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Examiners' report

FURTHER MATHEMATICS B (MEI)

H645
For first teaching in 2017

Y433/01 Summer 2019 series

Version 1

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the question paper can be downloaded from OCR.

Paper Y433 series overview

This is a minor option for A Level Further Mathematics B (MEI).

The exam involves understanding algorithms and applying algorithms to model and solve network problems and linear programming problems.

This was the first A Level paper on Modelling with Algorithms.

Much good work was seen, although some candidates did not seem to have taken on board that this is an A Level Further Maths paper so required a deeper level of engagement than the legacy D1 papers.

Written explanations were often done well. Explanations need only be brief and are usually best when supported with appropriate calculations or other question specific details.

Simple arithmetic errors (and missing or repeating values e.g. in Q1) could have been avoided by candidates checking their work as they proceed.

There was evidence that some candidates had run out of time and were not able to properly complete or check their responses to the last question.

Question 1 (a)

1 Fig. 1 shows a network. The weights on the arcs are distances.

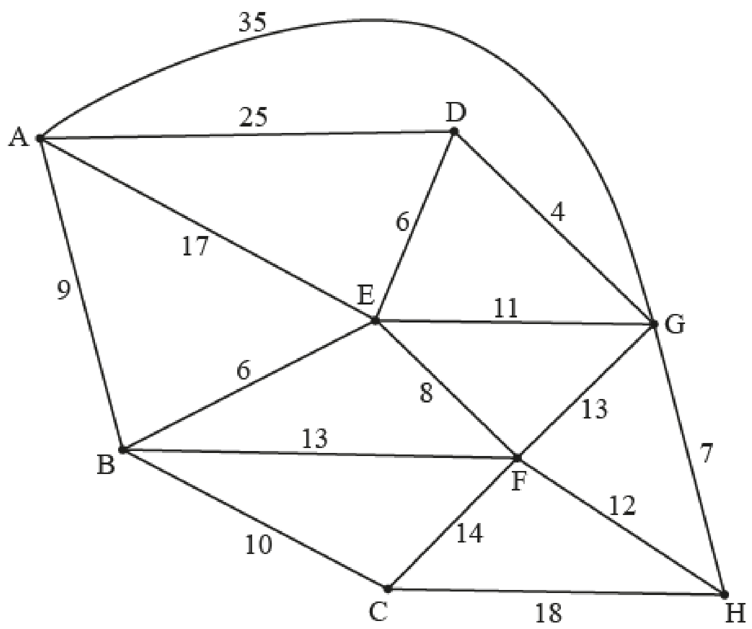


Fig. 1

(a) Apply Dijkstra's algorithm to the copy in the Printed Answer Booklet to find the shortest path from A to H.

State each of the following

- the shortest path and
- its length. **[6]**

	AfL	Centres should note that there is no need to record the vertex that has been used to achieve a temporary label. Temporary values should not be crossed out, unless they are errors. Candidates must show evidence of the correct use of Dijkstra's algorithm by updating temporary labels.

This part was usually answered well although there were some arithmetic errors. A few candidates showed extra temporary values and some started the order of labelling at 0 (0^{th}) instead of 1 (1^{st}). A few candidates stopped the algorithm when G was given a permanent label.

Question 1 (b)

The weights on the arcs in Fig. 1 are listed in descending order in the Printed Answer Booklet.

(b) Use the first fit decreasing algorithm to pack these weights into bins that have a capacity of 50. **[2]**

Mostly done well, but some candidates used next fit on the decreasing list instead of first fit and some repeated or omitted a value and some made arithmetic errors, such as switching the 14 and the first 13.

Question 1 (c)

(c) Determine, in the worst case, the complexity of the first fit decreasing algorithm in terms of n. **[3]**

	AfL	Complexity can be measured using various criteria, and these may give different orders. Item A11 of the specification states that for first fit decreasing algorithms the worst case is O(n^2). This is achieved by counting the number of comparisons when each item is compared with all previous items before being put into a new bin.

Exemplar 1

Worst case scenario where n weights needs n bins to pack.

~~1 comparison for first the seo~~

0 comparisons for first item, 1 ~~for~~ comparison for second item .... n−1 comparisons for n^{th} item

$0+1+2+...+n-1 = \frac{1}{2}(n-1)n$ comparisons total

Quadratic complexity. $O(n^2)$

Exemplar 1 is a good response.

