CAMBRIDGE INTERNATIONAL EXAMINATIONS

General Certificate of Education Advanced Level

FURTHER MATHEMATICS 9231/2

PAPER 2

MAY/JUNE SESSION 2002

3 hours

Additional materials:
- Answer paper
- Graph paper
- List of Formulae (MF10)

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

Where a numerical value is necessary, take the acceleration due to gravity to be $10\,\mathrm{m\,s^{-2}}$.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The use of a calculator is expected, where appropriate.

Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.

You are reminded of the need for clear presentation in your answers.

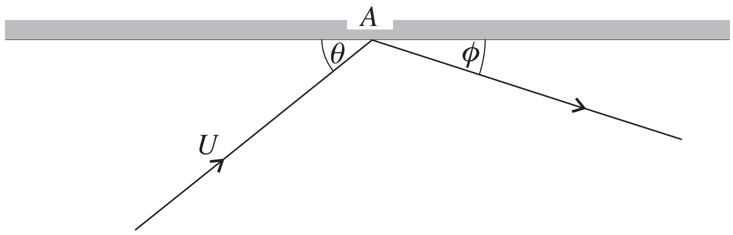
UNIVERSITY *of* CAMBRIDGE
Local Examinations Syndicate

1 When a bullet of mass 0.08 kg is fired horizontally from a stationary gun, a constant force of 2000 N acts forwards on the bullet for a period of 0.01 s. Find

(i) the impulse of this force, [2]

(ii) the speed of the bullet when the force ceases to act. [2]

2



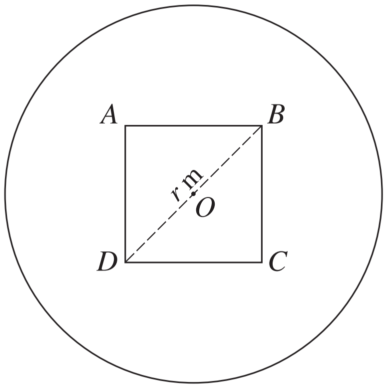
A snooker ball of mass m is moving with constant speed U in a straight line on a smooth horizontal table. It moves towards a straight smooth cushion, in a direction making an acute angle θ with the cushion, which it hits at the point A (see diagram). The coefficient of restitution between the ball and the cushion is e. Find the components, parallel and perpendicular to the cushion, of the velocity of the ball as it leaves the cushion, and show that the acute angle ϕ which this velocity makes with the cushion is given by

$$\tan\phi = e\tan\theta. \qquad [4]$$

Show that the kinetic energy lost in the impact is

$$\tfrac{1}{2}mU^2\sin^2\theta(1-e^2). \qquad [3]$$

3 A uniform square lamina $ABCD$ of mass m kg has its diagonals AC and BD each of length r m. Show that the moment of inertia of the lamina about an axis through its centre O and perpendicular to its plane is $\frac{1}{12}mr^2$ kg m^2. [2]



The square lamina is fixed to a coplanar uniform circular lamina of radius r m with its centre at O (see diagram). The two laminas have the same density. Show that the moment of inertia of the combined lamina about an axis through O perpendicular to its plane is

$$\frac{1}{12}Mr^2\left(\frac{1 + 12\pi}{1 + 2\pi}\right) \text{ kg m}^2,$$

where M kg is the total mass of the combined lamina. [4]

It is given that $r = 0.3$ and $M = 40$. The combined lamina is free to rotate in a vertical plane about a horizontal axis through O. A light inextensible string is wound round the circumference of the circular lamina and has one end attached to the circumference. A particle of mass 2 kg is attached to the other end of the string and hangs freely. The system is released from rest. Ignoring air resistance, find the angular speed of the combined lamina when it has turned through one revolution. [4]

 [Turn over

4 A simple pendulum of length l hangs inside a car from a point on the roof. The pendulum makes an angle θ with the vertical at time t.

(i) Given that the car is stationary, obtain the equation of motion of the pendulum in the form

$$l\frac{d^2\theta}{dt^2} = -g\sin\theta.$$ [2]

State the approximation that has to be made in this equation to reduce it to an equation of simple harmonic motion. [2]

(ii) The car now moves along a horizontal straight road with constant acceleration $\frac{7}{24}g$. You are given that the equation of motion of the pendulum is now

$$l\frac{d^2\theta}{dt^2} = -g(\sin\theta - \tfrac{7}{24}\cos\theta). \qquad \text{(A)}$$

[You are not expected to prove this result.]

