Modified Enlarged 18pt

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Friday 22 May 2020 – Morning

A Level Further Mathematics B (MEI)

Y420/01 Core Pure

Time allowed: 2 hours 40 minutes
plus your additional time allowance

YOU MUST HAVE:
the Printed Answer Booklet sent with the standard paper or any suitable paper provided by the centre. The Printed Answer Book may be enlarged by the centre.
the Formulae Booklet for Further Mathematics B (MEI) sent with the standard paper
a scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

INSTRUCTIONS

Use black ink. You can use an HB pencil, but only for graphs and diagrams.

If you use the Printed Answer Booklet write your answer to each question in the space provided. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.

If you use the Printed Answer Booklet fill in the boxes on the front of the Printed Answer Booklet.

Answer ALL the questions.

Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.

Give your final answers to a degree of accuracy that is appropriate to the context.

Do NOT send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

The total mark for this paper is 144.

The marks for each question are shown in brackets [ ].

ADVICE

Read each question carefully before you start your answer.

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SECTION A (36 marks)

Answer ALL the questions.

1 **Using standard summation of series formulae, determine the sum of the first n terms of the series**

$$(1 \times 2 \times 4) + (2 \times 3 \times 5) + (3 \times 4 \times 6) + \dots,$$

where n is a positive integer. Give your answer in fully factorised form. [6]

2 **(a) The matrices $\mathbf{M} = \begin{pmatrix} 0 & 1 & a \\ 1 & b & 0 \end{pmatrix}$ and $\mathbf{N} = \begin{pmatrix} b & -5 \\ -1 & c \\ -1 & 1 \end{pmatrix}$ are such that $\mathbf{MN} = \mathbf{I}$.**

Find a, b and c. [5]

(b) State with a reason whether or not $\mathbf{N}$ is the inverse of $\mathbf{M}$. [1]

3 In this question you must show detailed reasoning.

Find $\displaystyle\int_0^{\frac{1}{3}} \frac{1}{\sqrt{4-9x^2}}\,dx$, expressing your answer in terms of π. **[4]**
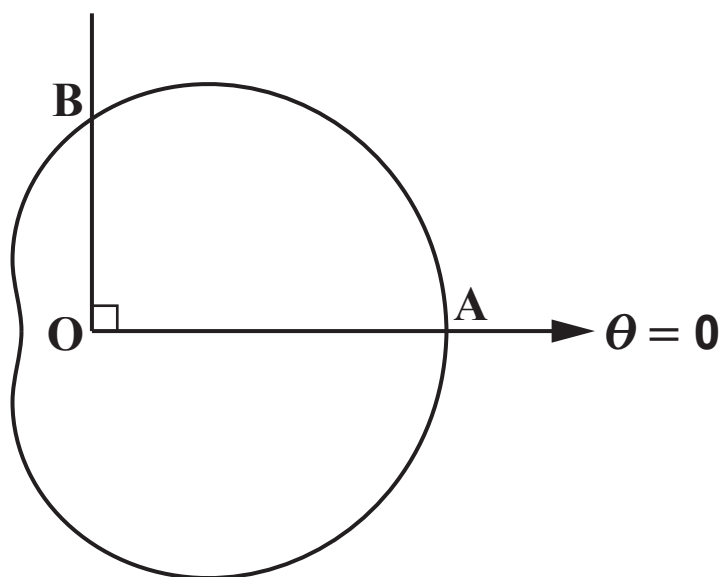
4 The roots of the equation $2x^3 - 5x + 7 = 0$ are α, β and γ.

(a) Find $\dfrac{1}{\alpha} + \dfrac{1}{\beta} + \dfrac{1}{\gamma}$. **[4]**

(b) Find an equation with integer coefficients whose roots are $2\alpha - 1$, $2\beta - 1$ and $2\gamma - 1$. **[4]**

5   Fig. 5 shows the curve with polar equation
     $r = a(3 + 2\cos\theta)$ for $-\pi \leqslant \theta \leqslant \pi$, where a is a constant.

