FURTHER MATHEMATICS

GCE Advanced Level

Paper 9231/01
Paper 1

General comments

The majority of candidates produced good work in response to at least half of the questions, though, in contrast, there were some who clearly were ill-prepared for this examination and so, overall, made very little progress. At the outset to this report, therefore, it should be emphasised that achievement at this level requires knowledge of a syllabus which is end on to that for A Level Mathematics and thus is not an immediate consequence of this basic knowledge, however well understood.

Clarity and legibility of working varied with Centres to a considerable extent so that it is worth emphasising at the beginning of the life of this syllabus that Examiners can only mark what can be read. In any case, it must be helpful to the candidate to work in an ordered way so that when a response runs into difficulties errors can be identified easily.

Related to the need for coherence is, of course, the absolute need for accuracy. In this respect there were many deficiencies in all but the most simple situations.

These negative effects were augmented further by rubric infringements in **Question 12** which provides 2 alternatives. Since all questions are to be attempted, this is the only part of the paper where any rubric infringement is possible, yet despite the obvious waste of time that would be involved by this strategy, there were, nonetheless, a substantial number of candidates who tried to better their lot in this way, but to no avail. It is to be hoped, therefore, that future candidates will promote their own interests by keeping strictly to what the question paper asks them to do.

Knowledge of the syllabus and understanding of the concepts involved were uneven. Thus the syllabus material covered by **Questions 1** to **5** and **Question 11** was well understood and, in consequence, a significant minority of candidates obtained most of their marks in this area. At the other extreme, particular questions for which responses frequently showed a conceptual void, were **Question 6**, involving induction, **Question 7** which requires the determination of $\dfrac{d^2y}{dx^2}$ in terms of the parameter t, **Question 8 (ii)** which requires the determination of the area of a surface of revolution about the y-axis, **Question 9** involving the use of complex numbers to sum a trigonometric series, **Question 10** which tests the basic ideas of linear spaces, and finally **Question 12 EITHER (ii)** and **Question 12 OR (iii)** on the use of the calculus in optimisation problems.

In summary, therefore, it can be said that lack of technical expertise together with inadequate syllabus coverage, both in extent and depth, were the main reasons why many candidates did not do well in this examination.

Comments on specific questions

Question 1

Almost all candidates obtained the correct characteristic equation and solved it accurately. Subsequently there followed a variety of possible eigenvectors but, almost without exception, these were correct.

Answer: Eigenvalues are 1, 2; eigenvectors can be any non-zero scaling of $\begin{pmatrix}2\\3\end{pmatrix}$, $\begin{pmatrix}1\\1\end{pmatrix}$.

Question 2

For the first part, the method of integration by parts was generally perceived to be an effective way to proceed. However, there were a number of errors, usually sign errors, in the working and also some cases of omission of limits, particularly in the '*u*-*v*' term. In contrast, it was good to see a number of correctly worked arguments based on a consideration of, for example, $D_x\,[(1-x)^n \cos x]$.

For the second part, candidates generally used the reduction formulae correctly to obtain, essentially, S_3 in terms of sin(1). However, in the context of numerical evaluation, about half of all candidates interpreted sin(1) as sin(1°).

Answer: $S_3 = 0.042886$.

Question 3

The majority of candidates began by writing $(2n-1)^3 = 8n^3 - 12n^2 + 6n - 1$ and then after applying standard summation formulae, worked accurately to obtain the displayed result. Only a minority of responses proceeded along the lines of $S_N = \sum_1^{2N} n^3 - 8\sum_1^{N} n^3 = \frac{1}{4}(2N)^2(2N+1)^2 - 2(N)^2(N+1)^2$, etc., from which, using obvious factorisations, the result follows immediately.

Most candidates began the second part of this question with a correct preliminary result such as $\sum_{n=N+1}^{2N} n^3 = 4N^2(8N^2-1) - N^2(2N^2-1)$ and then simplified this expression without apparent difficulty. A small minority of responses showed incorrect partitions which may be symbolised as $\sum_{N+1}^{2N} = \sum_1^{2N} - \sum_1^{N+1}$ or $\sum_{N+1}^{2N} = \sum_1^{2N} - \sum_1^{N-1}$.

A few candidates expanded $(2n-1)^3$ and then, again, applied standard summation formulae, but such a complicated strategy proved to be very error prone.

Answer: $3N^2(10N^2-1)$.

