UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

FURTHER MATHEMATICS **9231/21**

Paper 2 **October/November 2011**

3 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF10)

* 9 1 8 5 1 8 2 3 2 8 *

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value is necessary, take the acceleration due to gravity to be $10\,\mathrm{m\,s^{-2}}$.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **5** printed pages and **3** blank pages.

UNIVERSITY *of* CAMBRIDGE
International Examinations

1 A particle is moving in a circle of radius 2 m. At time t s its velocity is $(t^2 - 12)\ \mathrm{m\,s^{-1}}$. Find the magnitude of the resultant acceleration of the particle when $t = 4$. [4]

2 A particle P is moving in simple harmonic motion with centre O. When P is 5 m from O its speed is $V\ \mathrm{m\,s^{-1}}$, and when it is 9 m from O its speed is $\frac{3}{5}V\ \mathrm{m\,s^{-1}}$. Show that the amplitude of the motion is $\frac{15}{2}\sqrt{2}$ m. [4]

Given that the greatest speed of P is $3\sqrt{2}\ \mathrm{m\,s^{-1}}$, find V. [3]

3 A fixed hollow sphere with centre O has a smooth inner surface of radius a. A particle P of mass m is projected horizontally with speed $2\sqrt{(ag)}$ from the lowest point of the inner surface of the sphere. The particle loses contact with the inner surface of the sphere when OP makes an angle θ with the upward vertical.

(i) Show that $\cos\theta = \frac{2}{3}$. [5]

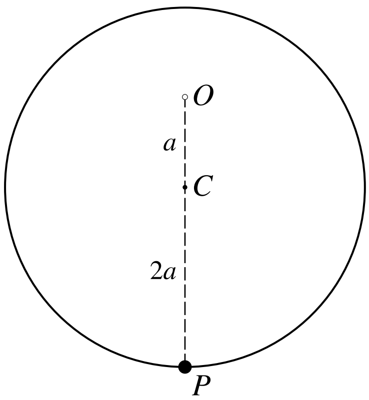
(ii) Find the greatest height that P reaches above the level of O. [4]

4 Two smooth spheres P and Q, of equal radius, have masses m and $3m$ respectively. They are moving in the same direction in the same straight line on a smooth horizontal table. Sphere P has speed u and collides directly with sphere Q which has speed ku, where $0 < k < 1$. Sphere P is brought to rest by the collision. Show that the coefficient of restitution between P and Q is $\dfrac{3k+1}{3(1-k)}$. [6]

One third of the total kinetic energy of the spheres is lost in the collision. Show that

$$k = \tfrac{1}{3}(2\sqrt{3} - 3).$$ [5]

5



A uniform solid sphere with centre C, radius $2a$ and mass $3M$, is pivoted about a smooth horizontal axis and hangs at rest. The point O on the axis is vertically above C and $OC = a$. A particle P of mass M is attached to the sphere at its lowest point (see diagram). Show that the moment of inertia of the system about the axis through O is $\frac{84}{5}Ma^2$. [4]

The system is released from rest with OP making a small angle α with the downward vertical. Find

(i) the period of small oscillations, [5]

(ii) the time from release until OP makes an angle $\frac{1}{2}\alpha$ with the downward vertical for the first time. [3]

6 The continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} 0 & x < 1, \\ \frac{1}{2} & 1 \leqslant x \leqslant 3, \\ 0 & x > 3. \end{cases}$$

Find the distribution function of X. [2]

The random variable Y is defined by $Y = X^3$. Find

(i) the probability density function of Y, [3]

(ii) the expected value and variance of Y. [3]

7 The lifetime, in hours, of a 'Trulite' light bulb is a random variable T. The probability density function f of T is given by

$$f(t) = \begin{cases} 0 & t < 0, \\ \lambda e^{-\lambda t} & t \geqslant 0, \end{cases}$$

where λ is a positive constant. Given that the mean lifetime of Trulite bulbs is 2000 hours, find the probability that a randomly chosen Trulite bulb has a lifetime of at least 1000 hours. [3]

A particular light fitting has 6 randomly chosen Trulite bulbs. Find the probability that no more than one of these bulbs has a lifetime less than 1000 hours. [3]

By using new technology, the proportion of Trulite bulbs with very short lifetimes is to be reduced. Find the least value of the new mean lifetime that will ensure that the probability that a randomly chosen Trulite bulb has a lifetime of no more than 4 hours is less than 0.001. [5]

8 A sample of 216 observations of the continuous random variable X was obtained and the results are summarised in the following table.