     FIG. 5

     (a) Write down the polar coordinates of the points A
         and B. [2]

     (b) Explain why the curve is symmetrical about the
         initial line. [2]

     (c) In this question you must show detailed
         reasoning.

         Find in terms of a the exact area of the region
         enclosed by the curve. [4]

6   The complex number z satisfies the equation
     $z^2 - 4iz^* + 11 = 0$.

     Given that $\mathrm{Re}(z) > 0$, find z in the form $a + bi$, where a
     and b are real numbers. [4]

SECTION B (108 marks)

Answer ALL the questions.

7 **Prove by mathematical induction that**

$$\sum_{r=1}^{n} (r \times r!) = (n+1)! - 1 \text{ for all positive integers } n. \text{ [6]}$$

8 (a) **Given that the lines** $\mathbf{r} = \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \\ 3 \end{pmatrix}$ **and**

$\mathbf{r} = \begin{pmatrix} -1 \\ 2 \\ k \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$ **meet, determine** k**. [5]**

   (b) **In this question you must show detailed reasoning.**

   Find the acute angle between the two lines. [4]

9   A linear transformation of the plane is represented by the matrix $M = \begin{pmatrix} 1 & -2 \\ \lambda & 3 \end{pmatrix}$, where λ is a constant.

(a) Find the set of values of λ for which the linear transformation has no invariant lines through the origin. [5]

(b) Given that the transformation multiplies areas by 5 and reverses orientation, find the invariant lines. [3]
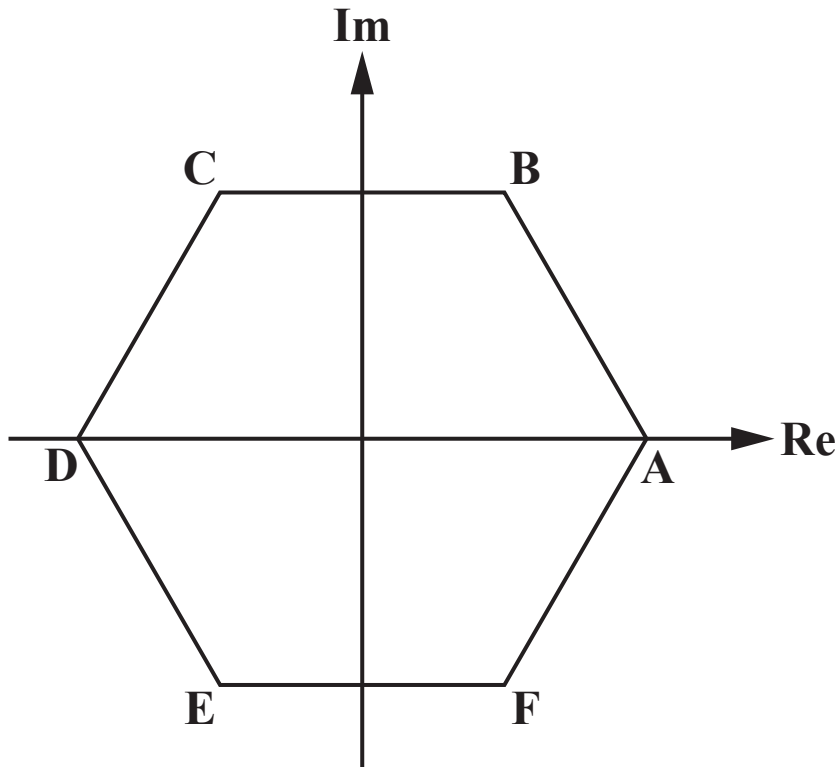
10 In this question you must show detailed reasoning.

The region in the first quadrant bounded by curve $y = \cosh\frac{1}{2}x^2$, the y-axis, and the line $y = 2$ is rotated through 360° about the y-axis.

Find the exact volume of revolution generated, expressing your answer in a form involving a logarithm. [7]

11 In this question you must show detailed reasoning.

In Fig. 11, the points A, B, C, D, E and F represent the complex sixth roots of 64 on an Argand diagram. The midpoints of AB, BC, CD, DE, EF and FA are G, H, I, J, K and L respectively.

