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A LEVEL

Examiners' report

FURTHER MATHEMATICS B (MEI)

H645
For first teaching in 2017

Y421/01 Summer 2019 series

Version 1

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects that caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the question paper can be downloaded from OCR.

Paper Y421 series overview

This is one of the two Major options for the reformed A Level Further Mathematics B (MEI). It is a two-hour fifteen-minute paper consisting of 120 marks. The paper consists of two sections, A and B. Section A will have between 25 and 35 marks and will comprise more straight-forward questions. Section B will have between 85 and 95 marks and will comprise a mixture of more and less straight-forward questions.

Inevitably, the report that follows will concentrate on aspects of the candidates' performance where improvement is possible, to assist centres on preparing candidates for future series. However, this should not obscure the fact that a significant number of candidates who sat this first assessment of Y421 produced solutions that were a pleasure for examiners to assess. Many candidates demonstrated a most impressive level of mathematical ability and insight that enabled them to meet the various challenges posed by this paper on all the associated mechanics content; precision, command of correct mathematical notation and excellent presentational skills were evident in many scripts.

The Specification (available from https://ocr.org.uk/alevelfurthermathsmei) includes some guidance about the level of written evidence required in questions; these were provided to reflect the increased functionality of currently available calculators and the changes to the Assessment Objectives, since there is a significant change in these from when the equivalent legacy qualifications were designed. There are a number of questions on this paper that began with 'In this question you must show detailed reasoning'; to quote the Specification, 'when a question includes this instruction candidates must give a solution which leads to a conclusion showing a detailed and complete analytical method. Their solution should contain enough detail to allow the line of their argument to be followed. This is not a restriction on a candidate's use of a calculator when tackling a question…but it is a restriction on what will be accepted as evidence of a complete method'. The Specification then goes on to include several examples that centres should consider to help future candidates understand exactly what is required when this request appears in future series. This command phrase also features in Questions 7 and 13(c) here, which centres should use for further guidance.

The command word 'determine' in a question does not imply that candidates should simply find the answer, but (again, from the Specification) 'this command word indicates that justification should be given for any results found, including working where appropriate'. This command word featured in Question 13(c). The phrase 'show that' generally indicates that the answer has been given and that candidates should respond with working that has enough detail to cover every step of their method. This command phrase features in Questions 9(a), 9(b), 10(a), 12(a), 12(c), 13(a) and 13(b).

In questions that use the command word 'find', while there is no specific level of working needed to justify answers, method marks may still be available for valid attempts that do not result in a correct answer. Standard advice included in the specification (in the 'Use of calculators' section) that candidates should state explicitly any expressions, integrals, parameters and variables that they use a calculator to evaluate (using correct mathematical notation rather than model specific calculator notation) also applies. There are several examples where the question specifically asks for an exact value or in surd form and hence an approximate decimal equivalent will not gain full credit; examples here are Questions 4(a) and 7. Regardless of the final accuracy required, candidates should be careful of not rounding prematurely, as well as also taking care to avoid over specifying rounded answers where the context does not support that level of accuracy.

Finally, unless told otherwise, the value that candidates should use for the acceleration due to gravity (g) is 9.8 and not 9.81 or 10 (and this value is stated explicitly on the front cover of the examination paper).

Section A overview

The majority of the five questions in this section worked as intended and provided candidates with a straight-forward start to the paper.

Question 1(a)

1 Three forces represented by the vectors $-4\mathbf{i}$, $\mathbf{i}+2\mathbf{j}$ and $k\mathbf{i}-2\mathbf{j}$ act at the points with coordinates (0, 0), (3, 0) and (0, 4) respectively.

(a) Given that the three forces form a couple, find the value of k. **[2]**

The concept of a set of forces forming a couple was surprisingly not known by many candidates and quite a number left this, and the next part, blank. As the three forces form a couple, all that was required to find *k* was to resolve in the direction of the **i**-component and obtain $-4+1+k=0$, so $k=3$.

Question 1(b)

(b) Find the magnitude and direction of the couple. **[3]**

Many candidates left this part blank and very few realised that the most efficient way of finding the magnitude of the couple was to take moments about O (as three of the five components of the forces acted through O). For some candidates the only mark they scored in this part was to correctly state the direction of the couple as clockwise.

Question 2

2 The Reynolds number, R, is an important dimensionless quantity in fluid dynamics; it can be used to predict flow patterns when a fluid is in motion relative to a surface.

The Reynolds number is defined as $R=\dfrac{\rho u l}{\mu}$,

where ρ is the density of the fluid, u is the velocity of the fluid relative to the surface, l is the distance travelled by the fluid and μ is the viscosity of the fluid.

Find the dimensions of μ. **[4]**

This question was answered extremely well, with the vast majority of candidates correctly stating the dimensions of ρ as ML^{-3} and the dimensions of u as LT^{-1}. Although most candidates correctly rearranged to make μ the subject and used the fact that *R* was dimensionless to correctly find the dimensions of μ, it was surprising that many candidates gave the dimension of μ as $M^{-1}LT$.

Question 3(a)

3 A ball of mass 2 kg is moving with velocity $(3\mathbf{i}-2\mathbf{j})\,\mathrm{m\,s^{-1}}$ when it is struck by a bat. The impulse on the ball is $(-8\mathbf{i}+10\mathbf{j})\,\mathrm{N\,s}$.