Interval	$0 \leqslant x < 1$	$1 \leqslant x < 2$	$2 \leqslant x < 3$	$3 \leqslant x < 4$	$4 \leqslant x < 5$	$5 \leqslant x < 6$
Observed frequency	1	3	15	31	59	107

It is suggested that these results are consistent with a distribution having probability density function f given by

$$f(x) = \begin{cases} kx^2 & 0 \leqslant x < 6, \\ 0 & \text{otherwise}, \end{cases}$$

where k is a positive constant. The relevant expected frequencies are given in the following table.

Interval	$0 \leqslant x < 1$	$1 \leqslant x < 2$	$2 \leqslant x < 3$	$3 \leqslant x < 4$	$4 \leqslant x < 5$	$5 \leqslant x < 6$
Expected frequency	1	7	a	b	c	91

(i) Show that $a = 19$ and find the values of b and c. [4]

(ii) Carry out a goodness of fit test at the 10% significance level. [7]

9 A random sample of five metal rods produced by a machine is taken. Each rod is tested for hardness. The results, in suitable units, are as follows.

524 526 520 523 530

Assuming a normal distribution, calculate a 95% confidence interval for the population mean. [5]

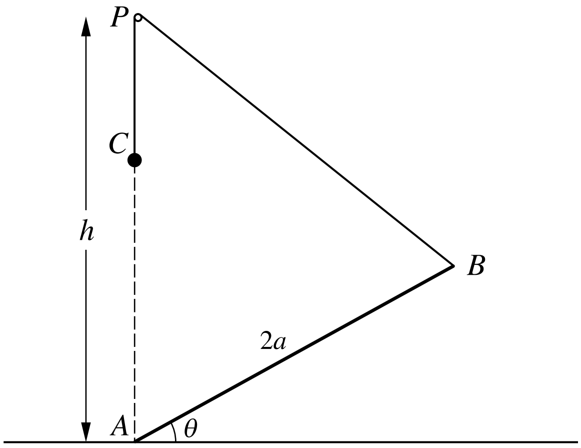
Some adjustments are made to the machine. Assume that a normal distribution is still appropriate and that the population variance remains unchanged. A second random sample, this time of ten metal rods, is now taken. The results for hardness are as follows.

525 520 522 524 518 520 519 525 527 516

Stating suitable hypotheses, test at the 10% significance level whether there is any difference between the population means before and after the adjustments. [8]

10 Answer only **one** of the following two alternatives.

EITHER



A uniform rod AB, of weight W and length $2a$, rests with the end A on a rough horizontal plane. A light inextensible string BC is attached to the rod at B and passes over a small smooth fixed peg P, which is at a distance h vertically above A. A particle is attached at C and hangs vertically. The points A, B and C are all in the same vertical plane. In equilibrium the rod is inclined at an angle θ to the horizontal (see diagram). The coefficient of friction between the rod and the plane is μ. Show that

$$\mu \geqslant \frac{2a\cos\theta}{h + 2a\sin\theta}.$$ [7]

Given that the particle attached at C has weight kW, angle $ABP = 90°$ and $h = 3a$, find

(i) the value of k, [4]

(ii) the horizontal component of the force on P, in terms of W. [3]

OR

The regression line of y on x obtained from a random sample of five pairs of values of x and y is

$$y = 2.5x - 1.5.$$

The data is given in the following table.

x	1	2	4	2	6
y	2	3	6	p	q

(i) Show that $p + q = 19$. [3]

(ii) Find the values of p and q. [6]

(iii) Determine the value of the product moment correlation coefficient for this sample. [3]

(iv) It is later discovered that the values of x given in the table have each been divided by 10 (that is, the actual values are 10, 20, 40, 20, 60). **Without any further calculation**, state

(a) the equation of the actual regression line of y on x,

(b) the value of the actual product moment correlation coefficient.

[2]

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