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Examiners' report

FURTHER MATHEMATICS B (MEI)

H645

For first teaching in 2017

Y433/01 Summer 2023 series

Version 1

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Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate answers is also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from OCR.

Paper Y433/01 series overview

This is a minor option paper for MEI Further Mathematics. It assesses Modelling with Algorithms under the three strands of algorithms, networks and linear programming.

The majority of candidates made some attempt at all the questions, although there does appear to be some evidence to suggest that a few candidates struggled to complete the paper in the time available and did not complete Question 6.

Candidates who did well on this paper generally:	Candidates who did less well on this paper generally:
• attempted all questions and gave responses that were appropriate for the number of marks available • worked neatly and explained their working where appropriate • answered written responses precisely and unambiguously.	• did not give written responses to questions asking for an explanation • did not set out work in a clear and logical way, with subsequent misreading of their own letters or numerical values • did not read the questions carefully enough.

Assessment for learning

Centres should advise candidates that erased work can sometimes still show through on the scanned copy and may lead to ambiguities especially when it has been overwritten or in diagrams.

Question 1 (a)

1 Ten suitcases are to be transported in containers. Each container can hold a maximum of n kg, where n is a positive integer.

The total weight of the ten suitcases is 216 kg.

You are given that at least 4 containers are needed to transport all ten suitcases.

(a) Determine the maximum value of n for which **any** set of 10 suitcases with total weight 216 kg needs at least 4 containers. **[2]**

Some candidates were distracted by the value ten. Most calculated either 216/4 = 54 or 216/3 = 72, these are the minimum sizes for 4 and 3 containers. A few candidates subtracted 1 (or 3) from 216 to account for the other containers.

Using containers of size 72 could give a packing with just 3 containers, so the maximum value for which at least 4 containers are needed is n = 71.

Question 1 (b)

The exact weights, in kg, of the suitcases are:

17	23	18	14	26	21	24	15	31	27

(b) Apply the quick sort algorithm to sort the list of numbers into **descending** order. You should use the first value as the pivot for each sublist. **[3]**

Most candidates applied the quick sort algorithm correctly. A few stopped after three passes, when the list was sorted but there was still a sublist of length 2.

Question 1 (c)

The first fit decreasing algorithm is applied to the sorted list of numbers. The following allocation of suitcases to containers is obtained.

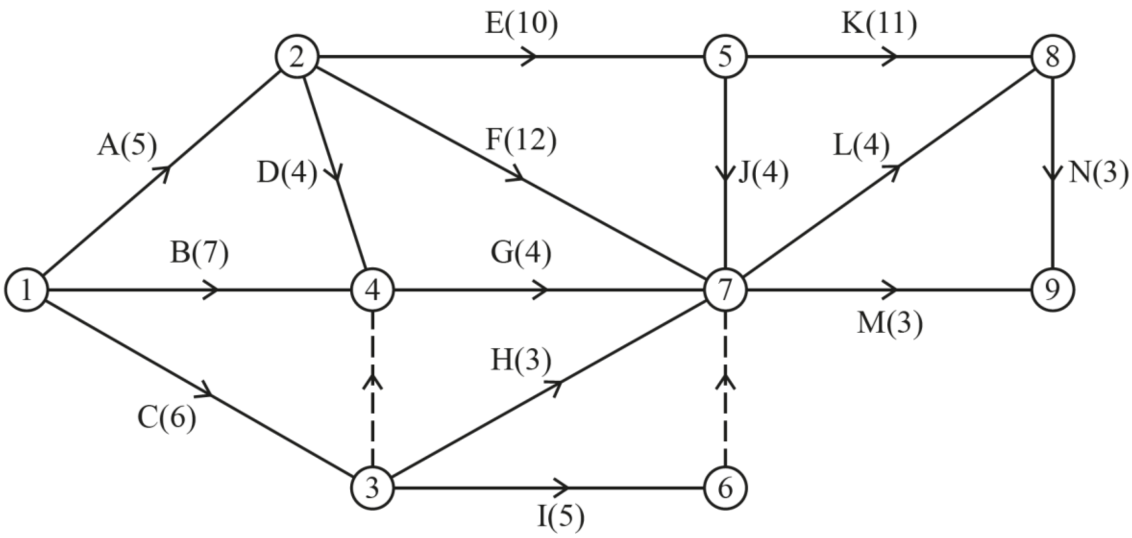
Container 1:	31	27	
Container 2:	26	24	18
Container 3:	23	21	17
Container 4:	15	14	

