GCE

Further Mathematics B (MEI)

Y433/01: Modelling with algorithms

Advanced GCE

Mark Scheme for June 2019

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

Subject-specific Marking Instructions for A Level Mathematics B (MEI)

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.

- When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value.
- When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. unless the question asks for a specific degree of accuracy.

Follow through should be used so that only one mark is lost for each distinct accuracy error.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>AOs</th><th>Guidance</th><th></th></tr>
<tr><td>1</td><td>(a)</td><td></td><td>
<table>
<tr><th>Node</th><th>Order of labelling</th><th>Permanent labels</th><th>Working values</th></tr>
<tr><td>A</td><td>1</td><td>0</td><td>(0)</td></tr>
<tr><td>B</td><td>2</td><td>9</td><td>9</td></tr>
<tr><td>C</td><td>4</td><td>19</td><td>19</td></tr>
<tr><td>D</td><td>5</td><td>21</td><td>25 21</td></tr>
<tr><td>E</td><td>3</td><td>15</td><td>17 15</td></tr>
<tr><td>F</td><td>6</td><td>22</td><td>22 (23)</td></tr>
<tr><td>G</td><td>7</td><td>25</td><td>35 26 25 (35)</td></tr>
<tr><td>H</td><td>8</td><td>32</td><td>37 34 32</td></tr>
</table>
Shortest path = ABEDGH
Length = 32</td><td>M1
A1

A1

A1ft

B1
B1ft
[6]</td><td>1.2
1.1a

1.1a

1.1

1.1
1.1</td><td>Correct working values at E
Working values / temporary labels
(values in brackets not penalised if seen)

Permanent labels

Order of labelling

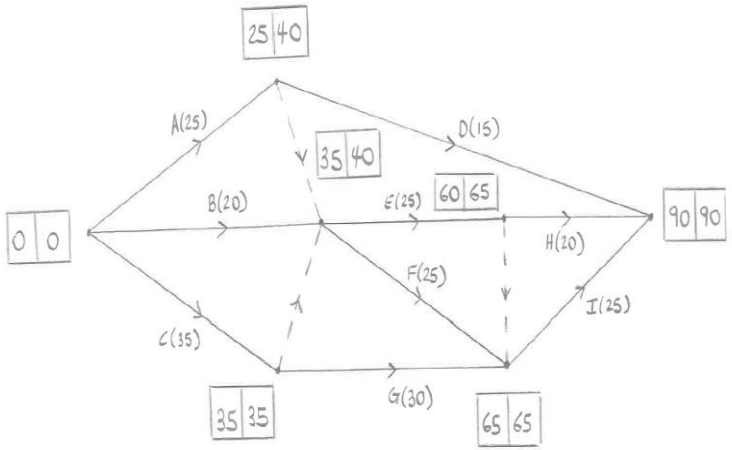
AB, BE, ED, DG, GH
Follow through their final value at H</td><td></td></tr>
<tr><td>1</td><td>(b)</td><td></td><td>Bin 1: 35 14
Bin 2: 25 18 7
Bin 3: 17 13 13 6
Bin 4: 12 11 10 9 8
Bin 5: 6 4</td><td>M1

A1
[2]</td><td>1.1

1.1</td><td>First six values placed correctly (up to and including the first 13)

cao</td><td></td></tr>
</table>

Question			Answer	Marks	AOs	Guidance	
1	(c)		In the worst case, placing the 2nd value consists of 1 comparison (with the first item), the 3rd value consists of 2 comparisons (with the first and second items), …, so total is $1 + 2 + 3 + \ldots + (n-1)$ comparisons OR in the worst case $n - 1$ comparisons with the 1st value, $n - 2$ comparisons with 2nd value, … so total is $(n - 1) + \ldots + 2 + 1$ comparisons	**M1**	**2.1**	Considers the correct worst case and attempts to sum the number of comparisons in the first $(n - 1)$ comparisons	
			$1+2+3+\ldots+(n-1)=\frac{1}{2}(n-1)(n)$ comparisons	**A1**	**1.1**	Correct sum seen or implied from correct simplified formula $\frac{1}{2}(n-1)n$ comparisons ⇒ M1, A1 $\frac{1}{2}n(n+1)$ comparisons ⇒ M1, A0	
			$O(n^2)$	**B1**	**2.2a**	o.e. e.g. quadratic order, order n^2 allow quadratic, allow complexity n^2	
				[3]			

