OCR
Oxford Cambridge and RSA

Wednesday 20 October 2021 – Afternoon

AS Level Further Mathematics B (MEI)

Y413/01 Modelling with Algorithms

Time allowed: 1 hour 15 minutes

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You must have:
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

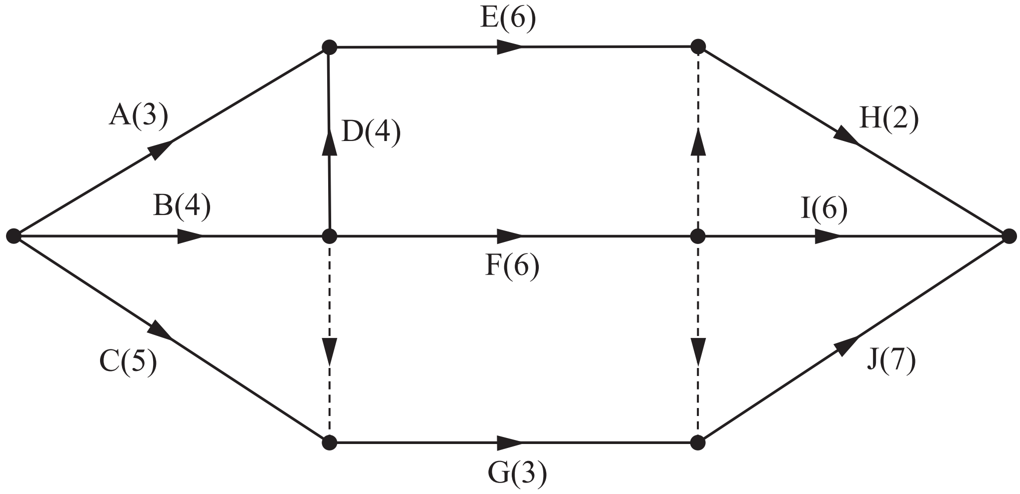
INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **12** pages.

ADVICE
- Read each question carefully before you start your answer.

Answer **all** the questions.

1



The diagram shows an activity network for a project. The numbers in brackets show the duration of the activities in hours.

(a) Complete the table in the Printed Answer Booklet to show the immediate predecessors for each activity. **[2]**

(b) **(i)** Carry out a forward pass and a backward pass through the activity network, showing the early event time and the late event time at each vertex of the network. **[3]**

(ii) State the critical activities of the project. **[1]**

(c) Calculate the total float for activity E. **[1]**

2 The list below shows the sizes of eleven items.

15 4 23 16 2 12 14 11 20 13 22

(a) Show the result of applying the first fit algorithm to pack items with the sizes listed above into bins that have a capacity of 40. **[2]**

(b) Apply the quick sort algorithm to sort the list of numbers above into **descending** order. You should use the first value as the pivot for each sublist. **[3]**

(c) Show the result of applying the first fit decreasing algorithm to pack items with the sizes listed above into bins that have a capacity of 40. **[2]**

(d) By considering the answers to parts **(a)** and **(c)** explain why first fit is an example of a heuristic algorithm. **[1]**

The quick sort algorithm is to be used to sort a different list of eight numbers into descending order. After one pass through the list the quick sort algorithm produces the following list

18 22 17 19 14 11 4 13

(e) State, with a reason, which number was used as a pivot for the first pass. **[1]**

3 The following LP problem in x, y and z is as follows.

Minimise $P = -x - 2y + z$

subject to
$$-2x + 3y + z \leqslant 45$$
$$x - y - 2z \leqslant 50$$
$$-x + y + 3z \leqslant 25$$
$$x, y, z \geqslant 0$$

A student sets up the following initial tableau in **Fig. 3.1** to solve the LP problem using the simplex method.

P	x	y	z	s_1	s_2	s_3	RHS
1	1	2	−1	0	0	0	0
0	−2	3	1	1	0	0	45
0	1	−1	−2	0	1	0	50
0	−1	1	3	0	0	1	25

Fig. 3.1

(a) Explain why the simplex algorithm cannot be applied to the initial tableau in **Fig. 3.1** to solve the LP problem. **[1]**

The tableau in **Fig. 3.1** is reformulated so the simplex method can be used to solve the LP problem.

The simplex method is applied to the reformulated problem. After two iterations a computer produces the tableau in **Fig. 3.2**.