(c) Determine the possible values of n that are consistent with this result from applying the first fit decreasing algorithm to the sorted list of these weights. **[2]**

Many candidates used the total for container 2 to deduce that n must be at least 68. However, fewer candidates spotted that if n = 71 then 21 would fit into container 2. The possible values of n were 68, 69 and 70.

Question 2 (a)

2 The diagram shows an activity network for a project. The arc weights show activity durations in hours. The numbers in circles are event numbers.



(a) Explain the significance of the dummy activity from event 3 to event 4. **[1]**

Candidates needed to be specific when describing the activities that needed to have finished before others could start.

Activity G cannot start until B, C and D have all finished; activities H and I also need C to have finished before they can start but do not depend on B or D.

Alternatively, activity G depends on both B and C, which both start from event 1, so for uniqueness (labelling) activities B and C cannot finish at the same event and a dummy is needed.

Question 2 (b)

(b) Using the diagram in the Printed Answer Booklet, carry out a forward pass and a backward pass through the entire network to find the following.

- The minimum completion time for the project.
- The critical activities.

[5]

There were many correct responses. Some candidates made slips in dealing with the dummy activities on the backward pass.

Question 2 (c)

The duration of activity J changes to x hours.

(c) Determine the following, in terms of x where necessary.

- The new early event time for event 7.
- The new late event time for event 7.

[3]

Most candidates realised that the new early event time for event 7 was $15 + x$ and some recognised that if x was small ($x \leq 2$) the early event time would be 17.

Several candidates said that the new late event time for event 7 was still 22, but some realised that for large values of x ($x \geq 7$) activity 7 would become critical and the late event time would be $15 + x$.

Question 2 (d)

It is given that the total float for activity K is now 5 hours.

(d) Find the value of x. **[1]**

Several candidates were able to write down the new value of x as 12. Some assumed that K having a float of 5 hours meant that x became 5 or that x increased by 5 to become 9.

If activity K has 5 hours of float then it is no longer critical and activities J and L become critical instead. The early event time for event 5 is still 15 and the duration of activity K is still 11, so the LET for event 8 must have changed from 26 to 31. This means that $19 + x = 31$ and hence $x = 12$.

Since the question asked candidates to 'find' it was sufficient to write the value 12.

Question 3 (a)

3 A **directed** network consists of eight nodes, A–H, and fifteen arcs.

A shortest path from A to H needs to be found for this network.

The objective for an LP formulation for finding the shortest path from A to H is to minimise the objective function given by

$$19\mathrm{AB}+37\mathrm{AD}+27\mathrm{AE}+41\mathrm{BC}+15\mathrm{BD}+23\mathrm{EF}+51\mathrm{AF}+20\mathrm{DC}+14\mathrm{DF}+32\mathrm{DG}+19\mathrm{CH}+10\mathrm{CG}$$
$$+42\mathrm{DH}+27\mathrm{FH}+6\mathrm{GH}$$

(a) Draw the network in the Printed Answer Booklet. **[3]**

Almost all candidates were able to draw the correct network although some did not show the arcs as being directed.

