UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

**FURTHER MATHEMATICS** **9231/21**

Paper 2 **May/June 2010**

**3 hours**

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF10)

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**READ THESE INSTRUCTIONS FIRST**

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value is necessary, take the acceleration due to gravity to be $10\,\text{m}\,\text{s}^{-2}$.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **5** printed pages and **3** blank pages.

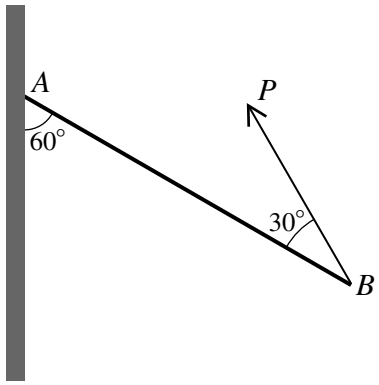


UNIVERSITY of CAMBRIDGE International Examinations

**[Turn over**

**1** A particle $P$, of mass 0.2 kg, moves in simple harmonic motion along a straight line under the action of a resultant force of magnitude $F$ N. The distance between the end-points of the motion is 0.6 m, and the period of the motion is 0.5 s. Find the greatest value of $F$ during the motion. [5]

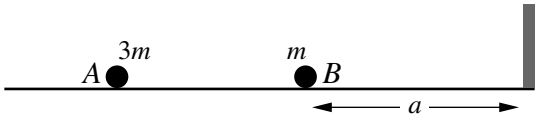
**2**




A uniform rod $AB$ of weight $W$ rests in equilibrium with $A$ in contact with a rough vertical wall. The rod is in a vertical plane perpendicular to the wall, and is supported by a force of magnitude $P$ acting at $B$ in this vertical plane. The rod makes an angle of 60° with the wall, and the force makes an angle of 30° with the rod (see diagram). Find the value of $P$. [3]

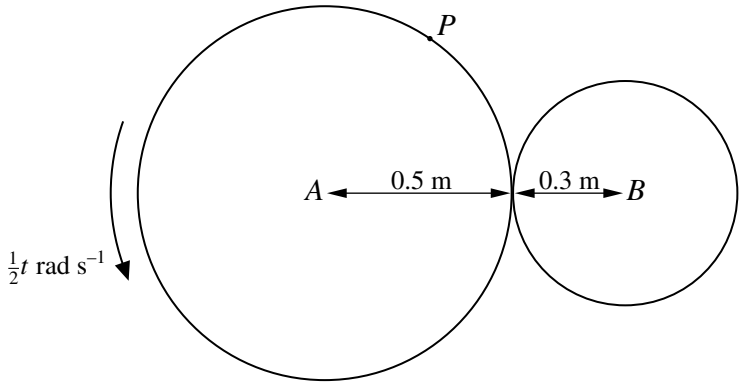
Find also the set of possible values of the coefficient of friction between the rod and the wall. [4]

**3**




Two perfectly elastic small smooth spheres $A$ and $B$ have masses $3m$ and $m$ respectively. They lie at rest on a smooth horizontal plane with $B$ at a distance $a$ from a smooth vertical barrier. The line of centres of the spheres is perpendicular to the barrier, and $B$ is between $A$ and the barrier (see diagram). Sphere $A$ is projected towards sphere $B$ with speed $u$ and, after the collision between the spheres, $B$ hits the barrier. The coefficient of restitution between $B$ and the barrier is $\frac{1}{2}$. Find the speeds of $A$ and $B$ immediately after they first collide, and the distance from the barrier of the point where they collide for the second time. [9]

**4**

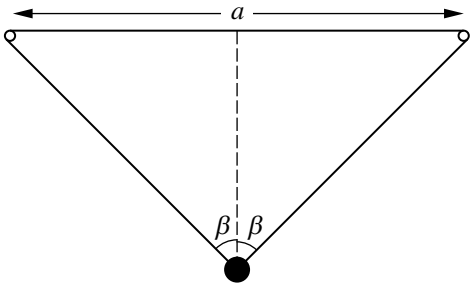



Two coplanar discs, of radii 0.5 m and 0.3 m, rotate about their centres $A$ and $B$ respectively, where $AB = 0.8$ m. At time $t$ seconds the angular speed of the larger disc is $\frac{1}{2}t$ rad s$^{-1}$ (see diagram). There is no slipping at the point of contact. For the instant when $t = 2$, find

**(i)** the angular speed of the smaller disc, [2]

**(ii)** the magnitude of the acceleration of a point $P$ on the circumference of the larger disc, and the angle between the direction of this acceleration and $PA$. [7]

**5**




A light elastic band, of total natural length $a$ and modulus of elasticity $\frac{1}{2}mg$, is stretched over two small smooth pins fixed at the same horizontal level and at a distance $a$ apart. A particle of mass $m$ is attached to the lower part of the band and when the particle is in equilibrium the sloping parts of the band each make an angle $\beta$ with the vertical (see diagram). Express the tension in the band in terms of $m$, $g$ and $\beta$, and hence show that $\beta = \frac{1}{4}\pi$. [4]