Question			Answer	Marks	AOs	Guidance	
2	(a)			**M1** **A1** **A1** **[3]**	**3.1b** **3.1a** **1.1**	Activity on arc, single start vertex from which A, B and C (only) emerge; D follows A; G follows C E and F follow B; H follows E; I follows F and G; D, H, I (only) merge at a single finish Three dummies correctly placed with correct directions, no extra dummies	
	(b)			**M1** **A1** **M1** **A1** **B1** **[5]**	**3.1b** **1.1** **1.1** **1.1** **1.1**	Forward pass at every vertex (network with at least 7 vertices) Forward pass correct correct structure, at most 1 extra dummy Backward pass at every vertex (network with at least 7 vertices) Backward pass correct correct structure, at most 1 extra dummy C, G, I (and no others)	
			Critical activities are C, G and I				

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>AOs</th><th>Guidance</th><th></th></tr>
<tr><td>2</td><td>(c)</td><td></td><td>
<table>
<tr><th>Activity</th><th>Number of workers</th></tr>
<tr><td>A</td><td>1</td></tr>
<tr><td>B</td><td>2</td></tr>
<tr><td>C</td><td>1</td></tr>
<tr><td>D</td><td>3</td></tr>
<tr><td>E</td><td>1</td></tr>
<tr><td>F</td><td>1</td></tr>
<tr><td>G</td><td>2</td></tr>
<tr><td>H</td><td>2</td></tr>
<tr><td>I</td><td>1</td></tr>
</table>
</td><td>M1

M1

A1

[3]</td><td>3.4

1.1

1.1</td><td>Any four rows correct

Any seven rows correct

All correct</td><td></td></tr>
<tr><td>2</td><td>(d)</td><td></td><td>e.g. Seven workers are required between time 35 and 40 and so activity D (which requires 3 workers) needs to be moved or E, F, G need to move 5 minutes

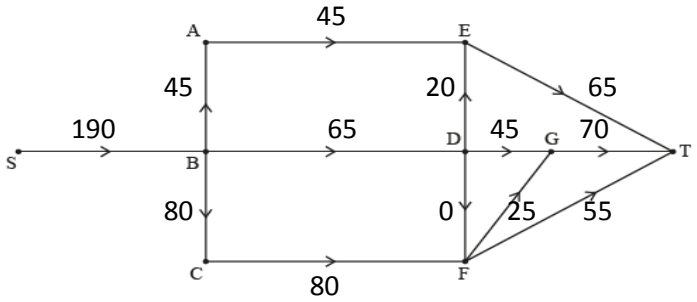
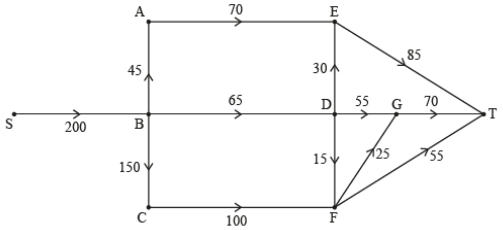
Move activity D to begin at time 80 and so the shortest time in which the project can be completed is 95 (minutes) (with only four workers)
Two possible solutions to assist with marking responses
<table>
<tr><td>A</td><td>A</td><td>A</td><td>A</td><td>A</td><td></td><td></td><td>E</td><td>E</td><td>E</td><td>E</td><td>E</td><td>H</td><td>H</td><td>H</td><td>H</td><td>D</td><td>D</td><td>D</td><td></td></tr>
<tr><td>B</td><td>B</td><td>B</td><td>B</td><td></td><td></td><td></td><td>F</td><td>F</td><td>F</td><td>F</td><td>F</td><td>H</td><td>H</td><td>H</td><td>H</td><td>D</td><td>D</td><td>D</td><td></td></tr>
<tr><td>B</td><td>B</td><td>B</td><td>B</td><td></td><td></td><td></td><td>G</td><td>G</td><td>G</td><td>G</td><td>G</td><td>G</td><td></td><td></td><td></td><td>D</td><td>D</td><td>D</td><td></td></tr>
<tr><td>C</td><td>C</td><td>C</td><td>C</td><td>C</td><td>C</td><td>C</td><td>G</td><td>G</td><td>G</td><td>G</td><td>G</td><td>G</td><td>I</td><td>I</td><td>I</td><td>I</td><td>I</td><td></td><td></td></tr>
</table>
<table>
<tr><td>A</td><td>A</td><td>A</td><td>A</td><td>A</td><td>D</td><td>D</td><td>D</td><td>E</td><td>E</td><td>E</td><td>E</td><td>E</td><td>H</td><td>H</td><td>H</td><td>H</td><td></td><td></td><td></td></tr>
<tr><td>B</td><td>B</td><td>B</td><td>B</td><td></td><td>D</td><td>D</td><td>D</td><td>F</td><td>F</td><td>F</td><td>F</td><td>F</td><td>H</td><td>H</td><td>H</td><td>H</td><td></td><td></td><td></td></tr>
<tr><td>B</td><td>B</td><td>B</td><td>B</td><td></td><td>D</td><td>D</td><td>D</td><td>G</td><td>G</td><td>G</td><td>G</td><td>G</td><td>G</td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>C</td><td>C</td><td>C</td><td>C</td><td>C</td><td>C</td><td>C</td><td></td><td>G</td><td>G</td><td>G</td><td>G</td><td>G</td><td>G</td><td>I</td><td>I</td><td>I</td><td>I</td><td>I</td><td></td></tr>
</table>
</td><td>M1