Question 3 (b)

(b) Complete the LP formulation associated with the shortest path problem by listing the constraints. **[3]**

There were several correct responses. Many candidates were able to deal with the source (AB + AD + AE + AF = 1) and the sink (CH + DH + FH + GH = 1), although some left out one of the arcs. Several candidates were able to deal with the intermediate vertices, apart from the occasional slip, but some treated the arcs as being undirected and some wrote the constraints as equations with variables on both sides.

Question 3 (c)

(c) Apply Dijkstra's algorithm to the completed network from part **(a)** to find the shortest path from A to H. **[5]**

There were many correct responses. Some candidates forgot to write the shortest path, and a few gave the length of the shortest path instead of the path itself (the route).

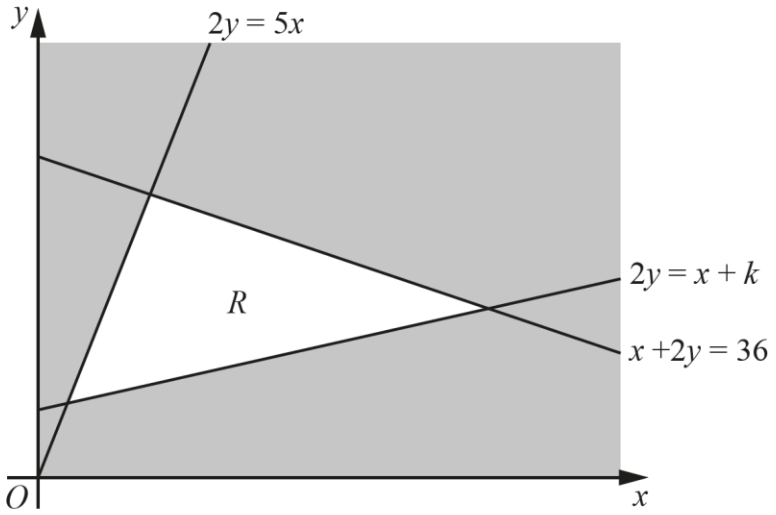
Some candidates left out one or more of the temporary labels, suggesting that they had not really used Dijkstra's algorithm. A few started the order of labelling from 0 (0 to 7 instead of 1 to 8), this was allowed but should be discouraged.

Assessment for learning

If a question asks for the use of an algorithm, the response must show sufficient working to make it clear that the algorithm has been used correctly.

Question 4

4



The diagram shows the constraints of a linear programming problem in which the objective is to maximise $P = x + ky$, where k is a positive constant.

The feasible region, R, is the unshaded region together with its boundaries.

You are given that the optimal value of P is 24.96.

Determine the following, explaining your reasoning.

- The possible value(s) of k.
- The corresponding coordinates of the optimal vertex.

[7]

With unstructured responses candidates need to set out their working coherently. Several candidates muddled themselves as to which lines they were using.

Candidates may have ignored the intersection of $2y = x + k$ and $2y = 5x$ at $(k/4, 5k/8)$

The intersection of $2y = 5x$ and $x + 2y = 36$ at (6, 15) could be found from a calculator or by solving the equations simultaneously. Most candidates found this vertex correctly.

The intersection of $2y = x + k$ and $x + 2y = 36$ at $(18 – k/2, 9 + k/4)$ could be found by solving the equations simultaneously. Several candidates found this vertex although some made slips in their working.

Having found the vertices these could then be substituted into the profit equation x + ky = 24.96 to give the values of k at the vertices. Negative solutions for k could be ignored. This gives k = 1.264 at (6, 15) and k = 0.8 at $(18 – k/2, 9 + k/4)$.

The final part of the solution was to substitute k = 0.8 into $(18 – k/2, 9 + k/4)$ to give the coordinates (17.6, 9.2) and then show that (6, 15) is not optimal, for example by calculating $P = x + ky$ at (6, 15) with k = 0.8 or at (17.6, 9.2) with k = 1.264.