P	x	y	z	s_1	s_2	s_3	RHS
1	0	0	−10	3	7	0	485
0	0	1	−3	1	2	0	145
0	1	0	−5	1	3	0	195
0	0	0	1	0	1	1	75

Fig. 3.2

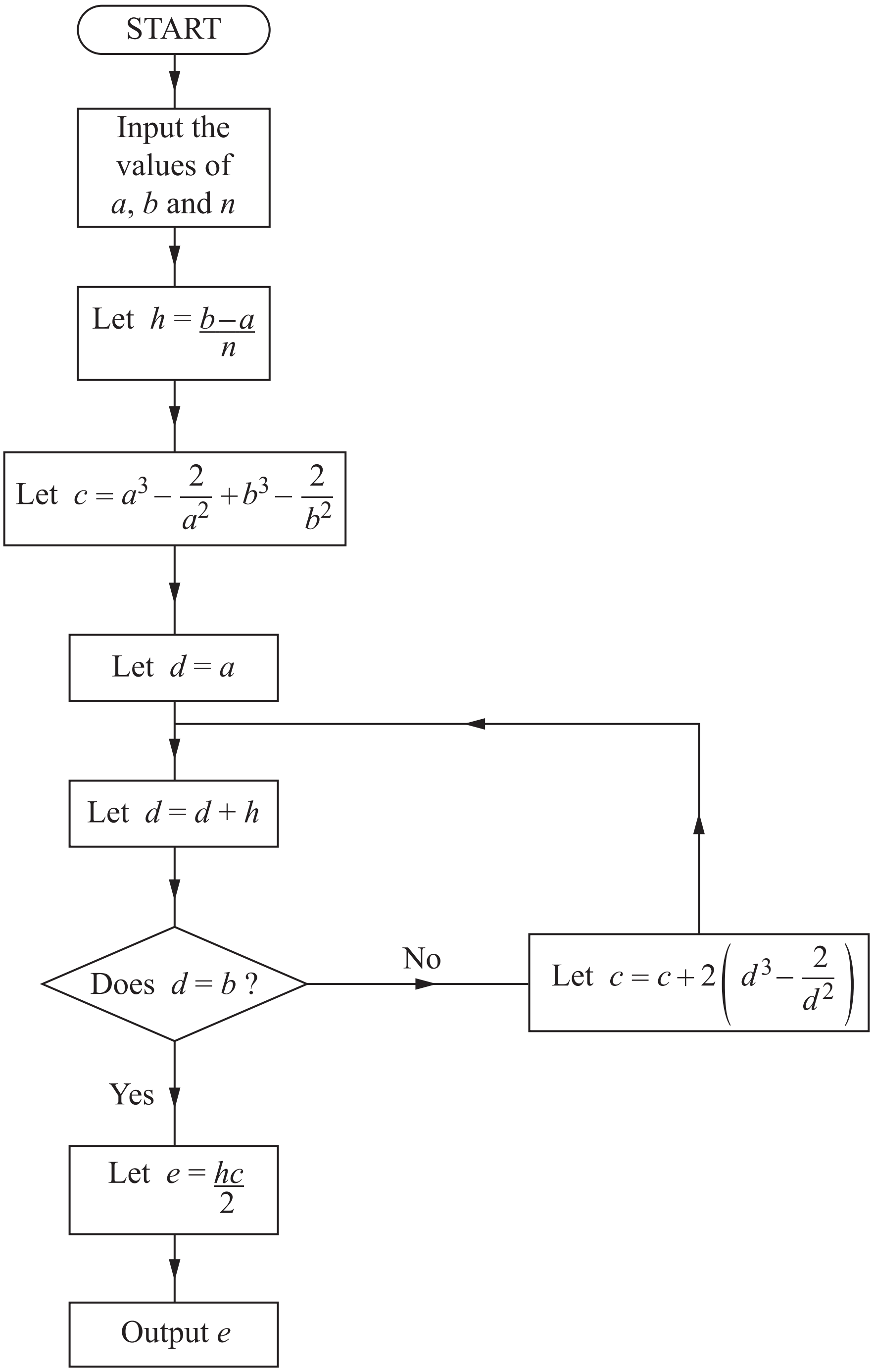
(b) Perform a third iteration of the simplex method. **[3]**

(c) Explain how the answer to part **(b)** shows that the solution obtained after the third iteration is optimal. **[1]**

(d) **(i)** State the optimal values of x, y and z. **[1]**

(ii) State the corresponding minimum value of P for the original LP problem. **[1]**

4 The flow chart shows an algorithm that produces a numerical approximation for the integral $\int_a^b \left(x^3 - \frac{2}{x^2}\right)\mathrm{d}x$