FIG. 11

(a) Write down, in exponential ($re^{i\theta}$) form, the complex numbers represented by the points A, B, C, D, E and F. **[2]**

(b) When these complex numbers are multiplied by the complex number w, the resulting complex numbers are represented by the points G, H, I, J, K and L.

Find w in exponential form. **[4]**

(c) You are given that G, H, I, J, K and L represent roots of the equation $z^6 = p$.

Find p. **[2]**

12 (a) Given that $z = \cos\theta + i\sin\theta$, express $z^n + \dfrac{1}{z^n}$ and $z^n - \dfrac{1}{z^n}$ in simplified trigonometric form. **[2]**

(b) By considering $\left(z + \dfrac{1}{z}\right)^3\left(z - \dfrac{1}{z}\right)^3$, find constants A and B such that $\sin^3\theta\cos^3\theta = A\sin 6\theta + B\sin 2\theta$.

[6]

13 (a) Using exponentials, prove that
$\sinh 2x = 2\cosh x \sinh x$. **[2]**

(b) Hence show that if $f(x) = \sinh^2 x$, then
$f''(x) = 2\cosh 2x$. **[2]**

(c) Explain why the coefficients of odd powers in the Maclaurin series for $\sinh^2 x$ are all zero. **[2]**

(d) Find the coefficient of x^n in this series when n is a positive even number. **[3]**

14 Solve the simultaneous differential equations

$$\frac{dx}{dt} + 2x = 4y, \qquad \frac{dy}{dt} + 3x = 5y,$$

given that when $t = 0$, $x = 0$ and $y = 1$. **[11]**

15 (a) Show that the three planes with equations

$$x + \lambda y + 3z = -12$$
$$2x + y + 5z = -11$$
$$x - 2y + 2z = -9$$

where λ is a constant, meet at a unique point except for one value of λ which is to be determined. [3]

(b) In the case $\lambda = -2$, use matrices to find the point of intersection P of the planes, showing your method clearly. [3]

The line l has equation $\dfrac{x-1}{2} = \dfrac{y-1}{-1} = \dfrac{z+2}{-2}$.

(c) Find a vector equation of l. [2]

(d) Find the shortest distance between the point P and l. [4]

(e) (i) Show that l is parallel to the plane $x - 2y + 2z = -9$. [3]

(ii) Find the distance between l and the plane $x - 2y + 2z = -9$. [2]

16 The population density P, in suitable units, of a certain bacterium at time t hours is to be modelled by a differential equation. Initially, the population density is zero, and its long-term value is A.

(a) One simple model is to assume that the rate of change of population density is directly proportional to $A - P$.

(i) Formulate a differential equation for this model. [1]

(ii) Verify that $P = A(1 - e^{-kt})$, where k is a positive constant, satisfies

this differential equation,

the initial condition,

the long-term condition. [3]

An alternative model uses the differential equation

$$\frac{dP}{dt} - \frac{P}{t(1 + t^2)} = Q(t),$$

where $Q(t)$ is a function of t.

(b) Find the integrating factor for this differential equation, showing that it can be written in the form $\dfrac{\sqrt{1 + t^2}}{t}$. [8]

(c) Suppose that $Q(t) = 0$.

    (i) Show that $P = \dfrac{At}{\sqrt{1+t^2}}$. [4]

    (ii) Find the time predicted by this model for the population density to reach half its long-term value. Give your answer correct to the nearest minute. [2]

(d) Now suppose that $Q(t) = \dfrac{te^{-t}}{\sqrt{1+t^2}}$.

Show that $P = \dfrac{At - te^{-t}}{\sqrt{1+t^2}}$. [You may assume that $\lim\limits_{t\to\infty} te^{-t} = 0$.] [5]

It is found that the long-term value of P is 10, and P reaches half this value after 37 minutes.

(e) Determine which of the models proposed in parts (c) and (d) is more consistent with these data. [2]

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