FURTHER MATHEMATICS   **9231/02**

Paper 2

May/June 2004

3 hours

Additional materials: Answer Booklet/Paper
                    Graph paper
                    List of Formulae (MF10)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value is necessary, take the acceleration due to gravity to be $10\,\mathrm{m\,s^{-2}}$.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

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International Examinations

[Turn over

1 A hammer of mass 3 kg falls vertically onto the head of a nail with speed $20\,\text{m s}^{-1}$ and rebounds vertically with speed $5\,\text{m s}^{-1}$. If contact with the nail lasts for 0.05 s, find the constant force exerted on the nail by the hammer. [4]

2 A circular flywheel of radius 0.5 m, with moment of inertia about its axis of $500\,\text{kg m}^2$, is rotating freely at an angular speed of $50\,\text{rad s}^{-1}$. A brake that exerts a tangential force of constant magnitude 2000 N is applied to its rim. Find the time taken for the flywheel to be brought to rest, and the energy lost in kilojoules. [6]

3 Three smooth uniform spheres, *A*, *B* and *C*, lie at rest in this order in a straight line on a smooth horizontal surface. Their masses are 0.6 kg, 1.2 kg and 0.6 kg respectively, and their sizes are equal. Sphere *A* is projected towards *B* with speed $4\,\text{m s}^{-1}$, and hits *B* directly. *B* moves on to hit *C* directly. Find the velocities of *A*, *B* and *C* after the second impact, given that the coefficient of restitution is $\frac{1}{2}$ for each impact. State whether there will be any further impacts between any of the spheres. [7]
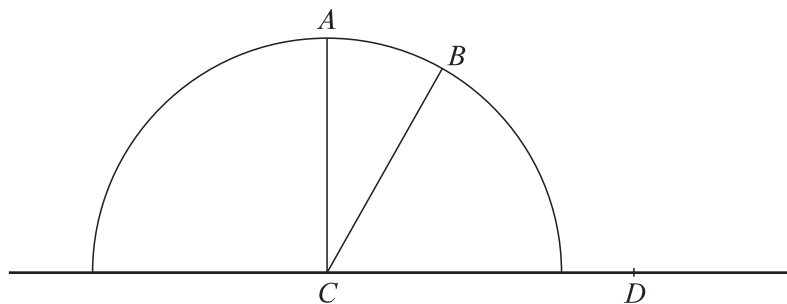
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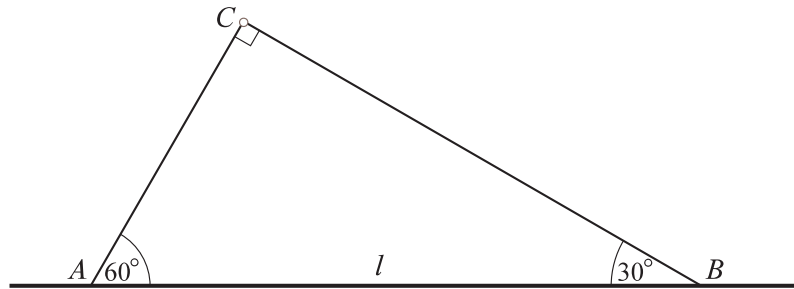
Two straight uniform ladders, *AC* and *BC*, have equal weights *W*. They stand in equilibrium with their feet, *A* and *B*, on rough horizontal ground and their tops freely hinged at *C*. Angle $CAB = 60°$, angle $CBA = 30°$ and $AB = l$ (see diagram). Find the vertical reactions at *A* and *B*. [6]

Find the magnitude and direction of the force exerted on *BC* by *AC*. [6]

5

A smooth semicircular cylinder of radius *a* is fixed on a horizontal table with its axis in the plane of the table. *C* is a point on the axis and *CA* is a vertical radius (see diagram). A particle is projected horizontally from *A*, with speed $\sqrt{(0.4ga)}$, in a direction perpendicular to the axis. Show that the particle leaves the surface with speed $\sqrt{(0.8ga)}$ at the point *B* where $\cos ACB = 0.8$. [7]