Question 2 (a) and (b)

2 The table in Fig. 2.1 lists the duration (in minutes) and immediate predecessors for each activity in a project.

Activity	Duration (minutes)	Immediate predecessors
A	25	–
B	20	–
C	35	–
D	15	A
E	25	A, B, C
F	25	A, B, C
G	30	C
H	20	E
I	25	E, F, G

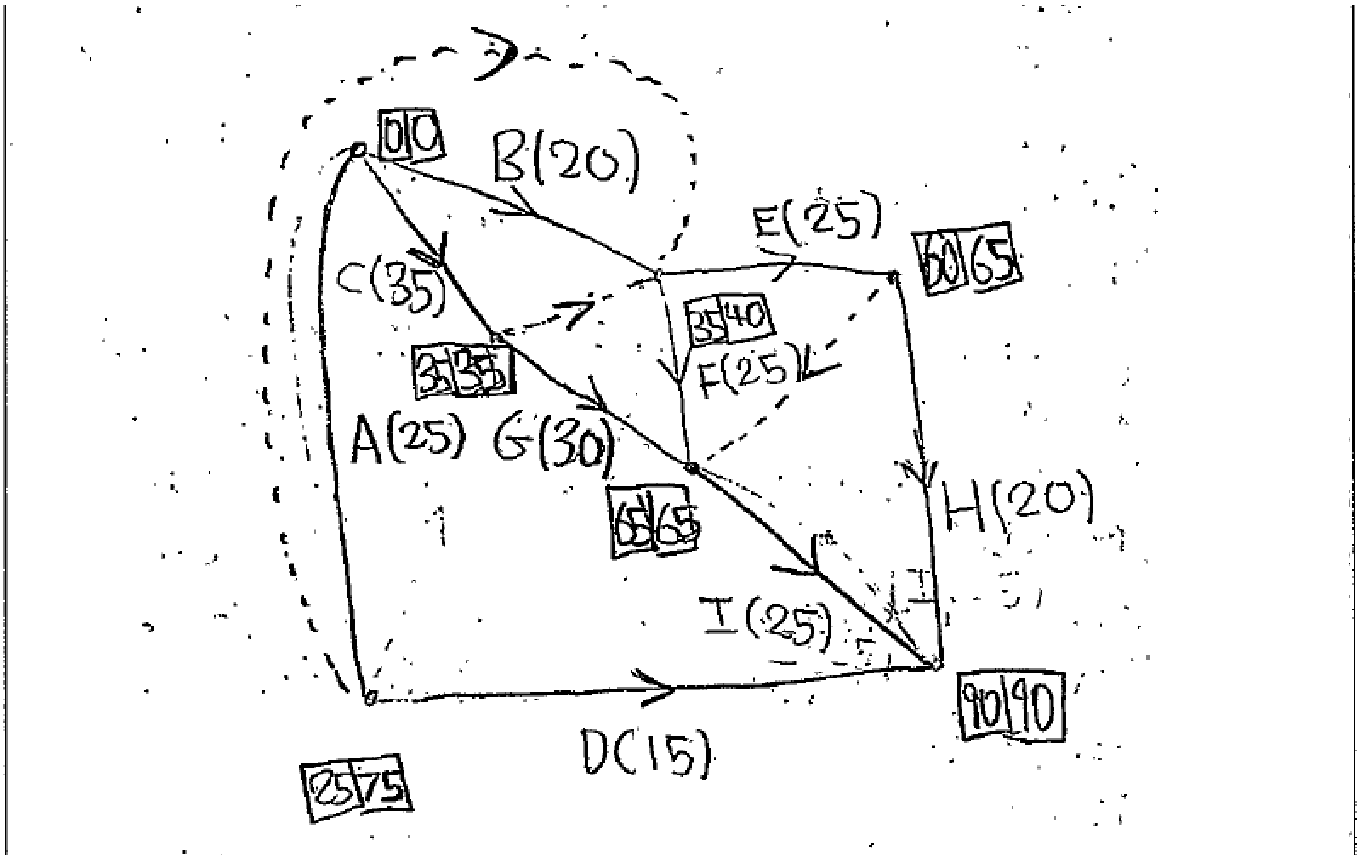
Fig. 2.1

(a) Draw an activity network, using activity on arc, to represent the project. **[3]**

(b)
- Carry out a forward pass and a backward pass through the activity network, showing the early event time and the late event time at each vertex of your network.
- List the critical activities. **[5]**

Many correctly drew an activity on arc networks, with only a very small number of candidates using activity on node. Some candidates used unnecessary extra dummy activities. The forward and backward passes were generally correct although several candidates did not cope with the backward pass when a dummy activity was involved. In particular, the latest finish time for A was often given as 75 or 35 rather than 40.