Some candidates used the profit equation much earlier in their solution, and sometimes this led to them eliminating k from the coordinates of the intersection of $2y = x + k$ and $x + 2y = 36$. Usually these candidates were able to recover to achieve most of the marks.

Some candidates found the coordinates and solved for k but did not use $k = 0.8$ to find the coordinates (17.6, 9.2) numerically. Several candidates found the values of k and found the coordinates numerically but did not deduce that the optimal solution was (17.6, 9.2) when $k = 0.8$.

Some candidates claimed that the optimum is always the point that is furthest from the origin, this is not true – it depends on the slope of the profit line.

Exemplar 1

$24.96 = x + ky$

$k = 2y - x$

$29.96 = x + (2y - x)y$

$\cancel{y = 36 - x}$

$24.96 = x + ky$ (1)

$k = 2y - x$ (2)

$x = 36 - 2y$ (3)

Sab (3)

$24.96 = 36 - 2y + hy$ (4)

$h = 2y - (36 - 2y)$ (5)

$h = 4y - 36$

(4)(5) $y = \frac{h}{4} + 9$ (4) $y = \frac{-11.04}{h-2}$

$\frac{h}{4} + 9 = \frac{-11.04}{h-2}$

$h(h-2) + 9(h-2) = -11.04 \times 4$

$h^2 + 7h + 13.84 = 0$

$h = \frac{-7 \pm \sqrt{7^2 - (4 \times 13.84 \times 1)}}{2}$

$h = \frac{-35 \pm \sqrt{159}i}{10}$

(answer space continued on the next page)

$$-11.04 = y(h-2)$$

$$y = \frac{h+36}{4}$$

$$y = \cancel{\frac{2-h}{11.04}} \quad \frac{11.04}{2-h}$$

$$\frac{h+36}{4} = 4\,\frac{11.04}{2-h}$$

$$(h+36)(2-h) - (11.04 \times 4) = 0$$

$$-h^2 - 34h - 44.16$$

$$h = -1.35,\ -32.64$$

$$h \geq 0$$

This candidate has started by using $2y = x + k$ to eliminate k from the profit equation and formed a new equation that must be satisfied by optimal solutions on the line $2y = x + k$. They have then used the equation $x + 2y = 36$ to eliminate x from the line $2y = x + k$. This gives them the y coordinate of the intersection of $2y = x + k$ and $x + 2y = 36$ and a second expression relating y and k at the optimal point.

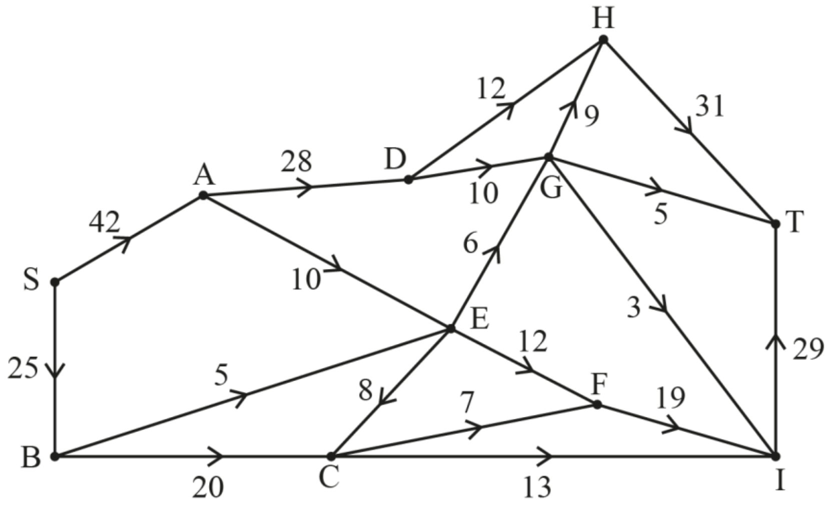
Next, they have eliminated y to form a quadratic equation in k and solved it but discovered that their solution is not real. This candidate has then redone this work, without the error this time, and achieved a correct expression $(k + 36)(2 - k) - 4(11.04) = 0$. In multiplying out the bracket the candidate has lost the constant 72 so their solutions are still wrong (which they know because both solutions are negative).