A1

[2]</td><td>2.4

2.2a</td><td>Identifying that D, E, F and G cannot happen together and trying to deal with this

95 from a valid schedule described, with correct numbers of workers and no more than 4 at any time</td><td></td></tr>
</table>

Question			Answer	Marks	AOs	Guidance	
3	(a)	(i)	[Capacity of cut α is] 210	B1 [1]	1.1		
3	(a)	(ii)	[Capacity of cut β is] 230	B1 [1]	1.1		
3	(b)		The maximum possible flow is ≤ 210 (litres per minute)	B1ft [1]	2.2a	Deduced from their least value given in (i) Must include 'less than or equal to'	
3	(c)		e.g. (may have different values on DE, DG, ET and GT)  	M1 A1 [2]	2.1 2.2a	Flow of 190 and a consistent flow pattern (flow in = flow out at each internal vertex) with flow through AE, BD and CF A correct flow with DF = 0 (flow $\leq$ capacity for each arc) For references, capacities are:  	
3	(d)		The capacity of the cut which partitions the vertices into the sets {S, B, C, F}, {A, D, E, G, T} Capacity of cut is 45 + 65 + 25 + 55 = 190 so min cut ≤ 190 By the maximum flow-minimum cut theorem the maximum flow is equal to the minimum cut and so therefore the maximum flow through the system is 190 litres per hour	M1 A1 [2]	3.1b 2.1	{S, B, C, F}, {A, D, E, G, T} either correct or shown on diagram and referenced here or (all) cut arcs listed here Or equivalent reasoning using theorem e.g. max flow = min cut, cut equals flow e.g. max flow = min cut, cut = 190	

Question			Answer	Marks	AOs	Guidance	
3	(e)		Flow is 190 + x up to x = 20 After that, flow is 210	M1 A1 B1 [3]	3.1a 2.2a 2.3	190 + x (as a possible maximum flow) Correct deduction of critical value x = 20 210 (as max possible flow) for (all) large x	
4	(a)	(i)	13AX + 12AY+…+12DZ is the total cost of transporting <u>all</u> the required stock (which is calculated by finding the sum of the product of the number of units transported along each route and the cost of using that route) We wish the total cost of transporting all the required stock to be as small as possible hence the use of minimise	B1 B1 [2]	3.3 1.1	Mention of total cost (oe) Relating minimum to the problem of transporting/delivering the goods	
4	(a)	(ii)	The line AX + BX + CX + DX = 24 constrains the total amount (number of units) sent (from the four suppliers) to depot X to 24, which is the demand at X	B1 [1]	3.3	Must mention 'X', demand (oe) and 24	
4	(b)	(i)	A sends 2 to X and 13 to Z, B sends 12 to Y, C sends 8 to X and D sends 14 to X OR X gets 2 from A, 8 from C and 14 from D, Y gets 12 from B and Z gets 13 from A	B1 [1]	3.4	These correct and no others (Total amount transported from each supplier to corresponding depot is) AX = 2, AZ = 13, BY = 12, CX = 8 and DX = 14	
4	(b)	(ii)	£50 700	B1 [1]	3.2a	Ignore lack of £ - must be 50700	
4	(c)		If the stock increases at B the problem is now unbalanced OR total supply > total demand Surplus/excess stock (of 4 units at B) OR the = needs to be ≤	B1 [1]	3.5b	oe e.g. if the amount supplied by B is more than 12 then the amount supplied by (at least one of) A, C, D will have to decrease, or similar reference to total unchanged or total must still be 49	

