GCE

Further Mathematics B (MEI)

Y413/01: Modelling with algorithms

Advanced Subsidiary GCE

2021 Mark Scheme (DRAFT)

This is a DRAFT mark scheme. It has not been used for marking as this paper did not receive any entries in the series it was scheduled for. It is therefore possible that not all valid approaches to a question may be captured in this version. You should give credit to such responses when marking learner's work.

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Text Instructions

1. Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
E	Explanation mark 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank page
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only previous M mark.
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

2. Subject-specific Marking Instructions for AS Level Mathematics B (MEI)

a Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)

- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.
A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words "Determine" or "Show that", or some other indication that the method must be given explicitly.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case, please escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such

cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)
We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.

- When a value is **given** in the paper only accept an answer correct to at least as many significant figures as the given value.
- When a value is **not given** in the paper accept any answer that agrees with the correct value to **2 s.f.** unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range. NB for Specification A the rubric specifies 3 s.f. as standard, so this statement reads "3 s.f"

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.
Candidates using a value of 9.80, 9.81 or 10 for *g* should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

g Rules for replaced work and multiple attempts:

- If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
- If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
- if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors. If a candidate corrects the misread in a later part, do not continue to follow through. E marks are lost unless, by chance, the given results are established by equivalent working. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" and "Determine. Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

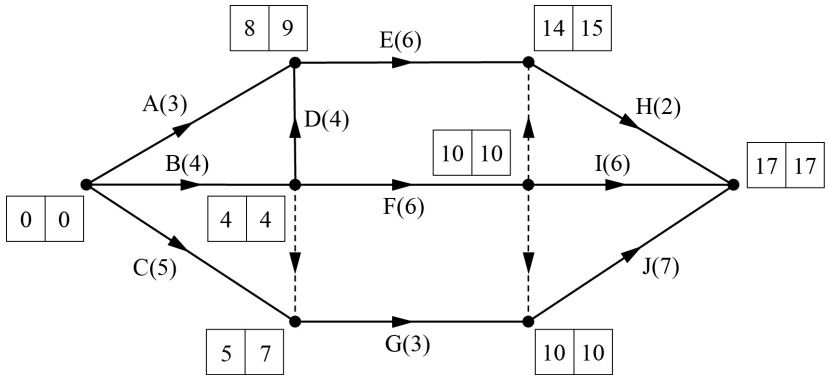
j If in any case the scheme operates with considerable unfairness consult your Team Leader.

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>AOs</th><th>Guidance</th></tr>
<tr><td>1</td><td>(a)</td><td></td><td>
<table>
<tr><th>Activity</th><th>Immediate Predecessor(s)</th></tr>
<tr><td>A</td><td>-</td></tr>
<tr><td>B</td><td>-</td></tr>
<tr><td>C</td><td>-</td></tr>
<tr><td>D</td><td>B</td></tr>
<tr><td>E</td><td>A, D</td></tr>
<tr><td>F</td><td>B</td></tr>
<tr><td>G</td><td>B, C</td></tr>
<tr><td>H</td><td>E, F</td></tr>
<tr><td>I</td><td>F</td></tr>
<tr><td>J</td><td>F, G</td></tr>
</table>
</td><td>B1

B1

[2]</td><td>1.1

1.1</td><td>Any 3 rows correct
(not including rows A, B, C)

Cao</td></tr>
<tr><td>1</td><td>(b)</td><td>(i)</td><td> 
</td><td>M1

M1

A1
[3]</td><td>1.1

1.1

1.1</td><td>Forward pass – numbers increasing from source to sink (allow one slip)

Backward pass – numbers decreasing from sink to source (allow one slip)

cao for backward and forward pass</td></tr>
<tr><td>1</td><td>(b)</td><td>(ii)</td><td>Critical activities: B, F and J</td><td>B1
[1]</td><td>1.1</td><td>cao</td></tr>
<tr><td>1</td><td>(c)</td><td></td><td>Total float for activity E = 15 – 8 – 6 = 1 (hour)</td><td>B1ft
[1]</td><td>1.1</td><td>Follow through their values for E</td></tr>
</table>