Question 4

This question was answered accurately by most candidates. Responses showed, almost without exception, a correct overall strategy and there were few scripts in which the correct general solution did not appear. In sharp contrast, very few candidates were able to provide a satisfactory explanation as to why, independently of the initial conditions, $y \approx 3x + 2$ when x is large and positive. In fact, something like the argument set out below was expected.

As $x \to \infty$, $e^{-x}[A \sin(2x) + B\cos(2x)] \to 0$, whatever the values of A and B and hence whatever the initial conditions. Thus independently of the initial conditions, $y \approx 3x + 2$ for large positive x.

Answer: General solution: $y = e^{-x}[A\sin(2x) + B\cos(2x)] + 3x + 2$.

Question 5

Responses to this question showed some suboptimal solution strategies and also many basic working errors.

In the first place, the required y-equation can be obtained expeditiously by noting that $y = \dfrac{x}{2x-1} \Rightarrow x = \dfrac{y}{2y-1}$ and so substituting for x in the given cubic leads at once to the required result for y.

For part **(i)**, it is then sufficient to observe that, as from the x and y cubic equations it is obvious that $\alpha\beta\gamma = -1$ and that $\alpha\beta\gamma/(\alpha-2)(\beta-2)(\gamma-2) = -\frac{1}{3}$, then $(\alpha-2)(\beta-2)(\gamma-2) = 3$.

For part **(ii)**, the optimal argument is also simple. Thus it is only necessary to write the following:

$\sum \alpha(\beta - 2)(\gamma - 2) = (\alpha - 2)(\beta - 2)(\gamma - 2) \sum \alpha/(\alpha - 2) = 3 \times 3 = 9$.

However, the majority of candidates got involved in more complicated arguments. Thus there were even some who first attempted to evaluate $\sum \alpha/(\alpha - 2)$, $\sum \alpha\beta/(\alpha - 2)(\beta - 2)$ and $\alpha\beta\gamma/(\alpha - 2)(\beta - 2)(\gamma - 2)$ from the given x-equation, and then started all over again in an attempt to find answers for parts **(i)** and **(ii)**. Such protracted arguments generated many errors.

Answers: **(i)** 3; **(ii)** 9.

Question 6

The quality of most responses to this question was not good. Even where the central part of the induction argument was present, it was common for there to be no clear statement of the inductive hypothesis nor of an unambiguous conclusion. In fact, only a minority of candidates produced a completely satisfactory response.

In this respect something like the following was required:

Let $P(k)$ be the statement, $u_k < 4$ for some k.
Then $P(k) \Rightarrow 4 - u_{k+1} = 4 - (5u_k + 4)/(u_k + 2) = (4 - u_k)/(u_k + 2) \Rightarrow u_{k+1} < 4$, since all u_n are given to be positive. Thus $P(k) \Rightarrow P(k + 1)$, and since also $P(1)$ is true, for it is given that $u_1 < 4$, then by induction it follows that $P(n)$ is true for all $n \geqslant 1$.

In the second part of the question, few candidates made significant progress. All that was required here was to write $u_{n+1} - u_n = \ldots = (4 - u_n)(u_n + 1)/(u_n + 2) > 0$, and thus as $0 < u_n < 4$ and all u_n are positive, then $u_{n+1} > u_n$.

In this context, one had the impression that some candidates were groping towards this kind of argument, but lacked the technical expertise to see it through.

Question 7

This standard exercise involving the obtaining of $\dfrac{d^2y}{dx^2}$ in a parametric context showed up at least one important conceptual error. Overall the quality of responses can only be described as disappointing.

In the first part of the question, the working was generally methodologically correct and accurate. It was in the remainder of the question that many responses fell apart. The most common error was the supposition that $\dfrac{d^2y}{dx^2} = D_t\left(\dfrac{dy}{dx}\right)$ Actually from this it is possible, in this case, to obtain the required values of t, but such arguments, which are essentially incorrect, obtained little credit. Another persistent, but less common, error was the writing of $\dfrac{d^2y}{dx^2}$ as $\dfrac{d^2y}{dt^2} \div \dfrac{d^2x}{dt^2}$. This result, of course, did not get any credit.

Answers: $\dfrac{dy}{dx} = t^4(t - 3)e^{-t}$; $\dfrac{d^2y}{dx^2} = (t^7 - 8t^6 + 12t^5)e^{-t}$; $t = 2, 6$.

