer of $kt - \ddot{x}$.

Question 13 (d)

(d) Show that x satisfies the differential equation

$$\ddot{x}+n^2x=kt+g.$$ **[3]**

The responses to this part were mixed. While some candidates correctly set up the equation of motion as $\frac{man^2x}{a}-mg=m(kt-\ddot{x})$ and were then successful in deriving the given differential equation many started with the acceleration as *a* or appeared to be trying to work backwards from the given result.

Question 13 (e)

(e) Verify that $x=\frac{1}{n^3}(knt+gn-k\sin(nt))$. **[4]**

While most candidates did indeed verify this solution by substituting it into the differential equation from part (d) and furthermore checked that when $t = 0$, $\frac{\mathrm{d}x}{\mathrm{d}t}=0$ and $x = gn^{-2}$, several candidates instead solved the given differential equation or did not check the initial conditions.

Misconception

The word 'Verify' does not have the same mathematical meaning as 'Show that' – many candidates in this part solved the second order differential equation and used the given conditions to calculate the two arbitrary constants. While this approach could score all 4 marks it was a time-consuming way of tackling the problem. Furthermore, the solution to differential equations (apart from those found via SHM) are not required for this unit and it may not always be possible to solve the differential equation that candidates are being asked to verify in this unit.

Exemplar 3

$\frac{d^2x}{dt^2} + n^2x = kt + g$

~~Use~~ $x = \frac{1}{n^3}(knt + gn - k\sin(nt))$

$= \frac{k}{n^2}t + \frac{g}{n^2} - \frac{k\sin(nt)}{n^3}$ $\quad \frac{k\sin(nt)}{n^3} = \frac{kt}{n^2} + \frac{g}{n^2} - x$

$\cancel{\frac{dx}{dt}}$ $\dot{x} = \frac{k}{n^2} - \frac{k}{n^2}\cos(nt)$ $\quad \frac{k\sin(nt)}{n} = kt + g - xn^2$

$\frac{d^2x}{dt^2} = \ddot{x} = \frac{k}{n}\sin(nt) = kt + g - xn^2$

$= \cancel{\left(\frac{kt}{n^2} + \frac{g}{n^2} - x\right)}$ $\quad \therefore \ddot{x} + n^2x = kt + g$

$= \cancel{\left(\frac{kt + g - xn^2}{n^2}\right)}$ as required.

$t = 0,\ x = \frac{g}{n^2}$

$x = \frac{1}{n^3}(0 + gn - 0)$

$= \frac{g}{n^2}$ as required.

This response scored 3 of the 4 marks available for this part. The candidate has correctly shown that the given expression for *x* satisfies the given differential equation and has also shown that at time *t* = 0, the expression for *x* is the required gn^{-2}. However, they did not verify that $\dot{x} = 0$ at time *t* = 0.

Question 13 (f)

(f) By considering $\dot{x}$ comment on the motion of P relative to the ceiling of the lift for all times after the lift begins to move. **[2]**

Very few candidates correctly stated that as $\dot{x} \geq 0$ for all values of *t* that it could then be inferred that P does not move closer to the ceiling of the lift in the subsequent motion. Even though the question explicitly directed candidates to do so, many gave an answer that did not include consideration of $\dot{x}$ at all.

Supporting you

Teach Cambridge

Make sure you visit our secure website **Teach Cambridge** to find the full range of resources and support for the subjects you teach. This includes secure materials such as set assignments and exemplars, online and on-demand training.

Don’t have access? If your school or college teaches any OCR qualifications, please contact your exams officer. You can forward them this link to help get you started.

Reviews of marking

If any of your students’ results are not as expected, you may wish to consider one of our post-results services. For full information about the options available visit the OCR website.

Access to Scripts

For the June 2023 series, Exams Officers will be able to download copies of your candidates' completed papers or 'scripts' for all of our General Qualifications including Entry Level, GCSE and AS/A Level. Your centre can use these scripts to decide whether to request a review of marking and to support teaching and learning.

Our free, on-demand service, Access to Scripts is available via our single sign-on service, My Cambridge. Step-by-step instructions are on our website.

Keep up-to-date

We send a monthly bulletin to tell you about important updates. You can also sign up for your subject specific updates. If you haven’t already, sign up here.

OCR Professional Development

Attend one of our popular CPD courses to hear directly from a senior assessor or drop in to a Q&A session. Most of our courses are delivered live via an online platform, so you can attend from any location.

Please find details for all our courses for your subject on **Teach Cambridge**. You'll also find links to our online courses on NEA marking and support.

Signed up for ExamBuilder?

ExamBuilder is the question builder platform for a range of our GCSE, A Level, Cambridge Nationals and Cambridge Technicals qualifications. Find out more.

ExamBuilder is **free for all OCR centres** with an Interchange account and gives you unlimited users per centre. We need an Interchange username to validate the identity of your centre’s first user account for ExamBuilder.

If you do not have an Interchange account please contact your centre administrator (usually the Exams Officer) to request a username, or nominate an existing Interchange user in your department.

Active Results

Review students’ exam performance with our free online results analysis tool. It is available for all GCSEs, AS and A Levels and Cambridge Nationals.

Find out more.

Need to get in touch?

If you ever have any questions about OCR qualifications or services (including administration, logistics and teaching) please feel free to get in touch with our customer support centre.

Call us on
01223 553998

Alternatively, you can email us on
support@ocr.org.uk

For more information visit
- **ocr.org.uk/qualifications/resource-finder**
- **ocr.org.uk**
- **facebook.com/ocrexams**
- **twitter.com/ocrexams**
- **instagram.com/ocrexaminations**
- **linkedin.com/company/ocr**
- **youtube.com/ocrexams**

We really value your feedback

Click to send us an autogenerated email about this resource. Add comments if you want to. Let us know how we can improve this resource or what else you need. Your email address will not be used or shared for any marketing purposes.

I like this **I dislike this**

Please note – web links are correct at date of publication but other websites may change over time. If you have any problems with a link you may want to navigate to that organisation's website for a direct search.

CAMBRIDGE
UNIVERSITY PRESS & ASSESSMENT