The particle is given a velocity of magnitude $\sqrt{(ag)}$ vertically downwards. At time $t$ the displacement of the particle from its equilibrium position is $x$. Show that, neglecting air resistance,

$$\ddot{x} = -\frac{2g}{a}x. \quad [3]$$

Show that the particle passes through the level of the pins in the subsequent motion, and find the time taken to reach this level for the first time. [6]

**6** The lifetime, $X$ days, of a particular insect is such that $\log_{10} X$ has a normal distribution with mean 1.5 and standard deviation 0.2. Find the median lifetime. [3]

Find also $P(X \geqslant 50)$. [2]

**7** The continuous random variable $X$ has distribution function given by

$$F(x) = \begin{cases} 0 & x < 0, \\ 1 - e^{-\frac{1}{2}x} & x \geqslant 0. \end{cases}$$

For a random value of $X$, find the probability that 2 lies between $X$ and $4X$. [3]

Find also the expected value of the width of the interval $(X, 4X)$. [4]

**8** An examination involved writing an essay. In order to compare the time taken to write the essay by students in two large colleges, a sample of 12 students from college $A$ and a sample of 8 students from college $B$ were randomly selected. The times, $t_A$ and $t_B$, taken for these students to write the essay were measured, correct to the nearest minute, and are summarised by

$$n_A = 12, \quad \Sigma t_A = 257, \quad \Sigma t_A^2 = 5629, \quad n_B = 8, \quad \Sigma t_B = 206, \quad \Sigma t_B^2 = 5359.$$

Stating any required assumptions, calculate a 95% confidence interval for the difference in the population means. [8]

State, giving a reason, whether your confidence interval supports the statement that the population means, for the two colleges, are equal. [1]

**9** A set of 20 pairs of bivariate data $(x, y)$ is summarised by

$$\Sigma x = 200, \quad \Sigma x^2 = 2125, \quad \Sigma y = 240, \quad \Sigma y^2 = 8245.$$

The product moment correlation coefficient is −0.992.

**(i)** What does the value of the product moment correlation coefficient indicate about a scatter diagram of the data points? [1]

**(ii)** Find the equation of the regression line of $y$ on $x$. [6]

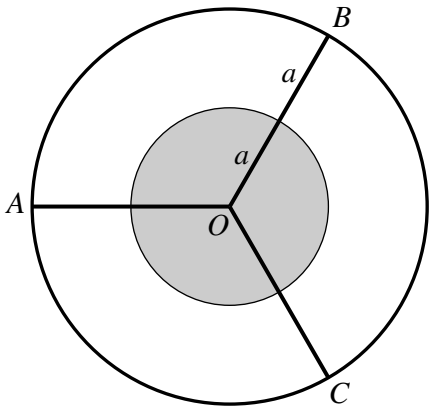
**(iii)** The equation of the regression line of $x$ on $y$ is $x = a' + b'y$. Find the value of $b'$. [2]

**10** Three new flu vaccines, $A$, $B$ and $C$, were tested on 500 volunteers. The vaccines were assigned randomly to the volunteers and 178 received $A$, 149 received $B$ and 173 received $C$. During the following year, 30 of the volunteers given $A$ caught flu, 29 of the volunteers given $B$ caught flu, and 16 of the volunteers given $C$ caught flu. Carry out a suitable test for independence at the 5% significance level. [10]

Without using a statistical test, decide which of the vaccines appears to be most effective. [3]

**11** Answer only **one** of the following two alternatives.

**EITHER**




A uniform disc, of mass $4m$ and radius $a$, and a uniform ring, of mass $m$ and radius $2a$, each have centre $O$. A wheel is made by fixing three uniform rods, $OA$, $OB$ and $OC$, each of mass $m$ and length $2a$, to the disc and the ring, as shown in the diagram. Show that the moment of inertia of the wheel about an axis through $A$, perpendicular to the plane of the wheel, is $42ma^2$. [5]

The axis through $A$ is horizontal, and the wheel can rotate freely about this axis. The wheel is released from rest with $O$ above the level of $A$ and $AO$ making an angle of $30^\circ$ with the horizontal. Find the angular speed of the wheel when $AO$ is horizontal. [3]

When $AO$ is horizontal the disc becomes detached from the wheel. Find the angle that $AO$ makes with the horizontal when the wheel first comes to instantaneous rest. [6]

**OR**

The continuous random variable $T$ has probability density function given by

$$\mathrm{f}(t) = \begin{cases} 0 & t < 2, \\ \dfrac{2}{(t-1)^3} & t \geqslant 2. \end{cases}$$

**(i)** Find the distribution function of $T$, and find also $\mathrm{P}(T > 5)$. [3]

**(ii)** Consecutive independent observations of $T$ are made until the first observation that exceeds 5 is obtained. The random variable $N$ is the total number of observations that have been made up to and including the observation exceeding 5. Find $\mathrm{P}\big(N > \mathrm{E}(N)\big)$. [3]

**(iii)** Find the probability density function of $Y$, where $Y = \dfrac{1}{T-1}$. [8]

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