(a) Find the speed of the ball immediately after the impact. **[4]**

Most candidates correctly applied the result that impulse equals change in momentum to obtain the velocity of the ball after impact as $-\mathbf{i}+3\mathbf{j}$. Many candidates however did not go on to calculate the corresponding speed and so scored only the first two marks in this part.

Question 3(b)

(b) State one modelling assumption you have used in answering part **(a)**. **[1]**

Several candidates gave a correct modelling assumption, for example that the impact of the ball and bat is assumed to be instantaneous or that the ball is modelled as a particle. Many candidates assumed however that the collision was elastic or no energy was lost in the collision.

Question 4(a)

4

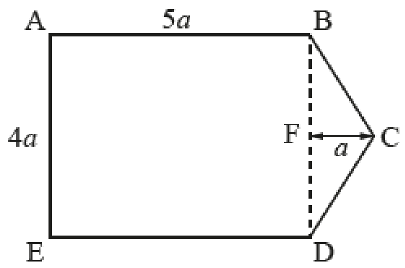


Fig. 4

Fig. 4 shows a uniform lamina ABCDE such that ABDE is a rectangle and BCD is an isosceles triangle. AB = $5a$, AE = $4a$ and BC = CD. The point F is the midpoint of BD and FC = a.

(a) Find, in terms of a, the exact distance of the centre of mass of the lamina from AE. **[4]**

This part was answered extremely well with most candidates correctly taking moments about the line of action through AE to find the distance of the centre of mass of the lamina from AE. The most common error was in taking the horizontal distance of the centre of mass of the triangular lamina as $5.5a$ from AE, instead of the correct $\frac{16}{3}a$. It was pleasing to note that most candidates correctly gave the distance as an exact answer as requested.

Question 4(b)

The lamina is freely suspended from B and hangs in equilibrium.

(b) Find the angle between AB and the downward vertical. **[2]**

Most candidates used either $\tan\theta = \frac{2a}{5a - \bar{x}}$ or its reciprocal to find the required angle between AB and the downward vertical. Only a small minority incorrectly used $\tan\theta = \frac{2a}{\bar{x}}$ or didn't show enough working to make it clear if they were obtaining a relevant angle.

Question 5(a)

5 A particle P of mass 4 kilograms moves in such a way that its position vector at time t seconds is $\mathbf{r}$ metres, where

$$\mathbf{r} = 3t\mathbf{i} + 2e^{-3t}\mathbf{j} .$$

(a) Find the initial kinetic energy of P. **[4]**

This question was probably the best answered on the whole paper, with nearly all candidates correctly differentiating **r** and obtaining the correct initial velocity as 3**i** – 6**j**. From this point nearly all candidates correctly applied the formula for kinetic energy and obtained the correct value of 90J.

Question 5(b)

(b) Find the time when the acceleration of P is 2 metres per second squared. **[3]**

Most candidates correctly equated their second derivative of **r** to 2 and correctly solved the equation $18e^{-3t} = 2$ by taking logs to find the time.

Section B overview

The questions in Section B were less straight-forward in nature than those in Section A and also provided candidates with an opportunity to apply their understanding of the mechanics content in a less structured setting.

Question 6

6

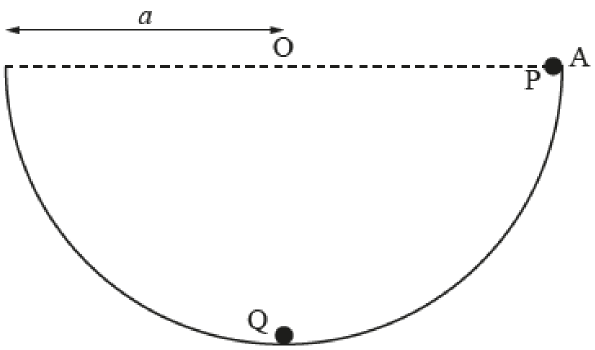


Fig. 6

The rim of a smooth hemispherical bowl is a circle of centre O and radius a. The bowl is fixed with its rim horizontal and uppermost. A particle P of mass m is released from rest at a point A on the rim as shown in Fig. 6.

When P reaches the lowest point of the bowl it collides directly with a stationary particle Q of mass $\frac{1}{2}m$. After the collision Q just reaches the rim of the bowl.

Find the coefficient of restitution between P and Q. **[7]**

	AfL	When a question involves multiple stages of working that include the need to introduce a certain number of variables that have not been defined in the question, it is the responsibility of the candidate to define the variables they use. In this question many candidates did not make it explicitly clear which letters were being used for the speed of P before and after impact and for the corresponding speed of Q after impact.

The responses to this question were mixed and while some candidates set their working out in a logical manner, examiners did report that at times it was difficult to follow some candidates' working.

Most candidates correctly calculated the expression for the speed of P before its impact with Q as $\sqrt{2ga}$ (via the principle of conservation of energy), but many candidates went on and applied the same conservation of energy principle for finding the speed of Q after impact incorrectly as $\frac{1}{2}mga = \frac{1}{2}mv_Q^{\ 2}$, instead of the correct $\frac{1}{2}\left(\frac{1}{2}m\right)v_Q^{\ 2} = \frac{1}{2}mga$. A small minority of candidates immediately realised that due to the symmetry of the problem, if the speed of P before impact with Q was $\sqrt{2ga}$ then the speed of Q after impact with P would also be $\sqrt{2ga}$. Most candidates did go and apply the conservation of linear momentum and Newton's experimental law correctly for the impact between P and Q and it was pleasing to see that many candidates correctly found the value of e as 0.5.

