Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

FURTHER MATHEMATICS **9231/12**

Paper 1 **May/June 2015**

3 hours

* 7 2 9 8 7 9 7 4 1 6 *

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF10)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **4** printed pages.

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1 Use the List of Formulae (MF10) to show that $\sum_{r=1}^{13}(3r^2 - 5r + 1)$ and $\sum_{r=0}^{9}(r^3 - 1)$ have the same numerical value. [4]

2 Find the value of the constant k for which the system of equations

$$\begin{aligned} 2x - 3y + 4z &= 1, \\ 3x - y \quad\quad &= 2, \\ x + 2y + kz &= 1, \end{aligned}$$

does not have a unique solution. [2]

For this value of k, solve the system of equations. [4]

3 The sequence $a_1, a_2, a_3, \ldots$ is such that $a_1 > 5$ and $a_{n+1} = \frac{4a_n}{5} + \frac{5}{a_n}$ for every positive integer n. Prove by mathematical induction that $a_n > 5$ for every positive integer n. [5]

Prove also that $a_n > a_{n+1}$ for every positive integer n. [2]

4 The roots of the cubic equation $x^3 - 7x^2 + 2x - 3 = 0$ are α, β and γ. Find the values of

(i) $\frac{1}{(\alpha\beta)(\beta\gamma)(\gamma\alpha)}$,

(ii) $\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha}$,

(iii) $\frac{1}{\alpha^2\beta\gamma} + \frac{1}{\alpha\beta^2\gamma} + \frac{1}{\alpha\beta\gamma^2}$.

[6]

Deduce a cubic equation, with integer coefficients, having roots $\frac{1}{\alpha\beta}$, $\frac{1}{\beta\gamma}$ and $\frac{1}{\gamma\alpha}$. [2]

5 The curves C_1 and C_2 have polar equations

$$C_1: \quad r = \frac{1}{\sqrt{2}}, \quad \text{for } 0 \leqslant \theta < 2\pi,$$

$$C_2: \quad r = \sqrt{\left(\sin \tfrac{1}{2}\theta\right)}, \quad \text{for } 0 \leqslant \theta \leqslant \pi.$$

Find the polar coordinates of the point of intersection of C_1 and C_2. [2]

Sketch C_1 and C_2 on the same diagram. [3]

Find the exact value of the area of the region enclosed by C_1, C_2 and the half-line $\theta = 0$. [4]

6 A curve has equation $x^2 - 6xy + 25y^2 = 16$. Show that $\frac{dy}{dx} = 0$ at the point $(3, 1)$. [4]

By finding the value of $\frac{d^2y}{dx^2}$ at the point $(3, 1)$, determine the nature of this turning point. [5]

7 Let $I_n = \int_0^{\frac{1}{2}\pi} x^n \sin x \, \mathrm{d}x$, where n is a non-negative integer. Show that

$$I_n = n\left(\tfrac{1}{2}\pi\right)^{n-1} - n(n-1)I_{n-2}, \quad \text{for } n \geqslant 2. \qquad [5]$$

Find the exact value of I_4. [4]

8 By considering $\sum_{r=1}^{n} z^{2r-1}$, where $z = \cos\theta + \mathrm{i}\sin\theta$, show that, if $\sin\theta \neq 0$,

$$\sum_{r=1}^{n} \sin(2r-1)\theta = \frac{\sin^2 n\theta}{\sin\theta}. \qquad [7]$$

Deduce that

$$\sum_{r=1}^{n} (2r-1)\cos\left[\frac{(2r-1)\pi}{2n}\right] = -\operatorname{cosec}\left(\frac{\pi}{2n}\right)\cot\left(\frac{\pi}{2n}\right). \qquad [4]$$

9 The curve C has parametric equations

$$x = 4t + 2t^{\frac{3}{2}}, \quad y = 4t - 2t^{\frac{3}{2}}, \quad \text{for } 0 \leqslant t \leqslant 4.$$

Find the arc length of C, giving your answer correct to 3 significant figures. [6]

Find the mean value of y with respect to x over the interval $0 \leqslant x \leqslant 32$. [5]

10 The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 2 & 2 & -3 \\ 2 & 2 & 3 \\ -3 & 3 & 3 \end{pmatrix}.$$

The matrix $\mathbf{A}$ has an eigenvector $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$. Find the corresponding eigenvalue. [2]

The matrix $\mathbf{A}$ also has eigenvalues 4 and 6. Find corresponding eigenvectors. [3]

Hence find a matrix $\mathbf{P}$ such that $\mathbf{A} = \mathbf{PDP}^{-1}$, where $\mathbf{D}$ is a diagonal matrix which is to be determined. [2]

The matrix $\mathbf{B}$ is such that $\mathbf{B} = \mathbf{QAQ}^{-1}$, where

$$\mathbf{Q} = \begin{pmatrix} 4 & 11 & 5 \\ 1 & 4 & 2 \\ 1 & 2 & 1 \end{pmatrix}.$$

By using the expression $\mathbf{PDP}^{-1}$ for $\mathbf{A}$, find the set of eigenvalues and a corresponding set of eigenvectors for $\mathbf{B}$. [5]

[Question 11 is printed on the next page.]

11 Answer only **one** of the following two alternatives.

EITHER

Show that the substitution $v = \dfrac{1}{y}$ reduces the differential equation

$$\frac{2}{y^3}\left(\frac{dy}{dx}\right)^2 - \frac{1}{y^2}\frac{d^2y}{dx^2} - \frac{2}{y^2}\frac{dy}{dx} + \frac{5}{y} = 17 + 6x - 5x^2$$

to the differential equation

$$\frac{d^2v}{dx^2} + 2\frac{dv}{dx} + 5v = 17 + 6x - 5x^2. \qquad [4]$$

Hence find y in terms of x, given that when $x = 0$, $y = \frac{1}{2}$ and $\dfrac{dy}{dx} = -1$. [10]

OR

The lines l_1 and l_2 have equations $\mathbf{r} = 8\mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j})$ and $\mathbf{r} = 5\mathbf{i} + 3\mathbf{j} - 14\mathbf{k} + \mu(2\mathbf{j} - 3\mathbf{k})$ respectively. The point P on l_1 and the point Q on l_2 are such that PQ is perpendicular to both l_1 and l_2. Find the position vector of the point P and the position vector of the point Q. [8]

The points with position vectors $8\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ and $5\mathbf{i} + 3\mathbf{j} - 14\mathbf{k}$ are denoted by A and B respectively. Find

(i) $\overrightarrow{AP} \times \overrightarrow{AQ}$ and hence the area of the triangle APQ,

(ii) the volume of the tetrahedron $APQB$. (You are given that the volume of a tetrahedron is $\frac{1}{3} \times$ area of base $\times$ perpendicular height.)

[6]