When an answer is not 'scaffolded' by the question, candidates need to make sure that they set out their response clearly so that they can follow what they have done and track how each step will progress to a solution.

This candidate has worked with the algebra and recognised that their attempts to solve the equations has not been successful. However no link has been made to the graph, with the x-coordinate of the intersection $2y = x + k$ and $x + 2y = 36$ not found, and no attempt to find the point (6, 15) where $2y = 5x$ intersects $x + 2y = 36$.

Question 5 (a)

5



The diagram represents a system of pipes through which a fluid flows continuously from a source to a sink. The weights on the arcs show the capacities of the pipes in litres per minute. All flows in pipes take integer values.

(a) By considering the **sink** node, explain why the maximum flow through the network cannot be greater than 65 litres per minute. **[1]**

Candidates needed to do more than just say that the maximum flow into T is 65. Most candidates gave the calculation 31 + 5 + 29 = 65 to show that the flow into T is 65 and some stated that the maximum flow in arcs GT, HT and IT is 65 litres per minute.

Some candidates involved other vertices to explain that 65 was not achievable, this was not necessary.

Some candidates found the flow from the source, S, and said that since this was 63 the maximum flow could not be 65. This was true but did not answer the question as set.

Question 5 (b) (i)

(b) **(i)** The cut α partitions the vertices into the sets {S, A, B}, {C, D, E, F, G, H, I, T}. Calculate the capacity of cut α. **[1]**

Most candidates were able to calculate the capacity as 28 + 10 + 5 + 20 = 63

Question 5 (b) (ii)

(ii) The cut β partitions the vertices into the sets $\{S, A, B, C, D\}$, $\{E, F, G, H, I, T\}$. Calculate the capacity of cut β. **[1]**

Most candidates were able to calculate the capacity as 12 + 10 + 10 + 5 + 7 + 13 = 57. A few candidates did not use a flow of 0, from the sink to the source, in arc EC.

Question 5 (c)

(c) Using **only** the capacities of cuts α and β explain what can be deduced about the maximum possible flow through the system. **[1]**

Most candidates realised that this required the smaller of their previous two answers.

Question 5 (d)

An LP formulation is set up to find the maximum flow through the network.

(d) Explain why a possible objective function for the LP formulation is $\mathrm{AD}+\mathrm{EG}+\mathrm{EF}+\mathrm{CF}+\mathrm{CI}$ which is to be maximised. **[1]**

Several candidates realised that this expression came from the cut that separated {S, A, B, C, E} from {D, F, G, H, I, T} and that the flow across any cut must equal the flow through the network.

Question 5 (e)

The complete LP was run in an LP solver, and it was found that only arcs AE, BE, CF, DH, DG, GH, GT, and IT were saturated and that there was zero flow through arc EC.

(e) By completing the diagram in the Printed Answer Booklet, determine the maximum value of the flow through the network. **[3]**

There were many correct responses. Most candidates were able to transfer the given information about saturated arcs and zero flow. Some candidates tried to flow 6 litres per minute along EG, which contradicted the statement that the given arcs were the only ones that were saturated.

Question 5 (f)

(f) Use a suitable cut to prove that this is the maximum flow. **[2]**

Most candidates realised that they needed to find a minimum cut and use the max flow = min cut theorem. Several were able to find the minimum cut but there were also several candidates who confused flows and capacities.

Some candidates used the given saturated arcs to find the cut {S, A, B, C, D, E, F, G, I}, {H, T} and some stated that this cut had capacity 55 but then did nothing with it.

Having found this cut with capacity 55 means that the capacity of the minimum cut must be less than or equal to 55 and from the previous part the maximum flow is greater than or equal to 55, and hence, using the maximum flow – minimum cut theorem, 55 litres per minute is the maximum flow.