Exemplar 1

For P at collision, $\frac{1}{2}mv^2 = mgah$

$v^2 = 2gah$

$v^2 = 2ga$ M1 A1

This is the same for Q just after the collision

PCLM : $M_pU_p + M_QU_Q = M_pV_p + M_QV_Q$

$M\sqrt{2ga} + \frac{1}{2}m \times 0 = MV_p + \frac{1}{2}m \times V_Q$ M1

NEL: $(V_P - V_Q) = e(U_P - U_Q)$

Conservation of energy: $M_p ga = KE(P) + PE(Q)$

$Mga = \frac{1}{2}mV_p^2 + \frac{1}{2}mga$

$\frac{1}{2}mV_p^2 = Mga - \frac{1}{2}mga$

$\frac{1}{2}mV_p^2 = \frac{1}{2}mga$

$V_p^2 = ga$

$V_p = \sqrt{ga}$, $V_Q = \sqrt{2ga}$, $U_p = \sqrt{2ga}$, $U_Q = 0$ B1

$V_Q - V_P = e(U_P - U_Q)$

$\sqrt{2ga} - \sqrt{ga} = e(\sqrt{2ga} - 0)$ M1 M0

$\sqrt{ga}(\sqrt{2} - 1) = \sqrt{2}e \times \sqrt{ga}$

$\sqrt{2} - 1 = \sqrt{2}e$

$e = \frac{\sqrt{2} - 1}{\sqrt{2}}$ A0

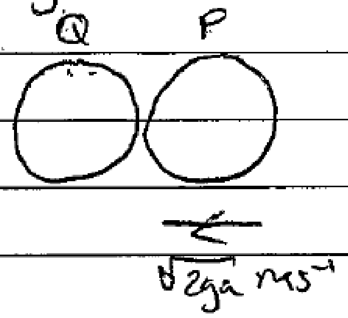
This response gained five out of the seven marks. The candidate's response is easy to follow with suitable detail at each stage indicating the methods (and notation) being employed. The candidate correctly derived the speed of P before impact and applies both the conservation of linear momentum and, towards the bottom of the response, Newton's experimental law correctly. They also make the correct comment that the speed of Q after impact is equal to the speed of P before impact. Their error is a conceptual one of using the conservation of energy to try to work out the speed of P after impact, when in fact energy is not conserved when an impact occurs; this error cost them the final two marks.

Exemplar 2

Speed of P at lowest point:

Conservation of energy:

$mga = \frac{1}{2}mv_P^2$, $ga = \frac{1}{2}v_P^2$ $\therefore v_P = \sqrt{2ga}$



Let U_P be speed of P just before collision

Let V_P be speed of P after collision parallel to line of centres.

Conservation of linear momentum along line of centres:

$mU_P = mV_P + \frac{1}{2}mV_Q$

$U_P = V_P + \frac{1}{2}V_Q$, $\sqrt{2ga} = V_P + \frac{1}{2}V_Q$. ①

Newton's Law of impact:

$eU_P = V_Q - V_P$

$\sqrt{2ga}\,e = V_Q - V_P$ ②

① + ② : $\sqrt{2ga} + \sqrt{2ga}\,e = \frac{3}{2}V_Q$.

$\sqrt{2ga}\,(1+e) = \frac{3}{2}V_Q$.

$\frac{1}{2}m(V_Q)^2 = mga$, as just reaches rim.

$\therefore V_Q = \sqrt{2ga}$.

$\sqrt{2ga}\,(1+e) = \frac{3}{2} \times \sqrt{2ga}$

$(1+e) = 1.5$

$\therefore e = 0.5$

This response scored full marks and showed all the correct detail required. The annotations help to follow the candidate's method and working towards the correct result.

Question 7

7 **In this question you must show detailed reasoning.**

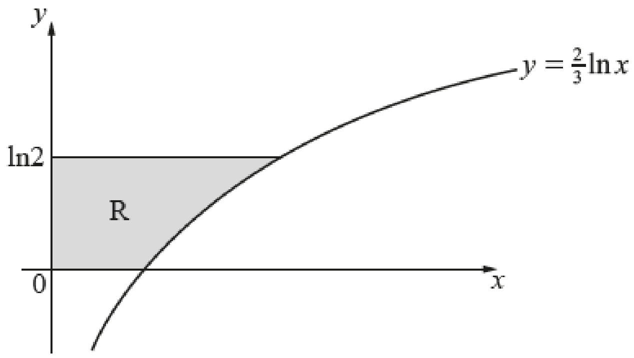


Fig. 7

Fig. 7 shows the curve with equation $y = \frac{2}{3}\ln x$. The region R, shown shaded in Fig. 7, is bounded by the curve and the lines $x = 0$, $y = 0$ and $y = \ln 2$. A uniform solid of revolution is formed by rotating the region R completely about the y-axis.

Find the exact y-coordinate of the centre of mass of the solid. **[8]**

This question was answered extremely well by most candidates with only a few not reading the question carefully enough and instead calculating the *y*-coordinate of the centre of mass of a corresponding uniform lamina instead of the required solid of revolution. As this question requested detailed reasoning it wasn't enough to simply state that the volume of the solid was given by $V = \pi\int_0^{\ln 2}\left(e^{\frac{3}{2}y}\right)^2 dx = \frac{7}{3}\pi$.

