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Examiners' report

FURTHER MATHEMATICS B (MEI)

H645

For first teaching in 2017

Y435/01 Summer 2023 series

Version 1

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Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate answers is also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from OCR.

Paper Y435/01 series overview

This was the second full sitting of this paper, which covers the Extra Pure content of H645, A Level Further Mathematics B (MEI), since June 2019 (2020 and 2021 exams were sat in the Autumn by very small cohorts of students).

The paper comprised five compulsory questions, covering almost every area of the specification, which nearly all candidates managed to attempt. There was little evidence of time pressure. Overall standards were quite high and, although presentation was variable, solutions were generally set out well.

Throughout the paper it was expected that candidates were able to demonstrate a high level of understanding of the (sometimes abstract) topics. Several questions required proof of a given result and the advice is that candidates should provide a detailed explanation of all their working; the examiner should not need to fill in any gaps of reasoning or calculation even if some of the steps appear obvious.

The following questions were answered well by a majority of candidates:

Questions 1, 2 (a), 2 (b) (i), 3, 4 (d), 5 (a), 5 (b) (i)

The following questions proved to be more challenging:

Questions 2 (b) (ii), 2 (c), 4 (a), 4 (b) (i) and (ii), 4 (c) (i) and (ii), 5 (b) (ii)

In particular, a large number of candidates seemed to have real difficulty in understanding the type and level of response required to do well when attempting Group Theory questions.

Candidates who did well on this paper generally:	Candidates who did less well on this paper generally:
• set out their working neatly, clearly and logically • explained their reasoning rigorously • included every step, even seemingly trivial ones • used correct technical language and notation • answered the question asked in the specified manner • followed standard methodologies.	• left out their working or presented it in a poorly or illogically structured way • skipped steps or explanations in their reasoning • lacked rigour in their explanations • used technical language or notation imprecisely or incorrectly • used incorrect or inappropriate methodology • did not answer the actual question asked.

Question 1

1 A surface is defined in 3-D by $z = 3x^3 + 6xy + y^2$.

Determine the coordinates of any stationary points on the surface. **[7]**

This question was generally well attempted and obtaining full marks was common. There was quite a large number of misreads, often of the powers of *x* and *y*, and candidates would be well advised, in general, to check that they have copied the information from any question correctly.

Key point call out – presentation of answers

Check what the question requires. If it asks for co-ordinates then do not give a position vector.

Question 2 (a)

2 A sequence is defined by the recurrence relation $4t_{n+1} - t_n = 15n + 17$ for $n \geqslant 1$, with $t_1 = 2$.

(a) Solve the recurrence relation to find the particular solution for t_n. **[7]**

This question was generally well attempted and obtaining full marks was common. Most candidates knew how to find the complementary function, which trial function to use and how to use it and how to combine the two. Apart from careless errors the most common mistake was to attempt to find the value of the arbitrary constant by substituting the initial conditions into the complementary function rather than the general solution.

Misconception

The initial condition must be substituted into the general solution, not the complementary function.

Assessment for learning

Candidates would be well advised to use *standard* techniques when dealing with recurrence relations.

Question 2 (b) (i)

Another sequence is defined by the recurrence relation $(n+1)u_{n+1} - u_n^2 = 2n - \frac{1}{n^2}$ for $n \geqslant 1$, with $u_1 = 2$.

(b) **(i)** Explain why the recurrence relation for u_n **cannot** be solved using standard techniques for non-homogeneous first order recurrence relations. **[1]**

This question was generally well attempted and the majority of candidates obtained the mark. Those that did not obtain the mark often wrote too much and ended up either stating something incorrect (e.g. 'the equation is homogeneous') or contradicting themselves in some way.

Assessment for learning

Candidates should judge the level of detail required in the response by the wording of the question, the tariff for the question and the amount of space available in the answer booklet. Knowledge of the specification is also important. In this case, with only 1 mark available and a limited amount of space, simply stating that the relation was 'non-linear' (as per the spec requirement for understanding the meaning of the word (Xs2)) was sufficient.

Question 2 (b) (ii)

(ii) Verify that the particular solution to this recurrence relation is given by $u_n = an + \frac{b}{n}$ where a and b are constants whose values are to be determined. **[5]**

Candidates seemed to find it difficult to carry out **verification** of given recurrence relation solutions. Candidates should bear in mind that a suggested solution must meet **two** conditions. Firstly, it must 'work' in the recurrence relation and secondly it must satisfy the initial conditions. Too many candidates simply focused on finding the correct values of *a* and *b* without either <u>completely</u> checking that the solution worked or that the initial condition was satisfied. Full marks on this question part were rarely credited.

Question 2 (c)

A third sequence is defined by $v_n = \frac{t_n}{u_n}$ for $n \geqslant 1$.

(c) Determine $\lim_{n \to \infty} v_n$. **[2]**

Candidates seemed to find it difficult to approach questions involving limits with the rigour required. It was rare for 2 marks to be credited here and it was common to see statements like $\lim(v_n) \to 5$ or $v_n = 5$.

Misconception

The limit of a sequence is a number. The limit itself does not tend to anything; it is just a number. Candidates can demonstrate understanding of these difficult concepts by using the notation and language accurately and correctly.

Question 3

3 A surface, S, is defined by $g(x, y, z) = 0$ where $g(x, y, z) = 2x^3 - x^2y + 2xy^2 + 27z$. The normal to S at the point $\left(1, 1, -\frac{1}{9}\right)$ and the tangent plane to S at the point $(3, 3, -3)$ intersect at P.

Determine the position vector of P. **[8]**

This question was generally well attempted and it was common for full marks to be credited. However, as with Question 1 misreads were surprisingly common. It was disappointing to note that many candidates were not properly aware of what the grad vector represents and many seemed to be confused with similar, but different, concepts from calculus in 2D.

Key point call out – the meaning of grad

Grad represents a normal vector to a surface at any point and can therefore be used directly either to find the equation of the normal line at the point or as the normal to the tangent plane to the point.

Question 4 (a)

4 The set G is given by $G = \{\mathbf{M}: \mathbf{M} \text{ is a real } 2\times2 \text{ matrix and } \det\mathbf{M} = 1\}$.

(a) Show that G forms a group under matrix multiplication, $\times$. You may assume that matrix multiplication is associative. **[5]**

The responses to this question were generally disappointing and the modal number of marks credited was only 2/5. It seems that many candidates lacked the rigour required to answer questions in Group Theory which is a highly abstract topic. Unfortunately, too often candidates relied on 'known' properties of matrices. For example, in this question it was not sufficient to state that 'an identity exists since **I** is known to be the identity matrix'. Candidates need to demonstrate that **I** is actually a member of the set G, so as a minimum they need to state explicitly that det**I** = 1 and so **I** is in G. They should also show that **I** does indeed satisfy the requirements of the identity axiom (i.e. that **AI** = **IA** = **A**).

The mark for the closure axiom was almost never given; candidates are being asked to show that if **M** and **N** are both in G then **MN** is also in G. Many candidates showed that **MN** is a real 2 by 2 matrix and that det(**MN**) = 1 but then neglected to mention that this meant that **MN** is in G which is surely the fundamental issue of closure so they never explicitly showed that they understood the closure axiom.

Exemplar 1

$ad - bc = 1$

identity $= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ for all real 2x2 matrix.

inverse $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1}$: M^{-1}

inverse exists for the matrix as long as there is a determinent.

~~[illegible]~~ Det M = 1 so inverse exists.

closure: $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix}$

yes it is closed as it forms a real 2x2 matrix with $G \in I$

associativity: ~~[illegible]~~ it has all other group axioms it is associative as it ~~[illegible]~~ says to assume that matrix multiplication is associative.

the set G forms a groups under matrix multiplication as it has all ~~4~~ group axioms.

This candidate has shown an understanding that 4 axioms are required to be satisfied for the set under the binary operation and these are listed and correct so the final mark is credited. However, the candidate has not correctly shown that an identity exists in the set (and nor is there explanation of the properties of an identity element which would usually be required). Nor has the candidate correctly demonstrated closure because (quite apart from the confusion with symbols) there is no statement that the product is within the set, nor complete justification of such a statement. The candidate has successfully demonstrated the existence of an inverse for each element of *G* and attained a mark for this. However, there is no statement of the fact that this inverse element is in *G*, nor justification of this fact (and nor an explanation of what properties an inverse element must have, which would usually be required).

Question 4 (b) (i)

(b) The matrix $\mathbf{A}_n$ is defined by $\mathbf{A}_n = \begin{pmatrix} 1 & 0 \\ n & 1 \end{pmatrix}$ for any integer n. The set S is defined by

$S = \{\mathbf{A}_n : n \in \mathbb{Z}, n \geqslant 0\}$.

(i) Determine whether S is closed under $\times$. **[2]**

Again, there was a distinct lack of rigour in the approach to this question. Many candidates did not consider two independent members of *S* and many candidates simply considered that *n* was an integer and missed the point that *n* was a non-negative integer. Finally, many candidates simply stated that *S* was closed under *x* without any justification. The command word was 'Determine' and so the onus is on candidates to provide justification.

OCR support

The meaning of the various command words is given on pages 9 – 11 of the specification.

Question 4 (b) (ii)

(ii) Determine whether S is a subgroup of $(G, \times)$. **[2]**

The idea behind this part was that really only the inverse axiom was required to be investigated (because associativity is given, closure comes from Question 4 (b) (i) and identity is straightforward) and that it fails for *S* so all that is required is a single counter-example. However, many candidates again did not apply sufficient rigour and concluded that *S* is actually a subgroup of *G*, which is not the case.

Careful consideration of the axioms is required and all too often this was not in evidence.

Assessment for learning

Rigour and familiarity are both essential for this topic. Candidates should be encouraged to do as many practice questions as possible and this should be marked scrupulously so that missing details are penalised.

Question 4 (c) (i)

(c) (i) Find a subgroup of $(G, \times)$ of order 2. **[2]**

It was pleasing to note that many candidates were able to find the subgroup of order 2 with apparent ease. In this question, correct set notation was required to obtain 2 marks and it was also pleasing to note that most candidates gave this as a matter of course.

Assessment for learning

Candidates should be encouraged to learn and use the correct notation at all times. Notation will be assessed throughout the series.

Question 4 (c) (ii)

(ii) By considering the inverse of the non-identity element in any such subgroup, or otherwise, show that this is the only subgroup of $(G, \times)$ of order 2. **[2]**

Although plenty of candidates understood the structure required (i.e. that the non-identity element must be self-inverse) most candidates again struggled to give the rigour necessary to complete the proof successfully. It is insufficient in such questions simply to state the answer with confidence. So the majority of candidates only obtained 1 mark out of 2 with only about a quarter of candidates gaining both marks.

Question 4 (d)

The set of all real 2×2 matrices is denoted by H.

(d) With the help of an example, explain why $(H, \times)$ is **not** a group. **[2]**

Pleasingly, the majority of candidates obtained both marks for this question part.

Key point call out – check the demand

If the question requires an example then candidates should provide an example or else they are liable not to obtain all available marks. In this case candidates needed to give an example of a matrix with no inverse in *H*, to explain why it did not have an inverse and also, then, to explain why this means that *H* is not a group.

Question 5 (a)

5 The matrix **P** is given by $\mathbf{P} = \begin{pmatrix} a & 0 \\ 2 & 3 \end{pmatrix}$ where a is a constant and $a \neq 3$.

(a) Given that the acute angle between the directions of the eigenvectors of **P** is $\frac{1}{4}\pi$ radians, determine the possible values of a. **[8]**

Candidates are clearly less comfortable finding eigenvalues and their associated eigenvector when unknowns are involved. The number of candidates who correctly produced a factorised characteristic equation (i.e. $(3 - \lambda)(a - \lambda) = 0$) and then, instead of simply writing down the solutions, expanded it and attempted to solve it using the quadratic formula was astonishingly high and very disappointing. Similarly, a large number of candidates could not make the correct deduction from the equation $ax = 3x$ (i.e. that $x = 0$, given that a is not 3) and hence that [0, 1] is an associated eigenvector. Similarly, too many candidates deduced, from $2x = (a – 3)y$, that the associated eigenvector is [2, a – 3] rather than [a – 3, 2]. Some candidates have clearly been taught to use $x = 1$ to find the eigenvector(s) but this approach simply does not work when x must be 0, as in this case. Some candidates seemed to think that signs were not important and so gave [3 – a, 2] as the eigenvector associated with an eigenvalue of a, rather than [a – 3, 2].

Key point call out – try to make the maths easier

If length is not important, as is often the case with eigenvectors, then don't introduce new unknowns but simply choose a component to make the components simple, e.g. [a – 3, 2] is easier to work with than [(a – 3)/2, 1] or [k(a – 3), 2k] or [1, 2/(a – 3)].