Question			Answer	Marks	AOs	Guidance
2	(a)		Bin 1: **15 4 16 2** Bin 2: **23 12** Bin 3: 14 11 13 Bin 4: 20 Bin 5: 22	**M1** **A1** **[2]**	**1.1** **1.1**	First two bins correct cao
2	(b)		15 4 23 16 2 12 14 11 20 13 22 23 16 20 22 **15** 4 2 12 14 11 13 (1st pass) **23** 16 20 22 **15** 12 14 11 13 **4** 2 (2nd pass) **23** 20 22 **16** **15** 14 13 **12** 11 **4** **2** (3rd pass) **23** 22 **20** **16** **15** **14** 13 **12** **11** **4** **2** (4th pass) **23** **22** **20** **16** **15** **14** **13** **12** **11** **4** **2** (5th pass)	**M1** **A1** **A1** **[3]**	**1.1** **1.1** **1.1**	15 used as the first pivot and in the correct position after the first pass First two passes correct – must be using quick (not slow) sort Correct sort with a fourth pass and then a fifth pass in which no changes are made or an indication that the sort is complete after a fourth pass **SC** M1 only for ascending
2	(c)		Bin 1: 23 16 Bin 2: 22 15 2 Bin 3: 20 14 4 Bin 4: 13 12 11	**M1** **A1** **[2]**	**1.1** **1.1**	First two bins correct cao
2	(d)		While first fit does find a solution efficiently to the problem of packing items with the given sizes into bins of capacity 40 it was not an optimal solution (as indicated by the fact that first fit required 5 bins while first-fit decreasing required only 4 bins) and hence first fit is an example of an heuristic algorithm	**B1** **[1]**	**1.2**	A clear understanding that a heuristic algorithm can find a solution efficiently but with no guarantee that it will be an optimal solution

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>AOs</th><th>Guidance</th></tr>
<tr><td>2</td><td>(e)</td><td></td><td>The number 14 was used as a pivot for the first pass as every number to the left of 14 is greater than 14 and every number to the right is less than 14</td><td>B1</td><td>3.1a</td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td>[1]</td><td></td><td></td></tr>
<tr><td>3</td><td>(a)</td><td></td><td>The simplex algorithm cannot be applied to the tableau in Fig. 2.1 because this tableau has been set up to maximise (rather than to minimise) the objective function</td><td>B1</td><td>3.5b</td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td>[1]</td><td></td><td></td></tr>
<tr><td>3</td><td>(b)</td><td></td><td>
<table>
<tr><th>P</th><th>x</th><th>y</th><th>z</th><th>s_1</th><th>s_2</th><th>s_3</th><th>RHS</th></tr>
<tr><td>1</td><td>0</td><td>0</td><td>0</td><td>3</td><td>17</td><td>10</td><td>1235</td></tr>
<tr><td>0</td><td>0</td><td>1</td><td>0</td><td>1</td><td>5</td><td>3</td><td>370</td></tr>
<tr><td>0</td><td>1</td><td>0</td><td>0</td><td>1</td><td>8</td><td>5</td><td>570</td></tr>
<tr><td>0</td><td>0</td><td>0</td><td>1</td><td>0</td><td>1</td><td>1</td><td>75</td></tr>
</table>
</td><td>M1

A1

A1</td><td>3.3

1.1

1.1</td><td>Bottom row unchanged and z-column correct
Any non-zero or 1 column correct (e.g. any slack variable or RHS column correct)
cao</td></tr>
<tr><td></td><td></td><td></td><td></td><td>[3]</td><td></td><td></td></tr>
<tr><td>3</td><td>(c)</td><td></td><td>The tableau in part (b) shows that the solution obtained after the third iteration is optimal as there are no negative values in the objective (top) row</td><td>B1</td><td>2.4</td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td>[1]</td><td></td><td></td></tr>
<tr><td>3</td><td>(d)</td><td>(i)</td><td>$x = 570$, $y = 370$ and $z = 75$</td><td>B1</td><td>3.4</td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td>[1]</td><td></td><td></td></tr>
<tr><td>3</td><td>(d)</td><td>(ii)</td><td>Minimum value of P is -1235</td><td>B1</td><td>2.2a</td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td>[1]</td><td></td><td></td></tr>
</table>