Nearly all candidates who correctly stated that $V\bar{y} = \pi\int_0^{\ln 2} y\left(e^{\frac{3}{2}y}\right)^2 dy$ completed the required integration by parts correctly and then went on to obtain the correct answer of $\bar{y} = \frac{8}{7}\ln 2 - \frac{1}{3}$ from correctly applying the formula $\bar{y} = \frac{V\bar{y}}{V}$

Question 8(a)

8 A car of mass 800 kg travels up a line of greatest slope of a straight road inclined at 5° to the horizontal.

The power developed by the car is constant and equal to 25 kW. The resistance to the motion of the car is constant and equal to 750 N.

The car passes through a point A on the road with speed $7\,\text{m s}^{-1}$.

(a) Find

- the acceleration of the car at A,
- the greatest steady speed at which the car can travel up the hill. **[5]**

Part (a) was answered extremely well, with nearly all candidates correctly applying Newton's second law parallel to the plane and using the result that $P = Dv$ to find both the acceleration of the car at A and also the greatest steady speed at which the car can travel up the hill.

Question 8(b)

The car later passes through a point B on the road where AB = 131 m. The time taken to travel from A to B is 10.4 s.

(b) Calculate the speed of the car at B. **[6]**

Most candidates realised that the way to calculate the speed of the car at B was to use the work-energy principle and most candidates correctly calculated the work done by the car, the work done by the resistive force, the change in potential and kinetic energies and combined these together into one equation. Basic errors involving signs were often seen by examiners, together with the common error of including $800g(131\sin 5)$ twice (once as part of a change in PE term and also as part of the work done by the resistive force) when it should only be included once. Several candidates attempted to use differential equations and while some obtained a correct integral equation of the form $\int_7^{v_1} \frac{800v}{\frac{25000}{v} - 750 - 800g\sin 5}\,dv = 131$ or $\int_7^{v_1} \frac{800}{\frac{25000}{v} - 750 - 800g\sin 5}\,dv = 10.4$, none were successful in obtaining the correct answer to the required degree of accuracy.

?	**Misconception**	Some candidates thought that it was possible to use the *suvat* equations in this part. The *suvat* equations can only be applied in circumstances when the acceleration is constant. As the power developed by the car is constant throughout the motion and the speed of the car is varying from point A to point B, this implies that the driving force is varying too (from the result that power is equal to the driving force times velocity) and hence, via Newton's second law, the acceleration is varying too.

Question 9(a)

9

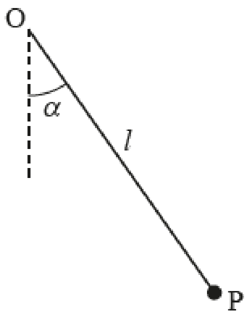


Fig. 9

A particle P of mass m is joined to a fixed point O by a light inextensible string of length l. P is released from rest with the string taut and making an acute angle α with the downward vertical, as shown in Fig. 9.

At a time t after P is released the string makes an angle θ with the downward vertical and the tension in the string is T. Angles α and θ are measured in radians.

(a) Show that

$$\left(\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)^2 = \frac{2g}{l}\cos\theta + k_1,$$

where k_1 is a constant to be determined in terms of g, l and α. **[4]**

This question discriminated extremely well between able and less able candidates, with many not realising that the best approach was to apply the conservation of energy when P is at the positions α and θ. Of those that correctly stated that $\frac{1}{2}mv^2 = mgl(\cos\theta - \cos\alpha)$, many did not realise that $v = l\frac{\mathrm{d}\theta}{\mathrm{d}t}$ and instead gave an answer of $\left(\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)^2 = 2gl(\cos\theta - \cos\alpha)$ even though this was clearly not of the correct form as stated in the question.

Question 9(b)

(b) Show that

$$T = 3mg\cos\theta + k_2,$$

where k_2 is a constant to be determined in terms of m, g and α. **[3]**

The responses seen in part (b) were slightly better than those seen in part (a), with many candidates correctly applying Newton's second law radially and obtaining $T - mg\cos\theta = ml\omega^2$. Only the most able candidates correctly substituted the answer from part (a) to obtain the correct result that $T = 3mg\cos\theta - 2mg\cos\alpha$.

Question 9(c)

It is given that α is small enough for α^2 to be negligible.

(c) Find, in terms of m and g, the approximate tension in the string. **[2]**

Due to most candidates failing to make any significant progress in the earlier parts of this question very few correctly set $\cos\alpha$ and $\cos\theta$ equal to 1 in their expression for the tension and obtained the result that $T \approx mg$.

Question 9(d)

(d) Show that the motion of P is approximately simple harmonic. **[3]**

Only a small minority who attempted this part used the method of differentiating their expression in part (a) with respect to t to find an expression for the angular acceleration of P in terms of θ . Those that did attempt this part mostly began by applying Newton's second law radially, but very few correctly stated that $ml\ddot{\theta} = -mg\sin\theta$. While some candidates did use the small angle approximation for sine correctly and obtain $\frac{d^2\theta}{dt^2} + \frac{g}{l}\theta \approx 0$, many did not then state that this differential equation for the motion of P was (approximately) simple harmonic.

Question 10(a)

10 A particle P, of mass m, moves on a rough horizontal table. P is attracted towards a fixed point O on the table by a force of magnitude $\frac{kmg}{x^2}$, where x is the distance OP.

The coefficient of friction between P and the table is μ.

P is initially projected in a direction directly away from O. The velocity of P is first zero at a point A which is a distance a from O.