Exemplar 2



In Exemplar 2 it is difficult to interpret where the start vertex occurs and it is missing a direction on the arc representing activity A. On the backward pass the value 75 comes from 90 – 15, but this should have been replaced by 40 by using the dummy arc.

	AfL	Candidates should show the activities (and dummy activities) as directed arcs, with dummy activities shown as dashed lines. Dummy activities should be used only where necessary to ensure correct precedence and uniqueness of labelling.

Question 2 (c)

The resource histogram in Fig. 2.2 shows the number of workers required when each activity begins at its earliest possible start time. It is given that each activity requires at least one worker and that when an activity is started it must be completed without interruption.

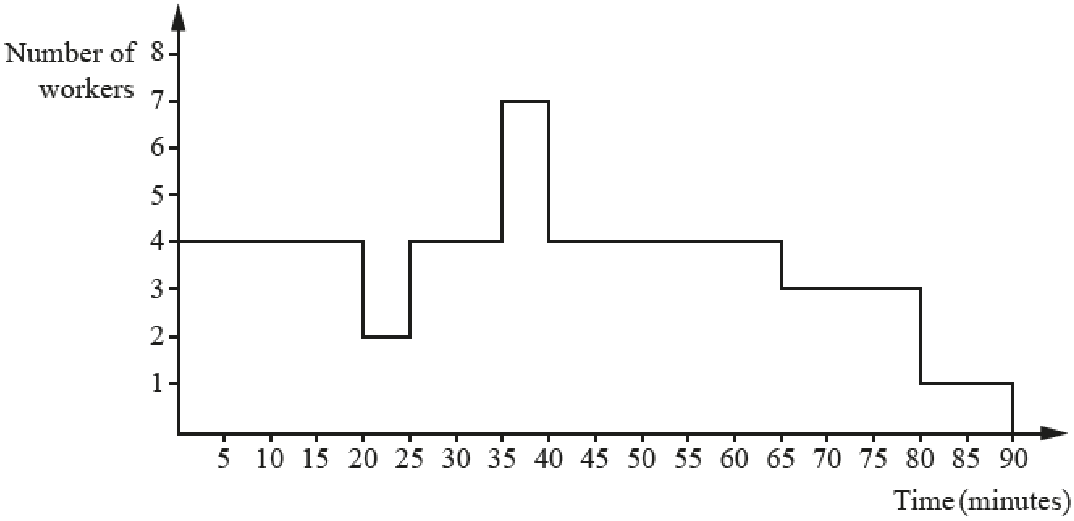


Fig. 2.2

(c) Complete the table in the Printed Answer Booklet showing the number of workers required for each activity in the project. **[3]**

Question 2 (d)

(d) Determine the shortest time in which the project can be completed when only four workers are available. **[2]**

Candidates often just gave what they believed to be the shortest time and did not explain how they had achieved this. The problem at 35 – 40 minutes needed to be identified and dealt with, including any consequences on later activities.

i	**OCR support**	The command words poster (available on the [OCR website]) gives the meaning of 'Determine' as 'Justification should be given for any results found, including working where appropriate.'

Question 3 (a) (i), (a)(ii) and (b)

3 Fig. 3 represents a system of pipes through which fluid flows continuously from a source node, S, to a sink node, T.
The weight on each arc shows the capacity of the corresponding pipe, in litres per minute.