<table>
<tr>
<td>4</td><td>(a)</td><td></td>
<td>
<table>
<tr><th>a</th><th>b</th><th>n</th><th>h</th><th>c</th><th>d</th><th>e</th></tr>
<tr><td>1</td><td>3</td><td>4</td><td>0.5</td><td>$\frac{232}{9}$</td><td>1</td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td>1.5</td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td>$\frac{123}{4}$</td><td>2</td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td>$\frac{183}{4}$</td><td>2.5</td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td>$\frac{1909}{25}$</td><td>3</td><td>$\frac{1909}{100}$</td></tr>
</table>
</td>
<td>M1

A1

A1

[3]</td>
<td>3.4

1.1

1.1</td>
<td>At least three rows of cells completed in columns c and d completed with a correct first row
cao – first, second and third rows correct – accept exact equivalent fractions
cao – fourth and fifth rows correct – accept exact equivalent fractions</td>
</tr>
<tr>
<td>4</td><td>(b)</td><td></td>
<td>$\int_1^3 \left(x^3 - \frac{2}{x^2}\right) \mathrm{d}x = \frac{56}{3}$</td>
<td>B1

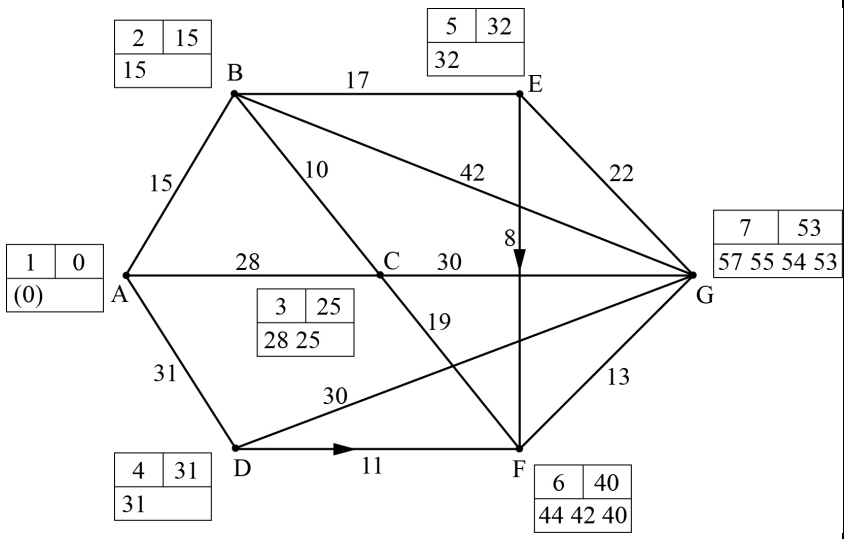
[1]</td>
<td>1.1</td>
<td>BC</td>
</tr>
<tr>
<td>4</td><td>(c)</td><td></td>
<td>Percentage difference = $\frac{\frac{1909}{100} - \frac{56}{3}}{\frac{56}{3}} \times 100$
= 2.27 % (3 sf) indicating that the algorithm gives a good approximation to the given integral</td>
<td>M1

A1

[2]</td>
<td>1.1

3.2b</td>
<td>Correct method using final output from (a)(ii) and (b)
Correct value (to at least 2 sf) and suitable comment (may mention that algorithm is trapezium rule)</td>
</tr>
<tr>
<td>4</td><td>(d)</td><td></td>
<td>In the line ‘Let $c = a^3 - \frac{2}{a^2} + b^3 - \frac{2}{b^2}$’ add 6 and add 3 in the bracket $\left(d^3 - \frac{2}{d^2}\right)$ in the line ‘Let $c = c + 2\left(d^3 - \frac{2}{d^2}\right)$’</td>
<td>B1