Misconception

? Some candidates confused the flow across a cut with the capacity of the cut (the maximum possible flow across the cut (without consideration of any other arcs)).

Question 6 (a)

6 The initial simplex tableau for a maximisation LP problem is shown in **Fig. 6.1**.

Fig. 6.1

P	x	y	z	s_1	s_2	s_3	RHS
1	−2	−1	−3	0	0	0	0
0	1	2	−1	1	0	0	b
0	−3	0	2	0	1	0	50
0	1	−1	2	0	0	1	55

It is given that b is a **positive** constant.

(a) Formulate the information given in **Fig. 6.1** as an LP problem by completing the following.

- State the objective function.
- List the constraints as simplified inequalities with integer coefficients.

[2]

Most candidates were able to state the objective function as $P = 2x + y + 3z$ and to give the constraints as inequalities without the slack variables. There were a few slips, such as writing $-3x + 2y \le 50$ instead of $-3x + 2z \le 50$.

Question 6 (b)

After **two** iterations of the simplex method the tableau shown in **Fig. 6.2** is produced.

Fig. 6.2

P	x	y	z	s_1	s_2	s_3	RHS
1	0	$-\frac{21}{8}$	0	0	$-\frac{1}{8}$	$\frac{13}{8}$	$\frac{665}{8}$
0	0	$\frac{15}{8}$	0	1	$\frac{3}{8}$	$\frac{1}{8}$	$b+\frac{205}{8}$
0	0	$-\frac{3}{8}$	1	0	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{215}{8}$
0	1	$-\frac{1}{4}$	0	0	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{5}{4}$

(b) Write down the value of each variable after the second iteration. **[1]**

Most candidates understood how to use the tableau to read off the values of the variables. Some only gave the values of *x*, *y* and *z* and some only gave the values of variables that were non-zero.

Question 6 (c)

(c) Explain how the tableau in **Fig. 6.2** shows that the solution obtained after the second iteration is not optimal. **[1]**

This was usually correct, although some candidates are still confused about the difference between 'negative' and 'not positive' and some referred to the objective function instead of the row.

Question 6 (d)

After a third iteration of the simplex method the resulting tableau does give an optimal solution to the problem. Furthermore, it is given that in this optimal solution the value of y is three times the value of x.

(d) By performing the third iteration of the simplex method, using the tableau in the Printed Answer Booklet, determine the optimal value of the objective function for this LP problem. **[6]**

Several candidates carried out the iteration correctly, although inevitably there were some slips, however some did not realise that the information in the stem ('in this optimal solution the value of *y* is three times the value of *x*') meant that they could then solve for *b* and hence find the numerical value of *P* at the optimal solution.

Assessment for learning

When performing an iteration of the simplex algorithm it is usually easier to use fractions rather than decimals, unless the decimals are exact to 2 d.p.

Rounded decimals can generate errors in subsequent iterations.

Question 6 (e)

The LP problem is modified so that b is now a **negative** constant.

(e) Explain why the simplex method cannot be used to solve this modified problem. **[1]**

Many candidates described why the tableau would not be in the required format rather than explaining why this means that the simplex method cannot be used without modification.

The standard simplex method starts with the origin as the initial basic feasible solution. However if b is negative then the constraint $x + 2y - z \leq b$ is not satisfied at (0, 0, 0) so the origin is not a feasible solution.

Question 6 (f)

(f) The two-stage simplex method is to be used to solve this modified problem. Complete the initial tableau in the Printed Answer Booklet so that the two-stage simplex method may be used to solve this modified problem. **[4]**

Many candidates were confused by b being negative and were reluctant to use $-b$ (or $|b|$) in the RHS column of their augmented tableau. Some defined a new variable to represent $-b$ but most just used b and appeared to have forgotten that it was negative.

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