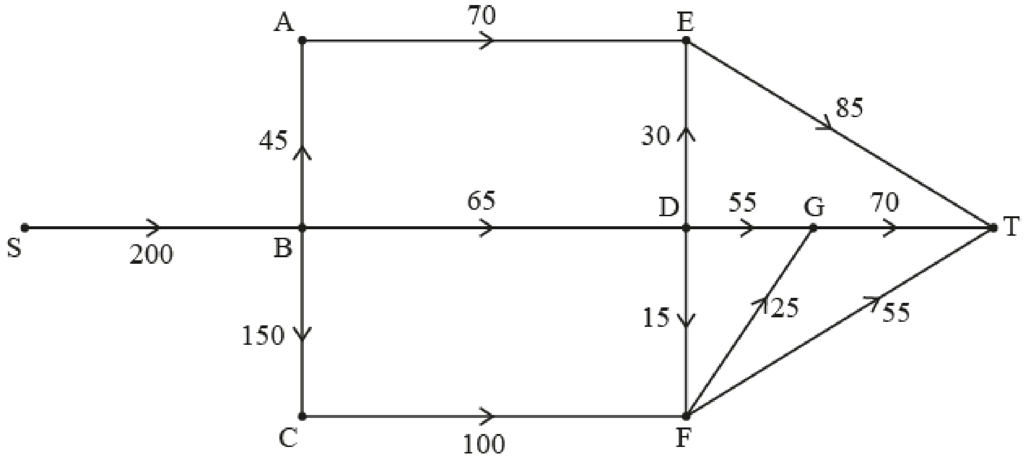


Fig. 3

(a) **(i)** The cut α partitions the vertices into sets {S, B, C}, {A, D, E, F, G, T}.
Calculate the capacity of cut α. **[1]**

(ii) The cut β partitions the vertices into sets {S, A, B, C, E, F}, {D, G, T}.
Calculate the capacity of cut β. **[1]**

(b) Using only the capacities of cuts α and β state what can be deduced about the maximum possible flow through the system. **[1]**

Most candidates correctly calculated the capacities of the given cuts, with only a minority not knowing how to deal with the arcs DE and DF where the direction of flow was 'backwards' across cut β.

Because we do not know the capacities of all possible cuts, what can be deduced is that the capacity of the minimum cut is at most 210 litres per minute, and hence, using maximum flow = minimum cut, that the maximum flow is less than or equal to 210 litres per minute (rather than equal to 210 as was often claimed).

Question 3 (c)

(c) Use the diagram in the Printed Answer Booklet to show how a flow of 190 litres per minute can be achieved. **[2]**

This should have been straightforward but some candidates forgot about the capacities of the arcs and some seemed to have used the diagram to answer part (e). The diagram needed to show a valid flow of 190 litres per minute, and not show excess capacities and potential backflows.

Question 3 (d)

(d) Use a suitable cut to prove that this is the maximum possible flow through the system. **[2]**

This required identification of a suitable saturated cut and an appropriate explanation. The proof needed to be more than just the statement 'maximum flow – minimum cut theorem'.

Exemplar 3

Cut
{S, B, C, F}, {A, E, D, G, T}
has capacity 190,

and by maximum cut = minimum flow,
if a cut and flow are
equal, they must be the
minimum cut, and maximum flow,
respectively.

This candidate in Exemplar 3 has given the cut and stated its capacity, stated the theorem and then explained that they have found a flow and a cut that are equal.

Question 3 (e)

An extra pipe is opened linking S to A. Let the capacity of this pipe be x litres per minute.

(e) Find, in terms of x where necessary, the maximum possible flow through the system. **[3]**

If the extra pipe is used with the flow from part (c), the flow becomes 190 + x litres per minute, provided this flow can get through the network.

The extra pipe is only useful for small values of x, for larger values of x the cut {S, A, B, C, D, E, F,G}, {T} imposes an upper limit of 210 litres per minute.

So the flow is 190 + x litres per minute up to x = 20 and 210 litres per minute for x greater than or equal to 20.

Question 4 (a) (i) and (a) (ii)

4 The table in Fig. 4.1 shows the unit cost, in hundreds of pounds, of transporting goods from each of four suppliers, A, B, C and D to each of three depots, X, Y and Z.
The margins of the table show the stock at each supplier and the demand at each depot.

	Demand	24	12	13
Stock		X	Y	Z
15	A	13	12	11
12	B	15	8	10
8	C	11	12	14
14	D	11	13	12

Fig. 4.1

The following LP formulation can be used to find the minimum total cost of delivering all the required stock.

Minimise $13AX + 12AY + 11AZ + 15BX + 8BY + 10BZ + 11CX + 12CY + 14CZ + 11DX + 13DY + 12DZ$

Subject to

$AX + AY + AZ = 15$

$BX + BY + BZ = 12$

$CX + CY + CZ = 8$

$DX + DY + DZ = 14$

$AX + BX + CX + DX = 24$

$AY + BY + CY + DY = 12$

$AZ + BZ + CZ + DZ = 13$

(a) Explain the purpose of each of the following lines from the LP formulation.