Determine the distance from *C* of the point *D* at which the particle hits the table, assuming that it moves freely under gravity from *B*. [7]

6 The continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} 1 & 1 \leqslant x \leqslant 2, \\ 0 & \text{otherwise,} \end{cases}$$

and the continuous random variable Y is defined by $Y = \dfrac{2}{X}$. Find the probability density function of Y. [4]

7 Anne is practising for the high jump. At each attempt the probability that she succeeds in clearing the qualifying height is $\frac{1}{3}$. The random variable N is the number of attempts she makes up to and including her first success. Find the value of $P(1 \leqslant N \leqslant E(N))$, assuming that the result of each attempt is independent of the results of previous attempts. [3]

Betty is also practising for the high jump, and at each attempt the probability that she succeeds is $\frac{1}{4}$. They jump alternately with Anne starting. Find the probability that Anne clears the qualifying height before Betty. [3]

8 The random variable X is the length of time, measured in days, between the occasions when Frank adjusts his watch. It is given that X has a negative exponential distribution with mean 120.

  (i) Find the distribution function for X, and hence find $P(X > 360)$. [3]

  (ii) Frank adjusts his watch at the start of day 1. Find the probability, P_n, that he next adjusts it at some time during day n, and show that the sequence $P_1, P_2, P_3, \ldots$ is a geometric progression. [3]

9 A random sample of 8 observations of a normal variable X gave the following summarised data:

$$\Sigma x = 233.2 \quad \text{and} \quad \Sigma(x - 30)^2 = 11.20.$$

Obtain a 95% confidence interval for the mean of X. [7]

10 A gardener plants 100 batches of 8 tulip bulbs and, when they come up, counts the number of yellow tulips in each batch, with the following results.

Number of yellow tulips in a batch	0	1	2	3	4	5	6	7	8
Number of batches	4	10	30	26	20	8	2	0	0

Fit a binomial distribution to these figures, and test the goodness of fit at the 5% level of significance. [8]

11 A statistics teacher takes two random samples, A and B, each of size 4, from her students and lists their recent scores, c (out of 20) for course work and e (out of 100) for an essay. See the tables below.

Sample A

c	6	11	14	18
e	52	74	61	84

Sample B

c	8	9	15	16
e	65	50	80	70

She chooses the sample with the higher product moment correlation coefficient (pmcc), and finds the regression line needed to predict e from c. Determine this regression line, and use it to predict e when $c = 12$. [5]

She takes a third random sample, C, of size 4 and discovers that, when she plots scatter diagrams for B and C on the same axes, the regression lines of e on c are parallel, and so are the regression lines of c on e. Deduce the pmcc of C, justifying your answer. [3]

She now takes A and B together to obtain a random sample of size 8. Use this combined sample to test, at the 5% significance level, whether the population pmcc is different from 0. [4]

12 Answer only **one** of the following two alternatives.

EITHER

A light spring of natural length 0.6 m stands upright on a horizontal floor. Its modulus of elasticity is 50 N. A stone of mass 2 kg is dropped onto the upper end T of the spring from a point A that is h m vertically above T. Once the stone makes contact with the spring at T, it remains attached to it. If h is small, show that the motion of the stone is simple harmonic, and find the height above the floor of the centre of this motion. [6]

Find the amplitude of this simple harmonic motion in terms of h. [5]

Hence show that, if the stone does not hit the floor, $h < 0.15$. [3]

OR

The numbers of road accidents per week in a certain town form an approximate normal distribution. A random sample of size 7 is

$$9, \quad 5, \quad 8, \quad 6, \quad 11, \quad 13, \quad 8.$$

Find a 95% confidence interval for the mean number of accidents per week. [5]

In a country area, a random sample of size 4 gives the numbers of road accidents per week as

$$7, \quad 3, \quad 10, \quad 2.$$

Assuming that both distributions are normal with a common variance, obtain a pooled estimate of that variance. [4]

Test, at the 5% level of significance, whether, on average, accidents occur more frequently in the town than in the country. [5]