Express the right hand side of equation (A) in the form $-R\sin(\theta - \alpha)$. [2]

Hence by writing $\phi = \theta - \alpha$, where ϕ is a small angle, show that equation (A) is equivalent to

$$l\frac{d^2\phi}{dt^2} = -\tfrac{25}{24}g\sin\phi.$$ [2]

Find the period of small oscillations and the value about which θ now oscillates. [3]

5 The side of a uniform square paving stone $ABCD$ is 0.6 m long and its weight is 500 N. The stone stands in a vertical plane with the side AB on rough horizontal ground, and the side AD at a distance 0.6 m from a vertical post HK. The point H is on the ground and $HK = 1.2$ m. A rope fixed to H goes through a small smooth light ring fixed to the stone at D, before passing over a small smooth pulley at K. The free end of the rope beyond K can be pulled so that the stone tilts about A.

(i)

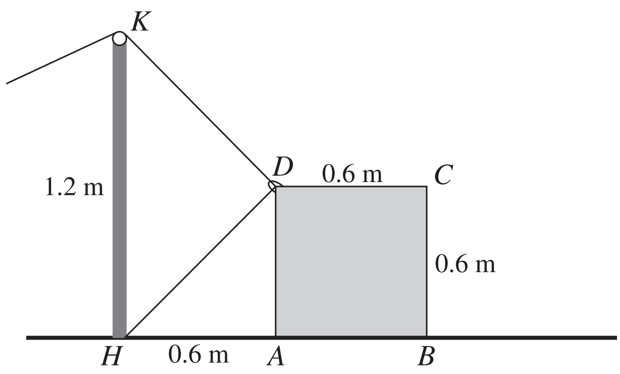


Fig. 1

Fig. 1 shows the stone in its original position. The tension T_1 N in the rope is such that the stone is on the point of turning about A. Find T_1, and the least possible value of the coefficient of friction at A for this to occur. [6]

(ii)

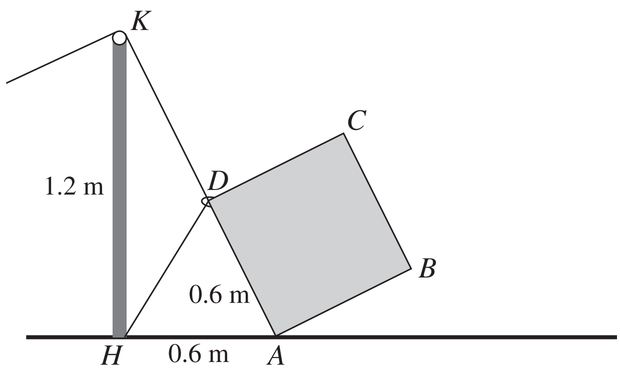


Fig. 2

The rope is now pulled until the stone has rotated about A so that it is in equilibrium when KDA is a straight line (see Fig. 2). The tension in the rope is now T_2 N. Find T_2. [5]

6 The manager of an army depot keeps records of diesel fuel issued to lorry drivers. A random sample of 12 of these records shows a mean issue of 58.14 gallons of diesel and an unbiased estimate of the population variance of 14.90 gallons^2. Obtain a 98% confidence interval for the mean of the population which the manager has sampled. State any assumption that you need to make for your method to be valid. [5]

7 A set of five coins is thrown 192 times and the numbers of heads obtained on each throw are summarised in the table.

Number of heads obtained	0	1	2	3	4	5
Number of throws	7	25	50	66	36	8

Test whether these results can be accepted at the 5% level of significance as consistent with the five coins being fair. [6]

8 A telephone engineer tests a random sample of mobile phones for accuracy in connecting to the desired number, and for sound quality when the number has been connected. He measures each property as Low, Medium or High. The numbers of phones in each category are given in the table.