(i) Minimise $13AX + 12AY + \ldots + 12DZ$. **[2]**

(ii) $AX + BX + CX + DX = 24$. **[1]**

Setting up the LP problem inevitably involves a lot of text, but once candidates understood the context they were able to make good progress. The purpose of each of the given lines needed to be given in context, so saying that (i) is the objective was not sufficient, responses needed to refer to minimising the total cost of delivering the goods, and for (ii) some reference to the suppliers needing to fulfil the demand of 24 units at depot X was required.

Question 4 (b) (i) and (b) (ii)

The LP was run in an LP solver and the output is shown in Fig. 4.2.

VARIABLE	VALUE
AX	2.000000
AY	0.000000
AZ	13.000000
BX	0.000000
BY	12.000000
BZ	0.000000
CX	8.000000
CY	0.000000
CZ	0.000000
DX	14.000000
DY	0.000000
DZ	0.000000

Fig. 4.2

(b) **(i)** Interpret the output to give a solution to the transportation problem. **[1]**

(ii) Find the minimum total cost of delivering all the required stock. **[1]**

There was some confusion about what constituted a unit, so some candidates talked about transporting multiple units of stock and some talked about making multiple delivery journeys.

Most candidates gave a suitable interpretation of the output and the majority were able to calculate the total cost as £50 700.

Question 4 (c), (d) (i) and (d) (ii)

It is later found that the amount of stock held by supplier B is 16. All other suppliers have the same level of stock as before and there are no changes to the demand at any of the three depots.

Supplier B says that the only required change in the LP formulation is that the line $BX + BY + BZ = 12$ needs to be replaced with $BX + BY + BZ = 16$.

(c) Explain why supplier B is incorrect. **[1]**

(d) State the effect this change for supplier B has on

(i) the objective function in the original LP formulation, **[1]**

(ii) the constraints in the original LP formulation. **[1]**

Some candidates had not read the stem and said in part (c) that there were only 12 units available, but most realised that there was now more stock available than the total demand (the problem is unbalanced), although they had various suggestions for how this impacted on the problem.

In part (d)(i) some candidates thought that the unit costs should be changed to reflect the excess stock, although there was no indication of this in the question.

Exemplar 4

The amounts sent out by suppliers become

$\leq$ inequalities:

$AX + AY + AZ \leq 15$

$BX + BY + BZ \leq 16$ 16 replaces 12.

$CX + CY + CZ \leq 8$

$DX + DY + DZ \leq 14$

Exemplar 4 is a complete response to part (d)(ii).

Question 4 (e)

The modified LP problem (where the amount of stock held by supplier B is increased from 12 to 16) is run in the same LP solver and the minimum total cost of delivering all the required stock is found to be £50 300. The only differences from the solution to the original LP problem shown in Fig. 4.2 is in the amount of stock transported from suppliers A and B to depot Z.

(e) Determine the amount of stock for the modified LP problem that is now transported from suppliers A and B to depot Z. **[3]**

The command word 'determine' meant that a full explanation was required.

The two pieces of information about the number of units transported from A to Z and from B to Z led to a pair of simultaneous equations from which the amounts could be deduced.

A few candidates deduced that 4 units were transferred from AZ to BZ but did not give the amounts transported, as asked for in the question.

Some candidates explained that each unit transported from B to Z instead of A to Z saved £1100 - £1000 = £100 and then used the saving of £400 to deduce the amounts.

Question 5 (a) and (b)

5 The initial tableau for a two-stage simplex solution for a maximisation LP problem is shown in Fig. 5.1.

Q	P	x	y	z	s_1	s_2	s_3	s_4	a_1	a_2	RHS
1	0	3	1	2	0	–1	–1	0	0	0	50
0	1	–2	–3	$-k$	0	0	0	0	0	0	0
0	0	1	3	1	1	0	0	0	0	0	30
0	0	0	2	1	0	–1	0	0	1	0	20
0	0	3	–1	1	0	0	–1	0	0	1	30
0	0	1	1	2	0	0	0	1	0	0	40

Fig. 5.1

(a) Formulate the information given in Fig. 5.1 as an LP problem by

- Stating the objective function for the original LP problem and
- listing the constraints as simplified inequalities with integer coefficients. **[3]**

(b) Show how the first numerical row of Fig. 5.1 was formed. **[2]**

This is a two-stage simplex in which the objective for the original LP problem was required, i.e. maximise $P = 2x + 3y + kz$. Some candidates gave expressions for both P and Q and some decided to make up a value for k.