Question			Answer	Marks	AOs	Guidance	
4	(d)	(i)	No change in the objective function	B1 [1]	1.1		
4	(d)	(ii)	The four <u>stock</u> constraints require $\leq$ inequalities e.g. $AX + AY + AZ \leq 15,...$	B1 [1]	3.5c	If demand constraints are mentioned then must be correct (either left unchanged or $AX + BX + CX + DX \geq 24,...$)	
4	(e)		$11AZ + 10BZ = 139$ $AZ + BZ = 13$ From A to Z is 9 and from B to Z is 4	M1 A1 B1 [3]	3.1a 1.1 1.1	Setting up one correct equation Both equations correct If M0 then Each unit transported to Z from B instead of A saves £100 $\Rightarrow$ SC1 AZ = 9, BZ = 4	
5	(a)		(max) $P = 2x + 3y + kz$ $x + 3y + z \leq 30$ $2y + z \geq 20$ $3x - y + z \geq 30$ $x + y + 2z \leq 40$ $(x \geq 0, y \geq 0, z \geq 0)$	B1 M1 A1 [3]	3.4 3.4 1.1	Two correct (non-trivial) inequalities not involving slack, artificial or surplus variables All four correct	
5	(b)		$Q = a_1 + a_2 \Rightarrow Q = (20 - 2y - z + s_2) + (30 - 3x + y - z + s_3)$ $Q + 3x + y + 2z - s_2 - s_3 = 50$	M1 A1 [2]	2.1 2.2a	Correct objective and attempt to substitute for, or eliminate, both artificial variables) Matching the form in the table	

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>AOs</th><th>Guidance</th><th></th></tr>
<tr><td>5</td><td>(c)</td><td></td><td>(The solution is feasible since) $Q = 0$
(The solution is optimal since) the only negative entries in objective row (row Q) are in the cols for the artificial variables (a_1 and a_2)</td><td>B1
B1

[2]</td><td>1.2
2.4</td><td>$Q = 0$ or RHS = 0
Or equivalent
e.g. at this stage there are no positive values in the objective row (row Q) (apart from the 1 in the Q column)</td><td></td></tr>
<tr><td>5</td><td>(d)</td><td></td><td>
<table>
<tr><th>P</th><th>x</th><th>y</th><th>z</th><th>s_1</th><th>s_2</th><th>s_3</th><th>s_4</th><th>RHS</th></tr>
<tr><td>1</td><td>0</td><td>0</td><td>0</td><td>$\frac{3}{2}-\frac{2}{7}k$</td><td>0</td><td>$-\frac{1}{2}+\frac{1}{7}k$</td><td>$-1+\frac{5}{7}k$</td><td>$20+\frac{110}{7}k$</td></tr>
<tr><td>0</td><td>1</td><td>0</td><td>0</td><td>$\frac{3}{14}$</td><td>0</td><td>$-\frac{5}{14}$</td><td>$-\frac{2}{7}$</td><td>$\frac{40}{7}$</td></tr>
<tr><td>0</td><td>0</td><td>1</td><td>0</td><td>$\frac{5}{14}$</td><td>0</td><td>$\frac{1}{14}$</td><td>$-\frac{1}{7}$</td><td>$\frac{20}{7}$</td></tr>
<tr><td>0</td><td>0</td><td>0</td><td>1</td><td>$-\frac{2}{7}$</td><td>0</td><td>$\frac{1}{7}$</td><td>$\frac{5}{7}$</td><td>$\frac{110}{7}$</td></tr>
<tr><td>0</td><td>0</td><td>0</td><td>0</td><td>$\frac{3}{7}$</td><td>1</td><td>$\frac{2}{7}$</td><td>$\frac{3}{7}$</td><td>$\frac{10}{7}$</td></tr>
</table>
</td><td>M1

M1

A1

B1

[4]</td><td>3.4

1.1

1.1

1.1</td><td>5 different basis columns (P, s_2, and three others)
Their pivot row correct (with RHS ≥ 0)
Other constraint rows correct (dependent on both M marks)
Objective row correct</td><td></td></tr>
<tr><td>5</td><td>(e)</td><td>(i)</td><td>Consideration of at least two of their
$\frac{3}{2}-\frac{2}{7}k \geq 0, -\frac{1}{2}+\frac{1}{7}k \geq 0. -1+\frac{5}{7}k \geq 0$

$\frac{7}{2} \leq k \leq \frac{21}{4}$</td><td>M1

A1
[2]</td><td>3.1a

2.2a</td><td>Dependent on linear expressions in terms of k in the objective row of the s_1, s_3, s_4 columns in part (d) – allow any inequality or equals
May be implied from correct final answer

cao</td><td></td></tr>
<tr><td>5</td><td>(e)</td><td>(ii)</td><td>$P_{\max} = 20 + \frac{110}{7}k$
$x = \frac{40}{7}, y = \frac{20}{7}, z = \frac{110}{7}$</td><td>B1

B1

[2]</td><td>1.1

1.1</td><td>cao (allow decimals to 3 sf or better)
$(10 + 15.7k)$
cao (allow decimals to 3 sf or better)
$x = 5.71$, $y = 2.86$, $z = 15.7$
(5.714…) (2.857…) (15.71…)</td><td></td></tr>
</table>

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