		Sound quality		
		Low	Medium	High
Accuracy	Low	12	23	76
	Medium	8	12	28
	High	13	30	112

Test, at the 5% level of significance, whether sound quality is independent of accuracy. [6]

If the number of phones with Low accuracy and High sound quality had been 0 instead of 76 (and all the other numbers in the table were unchanged) what problem would you have encountered in your solution? [2]

9 A company accountant is comparing the book values of assets owned by his company with their current values. He lists these values for 8 randomly chosen assets.

Book value (in \$1000): x	5	14	30	56	84	112	121	140
Current value (in \$1000): y	6	12	33	52	86	108	132	148

$$[\Sigma x = 562,\ \Sigma y = 577,\ \Sigma x^2 = 58\,098,\ \Sigma y^2 = 62\,361,\ \Sigma xy = 60\,112.]$$

(i) Calculate the equation of the regression line of y on x for these data, giving your answer in the form $y = a + bx$. [3]

(ii) Find the current value (to the nearest \$100), as given by the regression line, of an asset with a book value of \$80 000. [1]

(iii) The book values of two assets differ by \$2000. Find by how much their current values differ, as given by the regression line. [2]

(iv) Find the value of the product moment correlation coefficient of the data in the table, and comment on your answers to parts **(ii)** and **(iii)** with reference to this value. [3]

(v) Use your answers to parts **(i)** and **(iv)** to find the regression coefficient of x on y. [2]

10 A racing driver claims that he can bring his car to rest from a speed of $150\,\mathrm{km\,h^{-1}}$ in a shorter distance with radial-ply tyres than with other types of tyre. As evidence for this claim, he lists the stopping distances on a race track in twelve random trials, six with radial-ply tyres, and six with other types of tyre.

	Stopping distances (m)					
Radial-ply tyres	123	164	139	161	118	135
Other types of tyre	156	137	134	173	155	151

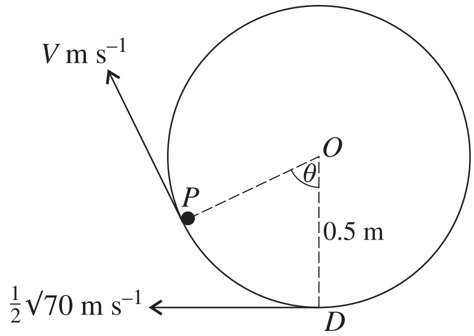
Test at the 5% significance level whether the racing driver's claim is justified, stating your null hypothesis and your alternative hypothesis clearly. Assume that the populations sampled are normal and have equal variances. [10]

Find a 95% confidence interval for the difference between the population means for radial-ply tyres and other types of tyre. [3]

[Question 11 is printed overleaf.]

11 Answer only **one** of the following two alternatives.

EITHER



A particle P of mass m kg is projected from the lowest point D of a fixed vertical circular hoop with centre O and radius 0.5 m. The speed of projection is $\frac{1}{2}\sqrt{70}\,\mathrm{m\,s^{-1}}$, and the particle travels along the smooth inside edge of the hoop. When the angle POD is θ the speed of P is $V\,\mathrm{m\,s^{-1}}$ (see diagram). Air resistance is to be neglected.

(i) Show that

$$V^2 = 7.5 + 10\cos\theta. \qquad [3]$$

(ii) Find, in terms of m and θ, the magnitude of the force which the hoop exerts on P, and show that P loses contact with the hoop when $\theta = 120°$. [5]

(iii) Show that, in its subsequent motion under gravity, P reaches the vertical through O at time $\sqrt{0.3}$ s after losing contact with the hoop. [3]

(iv) Show that the particle hits the hoop again at D. [3]

OR

A country bus driver picks up passengers randomly and independently at a mean rate of 12 per hour.

(i) Find, correct to 3 decimal places, the probability that he picks up at least 3 passengers in a period of 15 minutes. [2]

(ii) Show that the probability that he picks up at least one passenger in a period of t hours is

$$1 - \mathrm{e}^{-12t}. \qquad [2]$$

The time at which he picks up his first passenger is T hours. Explain why

$$\mathrm{P}(T < t) = 1 - \mathrm{e}^{-12t} \text{ for } t > 0. \qquad [3]$$

Hence find the probability density function of T, and find the values of $\mathrm{E}(T)$ and $\mathrm{Var}(T)$. [4]

Find also the median time at which the first passenger is picked up. [3]