[1]</td>
<td>3.5c</td>
<td></td>
</tr>
</table>

5	(a)		Indicator variables take the value of 1 if the corresponding arc is in the shortest path and 0 otherwise			
			The two constraints equal to 1 signify that one arc out of A and one arc into G must be in the shortest path	B1	2.4	Explaining why two of the constraints are equal to 1 – must indicate that these constraints refer to nodes A and G and that one arc in the shortest path must go from A and one arc must go into G
			At every other vertex either all the arcs have value 0 (indicating that the vertex is not in the shortest path) or two arcs have value 1 (one entering the vertex and one leaving it) and the others have value 0 (indicating that the vertex is in the shortest path) but in this case the 1's will cancel to give a total of 0 too	B1 [2]	2.4	Explaining why the other five constraints are equal to 0 – must consider both cases of a vertex being either in or not in the shortest path. Or for statement, 'Number of arcs being used into B = number of arcs from B, and similarly for C, D, E and F'
5	(b) (c)	(i)	 Length of shortest path from A to G is 53	M1 A1 [2] M1 A1 A1 A1 B1ft [5]	1.1 1.1 1.2 1.1a 1.1a 1.1 1.1	All arcs present (allow one absent/one extra) cao (including the directed arcs EF and DF) Correct working values at C Working values Labels Order of labelling

5	**(c)**	**(ii)**	Shortest path from A to G is ABEFG	**B1** **[1]**	**1.1**	
5	**(d)**		The objective requires + 5CX + 5XC + 4FX + 4XF + *x*XG adding to it An additional constraint equation CX + FX – XC – XF – XG = 0 is required Furthermore, + XC – CX needs adding to the constraint for C, + XF – FX needs adding to the constraint for F and + XG needs adding to the constraint for G	**B1** **B1** **B1** **[3]**	**3.5c** **2.2a** **2.4**	Correct additions to the objective function Correct indication that a further constraint equation is required (due to the addition of vertex X) and that the constraints for C, F and G need modifying too
5	**(e)**		Maximum value of *x* is 17	**B1** **[1]**	**3.1b**	
5	**(f)**		$t \approx 0.018\left(\frac{800}{8}\right)^2$ 180 (seconds)	**M1** **A1** **[2]**	**3.1a** **2.2b**	Using the fact that Dijkstra's algorithm has quadratic complexity

<table>
<tr><td>6</td><td>(a)</td><td></td><td>(Maximise) $P = 3x + 2y + z$
$2x + y + z \le 29$
$x + y + 2z \le 42$
$y + z \ge 16$
$4x + z = 28$</td><td>B1
M1
A1

B1

[4]</td><td>3.1a
1.1
1.1

3.1a</td><td>Correct objective
Any two correct
First three constraints correct

Correct equation (condone $4x + z \le 28$ and $4x + z \ge 28$)</td></tr>
<tr><td>6</td><td>(b)</td><td></td><td>$Q = a_1 + a_2$ where $y + z - s_3 + a_1 = 16$ and $4x + z - s_5 + a_2 = 28$

$Q = 16 - y - z + s_3 + 28 - 4x - z + s_5$
$\Rightarrow Q + 4x + y + 2z - s_3 - s_5 = 44$
<table><tr><td>Q</td><td>P</td><td>x</td><td>y</td><td>z</td><td>s_1</td><td>s_2</td><td>s_3</td><td>s_4</td><td>s_5</td><td>a_1</td><td>a_2</td><td>RHS</td></tr><tr><td>1</td><td>0</td><td>4</td><td>1</td><td>2</td><td>0</td><td>0</td><td>−1</td><td>0</td><td>−1</td><td>0</td><td>0</td><td>44</td></tr></table></td><td>M1