The constraints for the original LP were required in the form described, and not with slack or surplus variables still included. This part was often done well, although several candidates lost the coefficient of z in the final constraint.

Part (b) was about eliminating the artificial variables. Candidates needed to write three equations, one giving Q as the sum of the artificial variables and one giving each artificial variable as a function of x, y, z and the slack variables. Substituting for the artificial variables and rearranging then gave the required equation.

Exemplar 5

$2y + z - s_2 + a_1 = 20$

$a_1 = 20 + s_2 - 2y - z$

$3x + z - y - s_3 + a_2 = 30$

$a_2 = 30 + s_3 + y - z - 3x$

$Q = a_1 + a_2$

$= 20 + s_2 - 2y - z + 30 + s_3 + y - z - 3x$

$= 50 - y - 2z - 3x + s_2 + s_3$

$Q + 3x + y + 2z - s_2 - s_3 = 50$

Exemplar 5 is a complete response to part (b).

Question 5 (c)

After three iterations of the first stage of the two-stage simplex method the tableau shown in Fig. 5.2 was produced.

Q	P	x	y	z	s_1	s_2	s_3	s_4	a_1	a_2	RHS
1	0	0	0	0	0	0	0	0	–1	–1	0
0	1	0	0	0	$\frac{5}{2}-k$	$\frac{7}{3}-\frac{5}{3}k$	$\frac{1}{6}-\frac{1}{3}k$	0	$-\frac{7}{3}+\frac{5}{3}k$	$-\frac{1}{6}+\frac{1}{3}k$	$\frac{70}{3}+\frac{40}{3}k$
0	0	1	0	0	$\frac{1}{2}$	$\frac{2}{3}$	$-\frac{1}{6}$	0	$-\frac{2}{3}$	$\frac{1}{6}$	$\frac{20}{3}$
0	0	0	1	0	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$	0	$-\frac{1}{3}$	$-\frac{1}{6}$	$\frac{10}{3}$
0	0	0	0	1	–1	$-\frac{5}{3}$	$-\frac{1}{3}$	0	$\frac{5}{3}$	$\frac{1}{3}$	$\frac{40}{3}$
0	0	0	0	0	1	$\frac{7}{3}$	$\frac{2}{3}$	1	$-\frac{7}{3}$	$-\frac{2}{3}$	$\frac{10}{3}$

Fig. 5.2

(c) Explain how the tableau in Fig. 5.2 shows that the first stage has been completed. **[2]**

The two aspects here were that the solution for *Q* is feasible and optimal. This involved identifying that $Q = 0$ and that the only negative entries in the objective row are in the columns for the artificial variables. Several candidates did not seem to appreciate that 'negative' and 'non-positive' are not the same thing.

	AfL	'Positive' means 'greater than zero' and 'negative' means 'less than zero', in both cases excluding zero. 'Non-negative' means 'greater than or equal to zero' and 'non-positive' means 'less than or equal to zero'.

Question 5 (d), (e)(i) and (e)(ii)

It is given that the tableau in Fig. 5.2 does not give an optimal solution to the LP problem.

(d) Using the tableau in Fig. 5.2 as the starting point for the second stage

- reduce the tableau so that the second stage can be started and
- carry out one iteration of the second stage of the two-stage simplex method, using an entry in the s_2 column as the pivot element. **[4]**

Given that an optimal solution to the LP problem is found after one iteration of the second stage of the two-stage simplex method,

(e) **(i)** determine the range of values of k, **[2]**

(ii) state the optimal values of x, y and z and give, in terms of k, the corresponding maximum value of P. **[2]**

Most candidates drew up the tableau for the start of the second stage but some found dealing with the variable *k* was too much for them, and some ran out of time.

The candidates who identified the pivot, dealt with the basic columns and the constraint rows, but gave up on the non-zero entries in the objective row, were credited well, and could also achieve credit in part (e) (ii).

Candidates who attempted part (e) (i) needed to make sure that the expressions in the objective row were non-negative. Part (e) (ii) required the interpretation of the output from the final tableau.

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