(a) Work through the algorithm using the inputs $a = 1$, $b = 3$ and $n = 4$. Record the exact values of c and d every time they change. **[3]**

(b) Find the exact value of $\int_1^3 \left(x^3 - \frac{2}{x^2}\right) \mathrm{d}x$. **[1]**

(c) By considering the percentage difference between the exact value of the integral given in part **(b)** and the approximation found in part **(a)**, comment on the accuracy of the algorithm in this case. **[2]**

(d) Explain how to adapt the algorithm to produce a numerical approximation for the integral $\int_a^b \left(x^3 - \frac{2}{x^2} + 3\right) \mathrm{d}x$. **[1]**

5 An LP formulation to find the shortest path from A to G in a directed network with seven vertices, A, B, …, G is given below.

Minimise 15AB + 28AC + 31AD + 17BE + 17EB + 10BC + 10CB + 42BG + 30CG + 19CF + 19FC + 11DF + 30DG + 8EF + 22EG + 13FG

subject to

$$AB + AC + AD = 1$$
$$BG + CG + DG + EG + FG = 1$$
$$AB + CB + EB - BC - BE - BG = 0$$
$$AC + BC + FC - CB - CF - CG = 0$$
$$AD - DF - DG = 0$$
$$BE - EB - EF - EG = 0$$
$$CF + DF + EF - FC - FG = 0$$

(a) Explain why

- two of the constraint equations are equal to 1,
- five of the constraint equations are equal to 0. **[2]**

(b) Using the vertices given in the Printed Answer Booklet draw a directed network with 13 arcs to represent the information given in the formulation above. Two of the arcs are directed and the rest are undirected. The arcs from A and the arcs to G may be regarded as being undirected. **[2]**

(c) **(i)** Apply Dijkstra's algorithm to the network drawn in part **(b)**, to find the length of the shortest path from A to G. **[5]**

(ii) Write down the shortest path from A to G. **[1]**

An eighth vertex, X, is added to the network. The table in **Fig. 5.1** shows the corresponding weights between each pair of vertices for which there is a connecting arc.

	A	B	C	D	E	F	G
X	-	-	5	-	-	4	x

Fig. 5.1

It is given that x is a positive integer and all arcs from vertex X are undirected.

(d) State what changes need to be made to the original LP formulation when X is included as an eighth vertex in the network. **[3]**

For a certain value of x, the complete LP for the network with eight vertices was run in an LP solver and the output is shown in the table in **Fig. 5.2**.

Variable	Value
AB	1.000000
AC	0.000000
AD	0.000000
BC	1.000000
BE	0.000000
BG	0.000000
CB	0.000000
CF	0.000000
CG	0.000000
CX	1.000000
DF	0.000000
DG	0.000000
EB	0.000000
EF	0.000000
EG	0.000000
FC	0.000000
FG	0.000000
FX	0.000000
XC	0.000000
XF	0.000000
XG	1.000000

Fig. 5.2

(e) State the maximum value of x that is consistent with this solution. **[1]**

(f) A computer takes 0.018 seconds to solve a shortest path problem on a network with eight vertices using Dijkstra's algorithm.

Approximately how long will it take the computer to solve a shortest path problem on a network with eight hundred vertices using Dijkstra's algorithm? **[2]**

6 An incomplete initial tableau for a two-stage simplex solution for a maximisation integer linear programming (ILP) problem, in x, y and z where $x, y, z \geqslant 0$, is shown below.

Q	P	x	y	z	s_1	s_2	s_3	s_4	s_5	a_1	a_2	RHS
0	1	−3	−2	−1	0	0	0	0	0	0	0	0
0	0	2	1	1	1	0	0	0	0	0	0	29
0	0	1	1	2	0	1	0	0	0	0	0	42
0	0	0	1	1	0	0	−1	0	0	1	0	16
0	0	4	0	1	0	0	0	1	0	0	0	28
0	0	4	0	1	0	0	0	0	−1	0	1	28

(a) Formulate the information given in the tableau as an ILP problem by

- stating the objective function for the original ILP problem,
- listing the constraints in terms of x, y and z only. **[4]**

(b) In the Printed Answer Booklet, complete the top row of the initial tableau for the ILP problem. **[2]**

(c) Use a 2-D graphical method, with axes representing x and y, to determine

- the maximum value of P,
- the corresponding integer values of x, y and z. **[8]**

END OF QUESTION PAPER

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