(a) Show that the velocity v of P, when P is moving away from O, satisfies the differential equation

$$\frac{d}{dx}(v^2) + \frac{2kg}{x^2} + 2\mu g = 0.$$

[3]

While most candidates correctly applied Newton's second law and used the fact that $F = \mu R$ (so scoring the first two method marks) many did not know how to obtain the given answer from $a = -\frac{kg}{x^2} - \mu g$. Only the most able candidates re-wrote the acceleration as $v\frac{dv}{dx}$ and showed explicitly that this was equal to $\frac{1}{2}\frac{d}{dx}\left(v^2\right)$.

Question 10(b)

(b) Verify that

$$v^2 = 2gk\left(\frac{1}{x} - \frac{1}{a}\right) + 2\mu g(a - x).$$

[3]

While some candidates verified that when $x = a$, $v = 0$ and others verified that the given result in part (b) was a solution to the differential equation in (a), very few candidates did both.

?	**Misconception**	The word 'verify' does not have the same mathematical meaning as 'show that'. Many candidates in this part solved the differential equation by the method of separation of variables and used the given conditions to calculate the constant of integration. While this approach could score all three marks it was a time-consuming way of tackling the problem. Furthermore, the solution to differential equations (apart from those found via SHM) are not required content for this option and it may not always be possible to solve the differential equation that candidates are being asked to verify.

Question 10(c)

(c) Find, in terms of k and a, the range of values of μ for which P remains at A. **[2]**

Many candidates incorrectly thought that the way to solve this problem was to set $v = 0$ in either the given result in parts (a) or (b) instead of considering the relationship between the attractive force and frictional contact force. Of those that did, many had a strict inequality instead of the correct $\mu \geq \frac{k}{a^2}$.

Question 11(a)

11 Two uniform smooth spheres A and B have equal radii and are moving on a smooth horizontal surface. The mass of A is 0.2 kg and the mass of B is 0.6 kg.

The spheres collide obliquely. When the spheres collide the line joining their centres is parallel to **i**.

Immediately before the collision the velocity of A is $\mathbf{u}_A \, \mathrm{m\,s^{-1}}$ and the velocity of B is $\mathbf{u}_B \, \mathrm{m\,s^{-1}}$. The coefficient of restitution between A and B is 0.5.

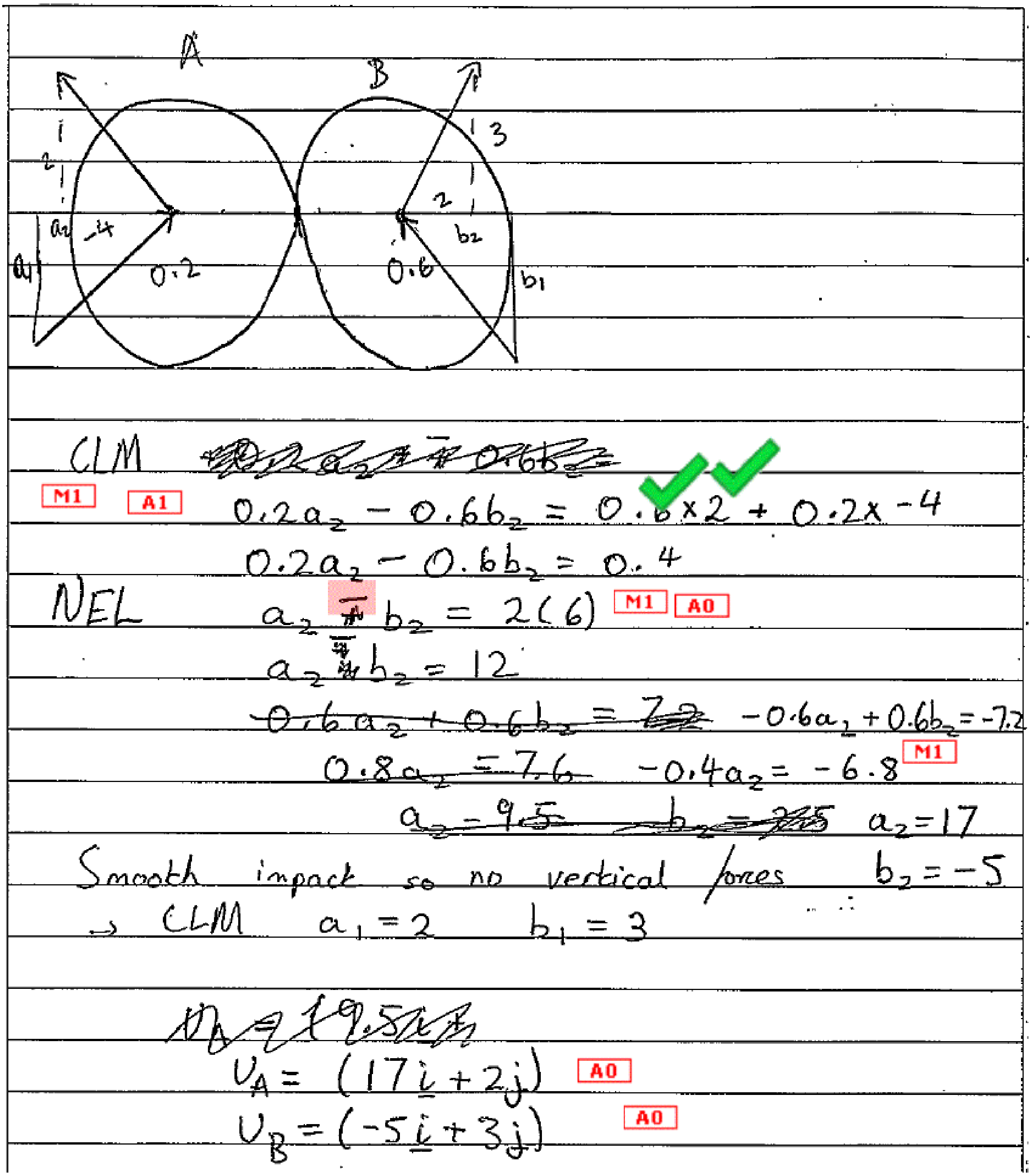
Immediately after the collision the velocity of A is $(-4\mathbf{i}+2\mathbf{j})\,\mathrm{m\,s^{-1}}$ and the velocity of B is $(2\mathbf{i}+3\mathbf{j})\,\mathrm{m\,s^{-1}}$.

(a) Find $\mathbf{u}_A$ and $\mathbf{u}_B$. **[7]**

What was supposed to be a relatively straight-forward application of the conservation of linear momentum (CLM) and Newton's experimental law (NEL) parallel to the line of centres at impact was made more difficult by candidates thinking they had to introduce unnecessary angles when applying these concepts. Several candidates produced a correct CLM equation, for example $0.2u_A + 0.6u_B = 0.2(-4) + 0.6(2)$, but they then had an inconsistent NEL equation, e.g. $4 + 2 = 0.5(u_A + u_B)$. Candidates are reminded as a check that in a situation like this the coefficient of u_B in the two equations should have different signs. Although many candidates correctly found the value of their u_A and u_B, many went on to find the speed of A and B after collision even though $\mathbf{u}_A$ and $\mathbf{u}_B$ were clearly defined as velocities in the question.

?	**Misconception**	Many candidates in this part wrote the equation pertaining to the conservation of linear momentum as $0.2u_A + 0.6u_B = 0.2\begin{pmatrix}-4\\2\end{pmatrix} + 0.6\begin{pmatrix}2\\3\end{pmatrix}$, with a similar set up for Newton's experimental law. This indicated to examiners a lack of understanding regarding the fundamental principle of oblique impact, this being that the linear momentum of each object is conserved in the direction perpendicular to the line of impulse. While candidates could obtain the 'correct' **i** components for the velocities of A and B before the impact, the **j** components (which should have been unchanged) were incorrect and due to the fundamental error in applying these two results, no marks were given in this part.

Exemplar 3



This response scored four out of a possible seven marks. The candidate correctly applied the conservation of linear momentum and obtained the equation $0.2a_2 - 0.6b_2 = 0.4$. When the candidate applied Newton's experimental law this was not consistent with their earlier equation (from the conservation of linear momentum) as they should have had $a_2 + b_2 = 12$ (as the signs of the coefficients of the b_2 terms should have been different in these two equations). The candidate did earn the final method mark for solving their simultaneous equations and having the correct **j** components for their two velocities.

Question 11(b)

After the collision B collides with a smooth vertical wall which is parallel to **j**.

The loss in kinetic energy of B caused by the collision with the wall is 1.152 J.

(b) Find the coefficient of restitution between B and the wall. **[3]**

While most candidates appreciated that if the vertical wall was parallel to **j** this implied that the velocity of B after impact would be $-2e\mathbf{i}+3\mathbf{j}$, many tried instead to use the speed of B as it hit the wall or the velocities before the impact between A and B and therefore they made little progress in this part.

Question 11(c)

(c) Find the angle through which the direction of motion of B is deflected as a result of the collision with the wall. **[4]**

Only the most able candidates realised that the deflected angle was given by $\arctan\left(\frac{2}{3}\right)+\arctan\left(\frac{2e}{3}\right)$.

Question 12(a)

12

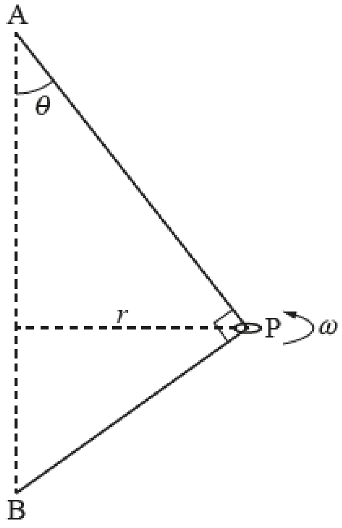


Fig. 12

The ends of a light inextensible string are fixed to two points A and B in the same vertical line, with A above B. The string passes through a small smooth ring of mass m. The ring is fastened to the string at a point P.

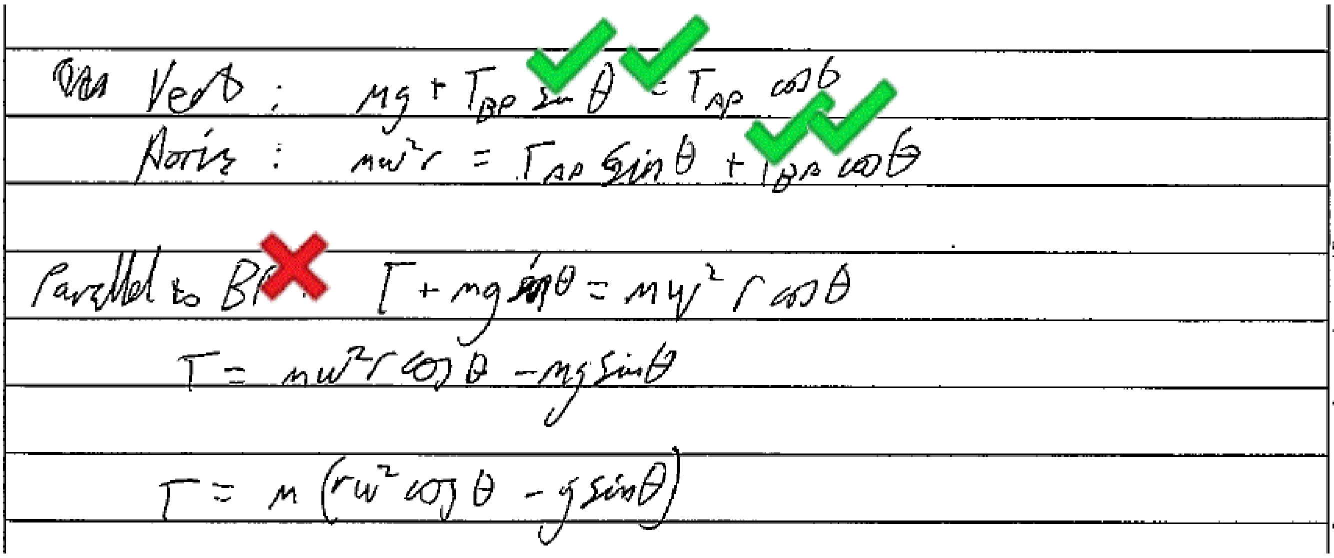
When the string is taut the angle APB is a right angle, the angle BAP is θ and the perpendicular distance of P from AB is r.

The ring moves in a horizontal circle with constant angular velocity ω and the string taut as shown in Fig. 12.

(a) By resolving horizontally and vertically, show that the tension in the part of the string BP is $m\left(r\omega^2 \cos\theta - g\sin\theta\right)$. **[6]**

The two most costly errors in this part were using T to represent the tension in both parts of the string or not reading the question carefully and instead resolved parallel to BP and not horizontally and vertically as requested. Of those that correctly resolved vertically and horizontally and obtained the equations $T_{AP}\cos\theta = T_{BP}\sin\theta + mg$ and $T_{AP}\sin\theta + T_{BP}\cos\theta = mr\omega^2$, most showed enough working (this is a 'show that' question) in eliminating the tension in AP to obtain the given result for string BP.

Exemplar 4



Vert : $mg + T_{BP}\sin\theta = T_{AP}\cos\theta$

Horiz : $m\omega^2 r = T_{AP}\sin\theta + T_{BP}\cos\theta$

Parallel to BP : $T + mg\sin\theta = m\omega^2 r\cos\theta$

$T = m\omega^2 r\cos\theta - mg\sin\theta$

$T = m(r\omega^2\cos\theta - g\sin\theta)$

This response scored the first four marks for correctly resolving vertically and horizontally. Instead of solving these two equations simultaneously the candidate then resolved parallel to BP, and even though they obtained the correct answer the final two marks could not be given.

Question 12(b)

(b) Find a similar expression, in terms of r, ω, m, g and θ, for the tension in the part of the string AP. **[2]**

This part was answered extremely well, with most candidates correctly obtaining an answer of the form $m\left(r\omega^2\sin\theta + g\cos\theta\right)$ (or an equivalent correct expression in sine and cosine only).

Question 12(c)

It is given that AB = $5a$ and AP = $4a$.

(c) Show that $16a\omega^2 > 5g$. **[3]**

Very few candidates used the given information in this part to obtain $r = 2.4a$ and one (or both) of $\sin\theta = 0.6$ or $\cos\theta = 0.8$. Of those that did, many did not make the connection that the given result came from realising that the tension in the part of the string BP had to be greater than zero.

Question 12(d)

The ring is now free to move on the string but remains in the same position on the string as before. The string remains taut and the ring continues to move in a horizontal circle.

(d) Find the period of the motion of the ring, giving your answer in terms of a, g and π. **[5]**

This part was often left blank or very little progress was made as many candidates believed that the period of the motion of the ring was somehow linked to the inequality from the previous part. Of those that realised that if the ring was now free to move on the string then the tension in both parts of the string was equal, most then went on to obtain the correct angular speed in terms of g and a and hence the period by using the formula $T = \frac{2\pi}{\omega}$.

Question 13(a)

13

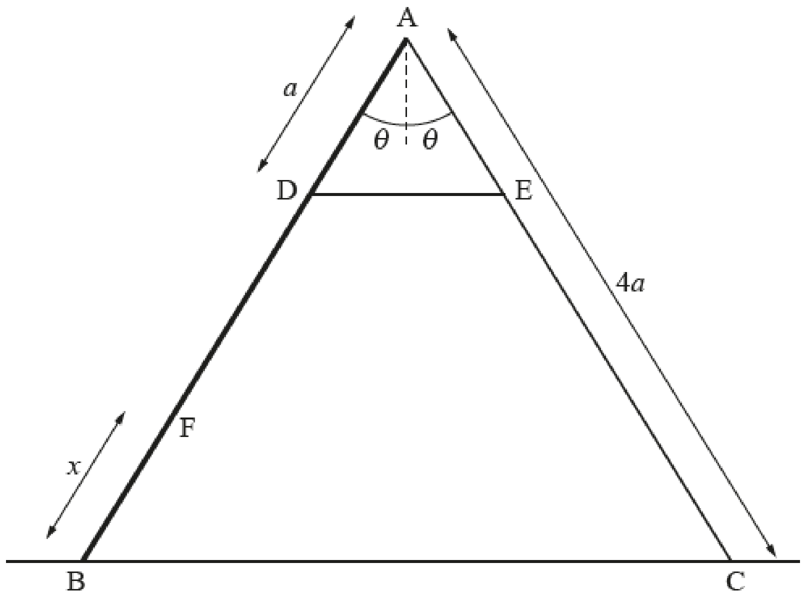


Fig. 13

A step-ladder has two sides AB and AC, each of length $4a$. Side AB has weight W and its centre of mass is at the half-way point; side AC is light.

The step-ladder is smoothly hinged at A and the two parts of the step-ladder, AB and AC, are connected by a light taut rope DE, where D is on AB, E is on AC and AD = AE = a.

A man of weight $4W$ stands at a point F on AB, where BF = x.

The system is in equilibrium with B and C on a smooth horizontal floor and the sides AB and AC are each at an angle θ to the vertical, as shown in Fig. 13.

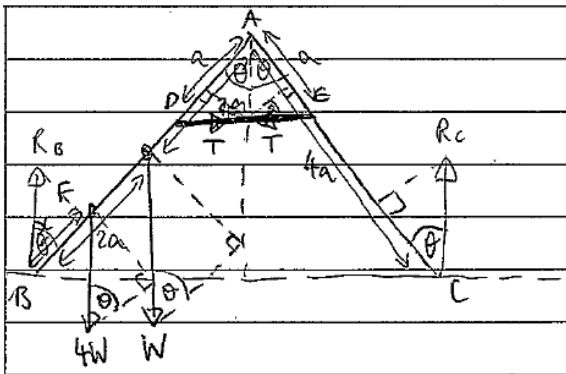
(a) By taking moments about A for side AB of the step-ladder and then for side AC of the step-ladder show that the tension in the rope is

$$W\left(1 + \frac{2x}{a}\right)\tan\theta.$$

[7]

Most candidates attempted this part and many correctly took moments about A for side AB and then for side AC and obtained two equations involving the correct number of terms. Some candidates did not realise that each equation needed to contain a normal contact force at the two points of contact with the ground and some who did realise this assumed that these were the same force. Some candidates in their moment equations did not have a force times a distance and so lost the corresponding method mark. Furthermore, some of the forces were not given as a component with cosine or sine θ and so lost the method mark too. Of those that correctly obtained the three equations $aT\cos\theta + 2aW\sin\theta + (4a - x)(4W\sin\theta) = 4aR_B\sin\theta$, $4aR_C\sin\theta = aT\cos\theta$ and $R_B + R_C = 4W + W$, only the most able candidates could correctly derive the tension as $W\left(1+\frac{2x}{a}\right)\tan\theta$.

Exemplar 5



A-C

Ⓐ $(T\cos\theta)(a) = (R_c\sin\theta)4a$

$T\cos\theta = 4R_c\sin\theta$

AB

Ⓐ: $(a)(T\cos\theta) + (2a)(W\sin\theta) + (4a-x)(4W\sin\theta) = (4a)R_B\sin\theta$

$aT\cos\theta + 2aW\sin\theta + 16aW\sin\theta - 4xW\sin\theta = 4aR_B\sin\theta$

$aT\cos\theta + 18aW\sin\theta - 4xW\sin\theta = 4aR_B\sin\theta$

R(↑) $R_B + R_C = 5W$

$\frac{aT\cos\theta + 18aW\sin\theta - 4xW\sin\theta}{4a\sin\theta} + \frac{T\cos\theta}{4\sin\theta} = 5W$

$aT\cos\theta + 18aW\sin\theta - 4xW\sin\theta + Ta\cos\theta = 5W(4a\sin\theta)$

$2Ta\cos\theta + 18aW\sin\theta - 4xW\sin\theta = 20aW\sin\theta$

$2Ta\cos\theta = 2aW\sin\theta + 4xW\sin\theta$

$T(2a\cos\theta) = 2W\sin\theta\,(a+2x)$

$T = \frac{2W\sin\theta}{2a\cos\theta}(a+2x)$

$= \frac{W\tan\theta}{a}(a+2x)$

$= W\tan\theta\left(1+\frac{2x}{a}\right)$ as required

This response was fully correct. Each line of working was extremely clear with the bonus that the candidate made it clear when (and about which point) they were taking moments and when they were resolving. The level of detail in the algebraic working to derive the given result was sound.

Question 13(b)

The rope is elastic with natural length $\frac{1}{4}a$ and modulus of elasticity W.

(b) Show that the condition for equilibrium is that

$$x = \frac{1}{2}a(8\cos\theta - \cot\theta - 1).$$

[5]

This part was answered very well, with many candidates correctly stating the extension in the string as $2a\sin\theta - 0.25a$ and then using Hooke's law to derive the given result.

Question 13(c)

In this question you must show detailed reasoning.

(c) Hence determine, in terms of a, the maximum value of x for which equilibrium is possible. **[5]**

It was extremely pleasing that most candidates did indeed show detailed reasoning and found the maximum value of *x* by differentiating the expression for *x* (with respect to theta) from part (a) and equating this to zero. Most then went on and solved this equation correctly to find θ and then stated the correct value of *x*. However, very few candidates earnt the final mark in this part for showing that this was in fact the maximum value of *x* and not just a stationary value.

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