Exemplar 2

$$P = \begin{pmatrix} a & 0 \\ 2 & 3 \end{pmatrix} \qquad P - \lambda I = \begin{pmatrix} a-\lambda & 0 \\ 2 & 3-\lambda \end{pmatrix}$$

$$\det(P - \lambda I) = (a-\lambda)(3-\lambda) = 0 \qquad \begin{pmatrix} a & 0 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix}$$

$$0 = 3a - a\lambda - 3\lambda + \lambda^2$$

$$0 = \lambda^2 - (3+a)\lambda + 3a \qquad ax = \dot{x}$$

$$2x + 3y = \dot{y}$$

$$\frac{3+a \pm \sqrt{(3+a)^2 - 4(3a)}}{2} = \frac{3+a \pm \sqrt{9-6a+a^2}}{2} \qquad 2x+3y = ax \; (2-a)x = -3y \; 3y = (a-2)x$$

$$= \frac{3+a \pm \sqrt{(a-3)^2}}{2} = \frac{(3+a) \pm (a-3)}{2} = \lambda$$

$$\lambda = a \quad \text{or} \quad \lambda = 3 \qquad \lambda = a, \; e_1 = \begin{pmatrix} 2 \\ a-3 \end{pmatrix}$$

$$\begin{pmatrix} a & 0 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \lambda \begin{pmatrix} x \\ y \end{pmatrix} \qquad \cancel{ax = ax}$$

$$2x + 3y = ay$$

$$2x = (a-3)y$$

$$\lambda = 3, \; e_2 = \begin{pmatrix} 0 \\ k_1 \end{pmatrix} \qquad x = \frac{(a-3)}{2}y$$

$$2x + 3y = 3y \qquad ax = 3x$$

$$x = 0$$

$$\begin{pmatrix} 0 \\ k_1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ a-3 \end{pmatrix} = \cancel{k_1} \times \sqrt{4+(a-3)^2} \cos\left(\frac{\pi}{4}\right) = \cancel{k_1}(a-3)$$

$$\frac{\sqrt{2}}{2}\sqrt{4+(a-3)^2} = (a-3) \qquad \to 4 + (a-3)^2 = (\sqrt{2}a - 3\sqrt{2})^2$$

$$4 + (a^2 - 6a + 9) = 2a^2 - 12 + 18$$

$$\sqrt{4+(a-3)^2} = \frac{2}{\sqrt{2}}(a-3) \quad \Rightarrow \quad a^2 + 6a - 7 = 0$$

$$a = 1 \quad \text{or} \quad a = -7$$

This candidate has obtained the factorised characteristic equation but, instead of simply reading off the eigenvalues, has expanded and used the formula to solve the equation. The eigenvector associated with an eigenvalue of 3 has been correctly deduced, although it would have been better to have expressed it as [0, 1] rather than the more general [0, k_1] – it is, after all, only the direction which is important. The second eigenvector, however, is incorrect since $2x = (a - 3)y$ does not imply that $x = 2$ and $y = a - 3$.

Question 5 (b) (i)

(b) You are given instead that $\mathbf{P}$ satisfies the matrix equation $\mathbf{I} = \mathbf{P}^2 + r\mathbf{P}$ for some rational number r.

(i) Use the Cayley-Hamilton theorem to determine the value of a and the corresponding value of r. **[4]**

A large number of candidates did not appear to be familiar with the Cayley-Hamilton theorem and so, even though many found the values of a and r by brute force, they did not gain any credit for this. Candidates who were familiar with the theorem generally knew what was required and found the values with little fuss.

Key point call out – do the question in the manner specified by the question

If the question requires that the question be done in a certain way then it must be done in this way or else full credit cannot be gained, even if the correct answer is found.

Question 5 (b) (ii)

(ii) Hence show that $\mathbf{P}^4 = s\mathbf{I} + t\mathbf{P}$ where s and t are rational numbers to be determined. You should **not** calculate $\mathbf{P}^4$. **[3]**

Generally speaking, those candidates who managed Question 5 (b) (i) also managed this, the final question of the paper. There were many different approaches to the question although, given that $\mathbf{P}^2$ was given in part (b) (i) and $\mathbf{P}^4$ was required in part (b) (ii) it was a little surprising that simply squaring the expression for $\mathbf{P}^2$ was not the most common.

Key point call out – check the demand of the question

In this question the actual demand is to show that $\mathbf{P}^4$ can be written in a certain way with the values of s and t being very much subsequential to this. Thus, candidates who did not actually express $\mathbf{P}^4$ in the required form did not obtain full credit.

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