A1

[2]</td><td>2.1

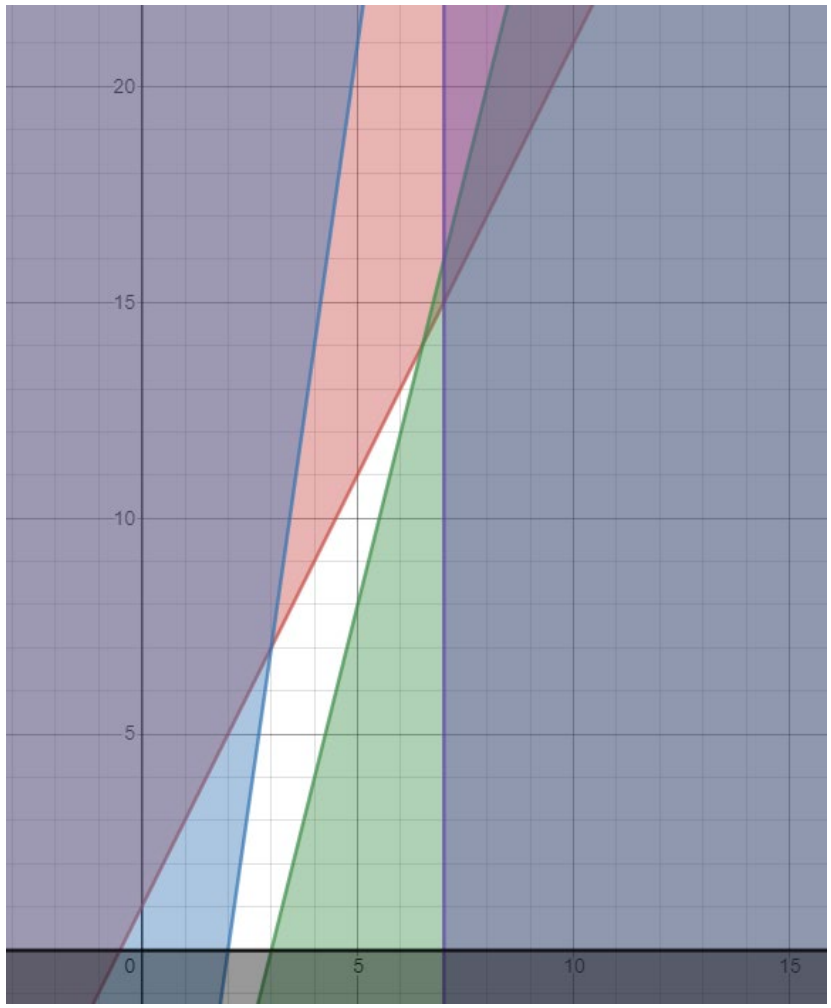
2.2a</td><td>Correctly indicates that $Q = a_1 + a_2$ and attempt to find equation with two equations involving the artificial variables

cao</td></tr>
<tr><td>6</td><td>(c)</td><td></td><td>$P = 3x + 2y + z \Rightarrow P = -x + 2y$ (+28)
$2x + y + z \le 29 \Rightarrow -2x + y \le 1$
$x + y + 2z \le 42 \Rightarrow 7x - y \ge 14$
$y + z \ge 16 \Rightarrow 4x - y \le 12$
$z \ge 0 \Rightarrow x \le 7$</td><td>B1
M1

A1</td><td>3.1a
1.1

1.1</td><td>
Eliminate z from at least two constraints using $4x + z = 28$

All four correct</td></tr>
</table>

				**B1** **B1** **B1**	**1.1** **1.1** **1.1**	Any one line drawn correctly Any two lines drawn correctly Correct region
			Maximum value of P is 48 $x = 6$, $y = 13$ and $z = 4$	**B1** **B1** **[8]**	**2.2a** **2.2a**	

OCR (Oxford Cambridge and RSA Examinations)
The Triangle Building
Shaftesbury Road
Cambridge
CB2 8EA

OCR Customer Contact Centre

Education and Learning
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk