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AS LEVEL

Examiners' report

FURTHER MATHEMATICS B (MEI)

H635
For first teaching in 2017

Y410/01 Summer 2019 series

Version 1

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the question paper can be downloaded from OCR.

Paper Y410 series overview

Y410 is the mandatory Pure Mathematics paper of H635 OCR B (MEI) AS Level Further Mathematics and assumes candidates know the content of AS Level Mathematics.

The overall standard of scripts seen was quite high, with most scoring between 40 and 50 marks. However, there were one or two quite demanding questions – in particular 6(a) and 6(b) – in which few candidates scored full marks, and as a consequence we saw fewer over 50 marks than last year's examination – around 18%. Conversely, only 12% candidates scored fewer than 25 marks. There was little evidence of time pressure – virtually all candidates answered all the questions. Presentation of scripts was good.

Most successful questions	***Least successful questions***
Q1 (difference method for summing a series) Q2 (roots of equations) Q4 (Intersection of planes)	Q3 (calculations with matrices) Q6 (transformations)

Candidates should pay extra care over the presentation of working when a question request includes any of the specification defined command words.

i	**OCR support**	A poster detailing the different command words and what they mean is available here: https://teach.ocr.org.uk/italladdsup

A number of good candidates did not show enough working for detailed reasoning questions such as 1, 3 and 8. Whilst there is no restriction on the use of calculators in this assessment, the bold statement signifies that candidates must give a solution which provides a complete analytical method that can be followed in a logical matter. (although checking answers using calculator methods should be encouraged).

Question 1

1 **In this question you must show detailed reasoning.**

Find $\sum_{r=1}^{100}\left(\frac{1}{r}-\frac{1}{r+2}\right)$, giving your answer correct to 4 decimal places. **[3]**

This was assumed to be a straightforward starter question on the difference method, but proved to be more discriminating than we expected.

A number of candidates wrote down the correct answer without justification, presumably using calculator methods.

However, half the candidates recognised the use of the difference method and identified the correct remaining terms.

?	**Misconception**	A small minority of candidates attempted to use the following misconception $\sum \frac{1}{r} = \frac{1}{\sum r}$

Exemplar 1

$$\sum_{r=1}^{100} \frac{1}{r} - \sum_{r=1}^{100} \frac{1}{r+2} = 1.4803 \quad 4\text{dp.}$$

The bold statement "**In this question you must show detailed reasoning**" is used to make clear that an answer obtained directly from the calculator, without any evidence of mathematical understanding of the concept, will not gain credit.

Question 2

2 The roots of the equation $3x^2-x+2=0$ are α and β.
Find a quadratic equation with integer coefficients whose roots are $2\alpha-3$ and $2\beta-3$. **[3]**

About half the candidates scored full marks for this question. Methods used were equally divided between substituting $x=\frac{(y+3)}{2}$ and simplifying the quadratic in y, and finding the sum and product of the roots $(2\alpha-3)$ and $(2\beta-3)$ from those of α and β. Marks were quite frequently lost for arithmetic slips, for example sign errors, or slips in clearing fractions using the first method.

Question 3 (a)

3 In this question you must show detailed reasoning.

A and **B** are matrices such that $\mathbf{B}^{-1}\mathbf{A}^{-1} = \begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix}$.

(a) Find **AB**. **[3]**

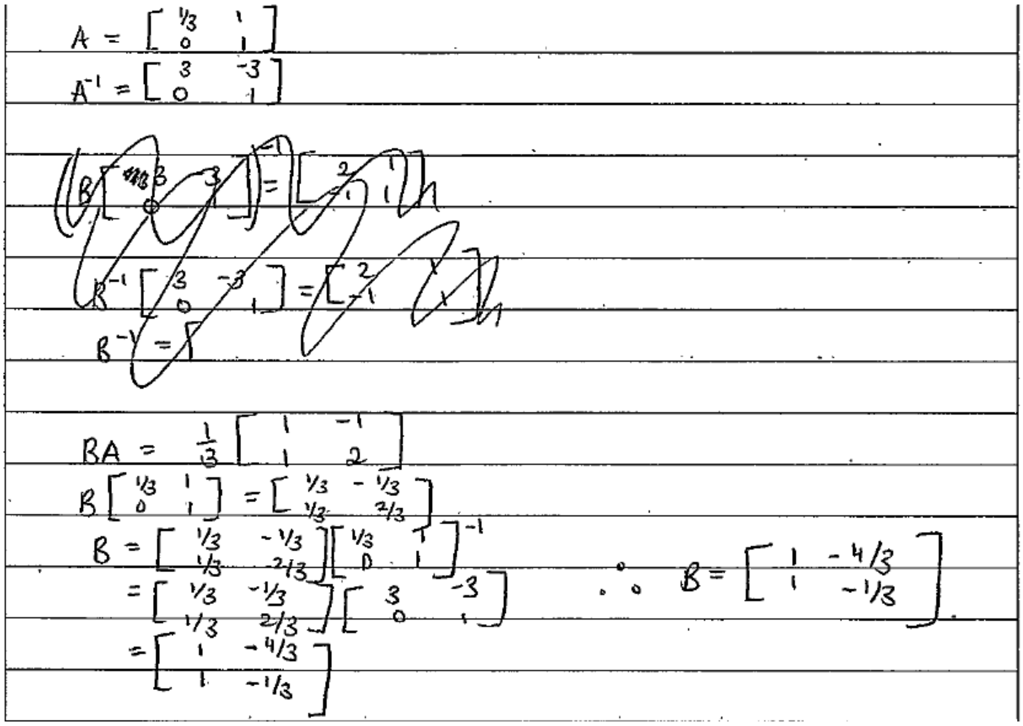
Most candidates scored full marks here. Nevertheless, a somewhat disappointing number of candidates seemed to assume that matrix multiplication is commutative, and that $\mathbf{B}^{-1}\mathbf{A}^{-1}$ = $\mathbf{(BA)}^{-1}$ rather than $\mathbf{(AB)}^{-1}$.

Question 3 (b)

(b) Given that $\mathbf{A} = \begin{pmatrix} \frac{1}{3} & 1 \\ 0 & 1 \end{pmatrix}$, find **B**. **[3]**

If using the pre-multiplication of **AB** by $\mathbf{A}^{-1}$, as this was a 'DR' question, we needed to see $\mathbf{A}^{-1}$ to get the full three marks

Exemplar 2



$A = \begin{bmatrix} 1/3 & 1 \\ 0 & 1 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 3 & -3 \\ 0 & 1 \end{bmatrix}$

$BA = \frac{1}{3}\begin{bmatrix} 1 & -1 \\ 1 & 2 \end{bmatrix}$

$B\begin{bmatrix} 1/3 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/3 & -1/3 \\ 1/3 & 2/3 \end{bmatrix}$

$B = \begin{bmatrix} 1/3 & -1/3 \\ 1/3 & 2/3 \end{bmatrix}\begin{bmatrix} 1/3 & 1 \\ 0 & 1 \end{bmatrix}^{-1}$

$= \begin{bmatrix} 1/3 & -1/3 \\ 1/3 & 2/3 \end{bmatrix}\begin{bmatrix} 3 & -3 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & -4/3 \\ 1 & -1/3 \end{bmatrix}$

$\therefore B = \begin{bmatrix} 1 & -4/3 \\ 1 & -1/3 \end{bmatrix}$

Commutativity errors also caused marks to be lost here, as in the above exemplar, where the candidate has started by assuming that the matrix BA is the matrix AB, and post-multiplying this by A^{-1} to find B.

Question 4 (a)

4 **(a)** Find $\mathbf{M}^{-1}$, where $\mathbf{M} = \begin{pmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ -2 & 1 & 2 \end{pmatrix}$. **[1]**

Students proved to be adept at using matrix calculators here, and 90% got this correct.

Question 4 (b)

(b) Hence find, in terms of the constant k, the point of intersection of the planes

$$x + 2y + 3z = 19,$$
$$-x + y + 2z = 4,$$
$$-2x + y + 2z = k.$$

[3]

Again this was well answered, although a significant minority made arithmetic errors in the matrix multiplication.

Question 4 (c)

(c) **In this question you must show detailed reasoning.**

Find the acute angle between the planes $x + 2y + 3z = 19$ and $-x + y + 2z = 4$. **[4]**

Over 80% of candidates scored full marks here.

Question 5

5 Prove by induction that, for all positive integers n, $\sum_{r=1}^{n} \frac{1}{3^r} = \frac{1}{2}\left(1 - \frac{1}{3^n}\right)$. **[6]**

The method of mathematical induction was well rehearsed by the majority of candidates. However, the inductive step proved to be tricky for many, who were unable to successfully combine the fractions $\frac{1}{3^k}$ and $\frac{1}{3^{k+1}}$. This proved rather costly, as this step was required to achieve the concluding A1 mark.

	AfL	The use of precise language to support the algebraic manipulation is a key part of any proof by induction. It is also important that candidates understand that the truth for n = k is an assumption, not a given.

Exemplar 3

$$\sum_{r=1}^{n} \frac{1}{3^r} = \frac{1}{2}\left[1-\frac{1}{3^n}\right]$$

for $n=1$ LHS $= \frac{1}{3}(1) = \frac{1}{3}$ $\left(\sum_{r=1}^{1} \frac{1}{3^r} = \frac{1}{2}\left[1-\frac{1}{3^n}\right]\right)$

RHS $= \frac{1}{2}\left[1-\frac{1}{3^1}\right] = \frac{1}{2}\left[1-\frac{1}{3}\right] = \frac{1}{2}\left[\frac{2}{3}\right] = \frac{1}{3}$

so LHS = RHS $= \frac{1}{3}$ so true for $n=1$

Assume true for $n=k$ $= \frac{1}{2}\left[1-\frac{1}{3^k}\right]$

Assume true for $n=k+1$ $= \frac{1}{2}\left[1-\frac{1}{3^{k+1}}\right]$

$= \frac{1}{2}\left[1-\frac{1}{3^k}\right] + \frac{1}{3}(k+1)$

~~$\frac{1}{2} - \frac{1}{6^k} + \frac{1}{3}(k+1) =$~~

$k+1 = \frac{1}{2}\left[1-\frac{1}{3\times3^k}\right] = \frac{1}{2}\left[1-\frac{1}{3^{k+1}}\right]$

If true for $n=k$ and true for $n=k+1$, and since true for $n=1$, must be true for all positive integers

This candidate has assumed the truth of the statement for $n = k+1$, instead of attempting to prove its truth assuming the truth for $n = k$. The inductive step is missing, and the final statement is incorrect.

Question 6 (a) (i)

6 A linear transformation T of the x-y plane has an associated matrix **M**, where $\mathbf{M} = \begin{pmatrix} \lambda & k \\ 1 & \lambda - k \end{pmatrix}$, and λ and k are real constants.

(a) You are given that $\det \mathbf{M} > 0$ for all values of λ.

(i) Find the range of possible values of k. **[3]**

This was perhaps the hardest question on the paper, as the argument is quite subtle. For this reason, very few candidates presented a fully correct solution. Most candidates successfully wrote down the determinant, but very few thought about using the discriminant of the quadratic to find the range of *k* for which this is positive. Of those who did see the significance of the discriminant, most assumed this was positive rather than negative.

Question 6 (a) (ii)

(ii) What is the significance of the condition $\det \mathbf{M} > 0$ for the transformation T? **[1]**

Many recognised the significance of the condition det **M** > 0 for the area of the image, but few thought of the orientation, which is the distinguishing factor between this condition and det **M** < 0.

Exemplar 4

If det M > 0 there is 1 unique solution (means transformation T has an inverse).

Here, the candidate correctly states that the linear equations associated with the matrix have a unique solution if det **M** > 0, but this does not describe the significance for the transformation itself.

Question 6 (b)

For the remainder of this question, take $k = -2$.

(b) Determine whether there are any lines through the origin that are invariant lines for the transformation T. **[4]**

This proved to be a demanding question. A rather disappointing number of candidates made a slip in substituting $k = -2$ into the matrix, getting $\lambda - 2$ instead of $\lambda + 2$. This lost the B1 mark.

Those who got as far as the second M1 made life harder for themselves by choosing to use $y = mx + c$ instead of $y = mx$ (the question explicitly refers to lines through the origin), which simplifies the algebra considerably. Those few candidates who identified the condition $2m^2 + 2m + 1 = 0$ were required to solve this or mention the discriminant to get the final 'A' mark.

	AfL	As seen in previous assessments, the ability to distinguish between a line of invariant points and invariant lines is important.

Question 6 (c)

(c) The transformation T is applied to a triangle with area 3 units^2. The area of the resulting image triangle is 15 units^2.
Find the possible values of λ. **[3]**

This question was correctly answered by half the candidates. The relationship between the determinant of the matrix and the area scale factor was well understood.

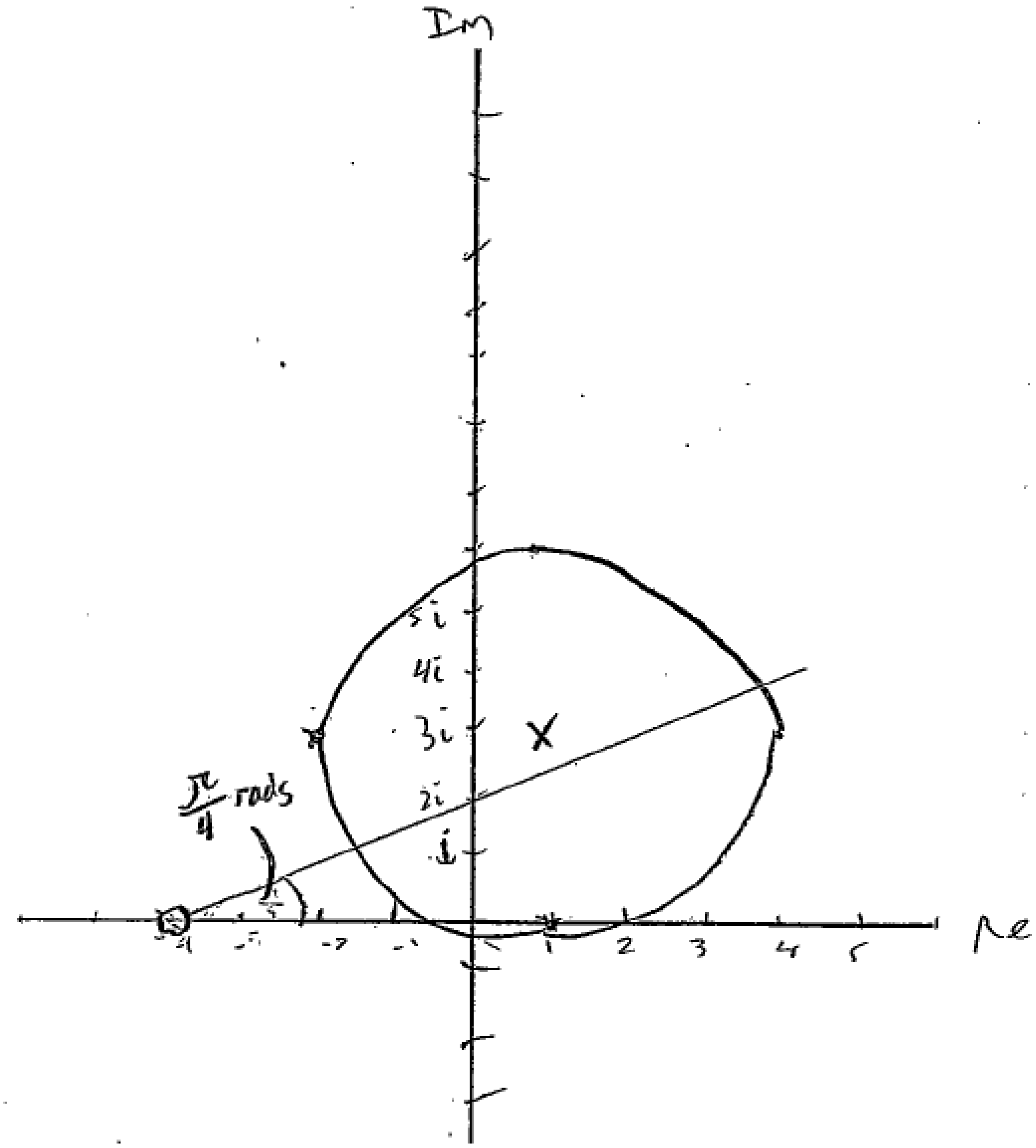
Question 7 (a) (i)

7 **(a)** Sketch on a single Argand diagram

(i) the set of points for which $|z-1-3i|=3$, **[3]**

This proved to be an easy three marks, although some solutions did not specify the centre of the circle effectively, and in poor freehand drawings it was sometimes difficult to see if the circle touched the real axis as required.

Exemplar 5



The rough nature of this candidate's sketch, which appears to cross the x-axis twice, making it difficult to award full marks for 7(a) (i).

Question 7 (a) (ii)

(ii) the set of points for which $\arg(z+4) = \frac{1}{4}\pi$. **[3]**

Again, this was well answered. Occasionally the point –4 was incorrect. However, the fact that this is a half line was well understood.

	AfL	The specification provides guidance on the distinction between 'Plot', 'Sketch' and 'Draw'. For questions requiring a sketched curve candidates need to make clear the main features, including any intersections with axes.

Question 7 (b)

(b) Find, in exact form, the two values of z for which $|z-1-3\mathrm{i}| = 3$ and $\arg(z+4) = \frac{1}{4}\pi$. **[6]**

Those candidates who did not make the connection with cartesian geometry scored no marks here, and, for this reason, the most common scores were either 0 or 6. The two equations were solved correctly by quite a lot of the rest, although the final mark, for re-stating the coordinates in complex number form, was sometimes missing.

Question 8 (a)

8 In this question you must show detailed reasoning.

You are given that i is a root of the equation $z^4 - 2z^3 + 3z^2 + az + b = 0$, where a and b are real constants.

(a) Show that $a = -2$ and $b = 2$. **[4]**

The most efficient (and expected) solution here was to substitute $z = \mathrm{i}$ into the equation and equate real and imaginary parts. However, some candidates used methods based on the symmetric properties of roots or factoring out $z^2 + 1$ and completing the multiplication to deduce the missing coefficients, as detailed by the mark scheme. This led to a degree of overlap with part (b), and marks were carried over as appropriate. Some candidates tried to work back from the roots (presumably found from a calculator); however, this assumed the values of a and b and did not achieve full marks (this would have been appropriate for a 'Verify' style question. Overall, over 60% achieved full marks.

Question 8 (b)

(b) Find the other roots of this equation. **[7]**

As noted in part (a), some solutions overlapped with part (a). Candidates usually scored well here, either using polynomial division by $z^2 + 1$, or using the symmetric properties of roots to get $\alpha + \beta = 2$, $\alpha\beta = 2$, and solving these simultaneously. As a 'detailed reasoning' question, to get full marks, candidates needed to show their method for obtaining the solution of the quadratic; whether by completion of square, formula or factorising.

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