Question 8

This question was not answered as well as might be expected and certainly one persistent cause of failure was lack of technical competence.

In part **(i)**, most responses showed a correct integral representation of the arc length, but nearly a half of all candidates did not recognise that $\sqrt{\left[\frac{1}{4}\left(x^{1/3} - x^{1/3}\right)^2 + 1\right]} = \frac{1}{2}\left(x^{1/3} + x^{-1/3}\right)$ and so made no further progress. However, most of those who did get through this stage did go on to obtain the required result.

In part **(ii)**, it was clear that many candidates were put off by having to consider the surface area S generated by the rotation of C about the y-axis. Thus the (correct) integral representation of S by $\pi \int_1^8 x(x^{1/3} + x^{-1/3})\mathrm{d}x$ appeared in only a minority of scripts, though usually this was evaluated accurately.

Answers: **(i)** $\frac{63}{8}$; **(ii)** $\frac{2556\pi}{35}$.

Question 9

There were very few good quality responses to this question.

In the first part, a common error was the supposition that the given series is geometric with common ratio $\frac{1}{3}\cos 2\theta$. Thus, within the scope of this view of the question, complex numbers did not feature at all. In those responses which did show an attempt to determine the real part of $S_N = \frac{1-z^n}{1-z}$ where $z = \frac{1}{3}e^{2i\theta}$, there was much suppressed detail and erroneous working to be found. Thus although there was some appreciation of how to proceed, there were relatively few who could produce a completely accurate proof of the given result.

In the last part of this question, only a small minority of candidates comprehended that since $3^{-N+1} \to 0$, $3^{-N+2} \to 0$, as $N \to \infty$, then the given infinite series is convergent. Even where such statements appeared in responses, it was not always the case that the correct sum to infinity emerged from the working.

Answer: $S_\infty = \frac{9 - \cos 2\theta}{10 - 6\cos 2\theta}$.

Question 10

The majority of those candidates who produced serious work in response to this question established the linear independence of the vectors $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$ by the use of equations and likewise for the vectors $\mathbf{b}_1$, $\mathbf{b}_2$, $\mathbf{b}_3$. In contrast, a minority reduced the 4×3 matrices $(\mathbf{a}_1\ \mathbf{a}_2\ \mathbf{a}_3)$ and $(\mathbf{b}_1\ \mathbf{b}_2\ \mathbf{b}_3)$ to the echelon form. This is extremely easy to effect yet, surprisingly, there were errors even at this very basic level of operation.

There were also those that argued that, as it is given that the three vectors $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$ span V_1, then V_1 must be of dimension 3, and likewise for V_2. Such arguments implicitly ignore the possibility of linear dependence and as such are worthless.

Only a minority appeared to comprehend that $\dim(V_3) = 2$ and some gave 3 or even 4 vectors as a basis for this subspace. Thus again there was clear evidence of a general lack of understanding of the basic concepts of linear spaces.

About half of all candidates were able to produce 2 linearly independent vectors which belong to W, as required in part **(i)**, but in part **(ii)**, few could produce a satisfactory argument to show that W is not a linear space. This is most easily effected by showing closure does not hold.

A basis for V_3 is $\left\{ \begin{pmatrix}1\\0\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\\0\end{pmatrix} \right\}$; **(i)** $\left\{ \begin{pmatrix}0\\0\\1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\0\\1\end{pmatrix} \right\}$; **(ii)** $\begin{pmatrix}0\\0\\1\\0\end{pmatrix} + \begin{pmatrix}0\\0\\0\\1\end{pmatrix} = \begin{pmatrix}0\\0\\1\\1\end{pmatrix} \notin V_1 \cup V_2 - V_1 \cap V_2 = W$.

Question 11

Most responses showed correct methodology and accurate working to the extent that it can be said that this question was well answered by the majority of candidates.

In part **(i)** the vector product was used in a relevant way. Almost all failures to produce a correct result were due to accuracy errors.

The working in part **(ii)** was generally accurate. Most responses showed the position vector of a general point on l_3 in terms of a single parameter, s. Subsequently, 3 linear equations in s and t appeared and it was good to see that, almost always, the values of s and t obtained from 2 of the equations were checked out in the third.

In part **(iii)** since $l_1 \cap l_3 \equiv \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$, then the shortest distance, p, between l_1 and l_2 can be evaluated by using $l_2 \cap l_3 \equiv 4\mathbf{i} - \mathbf{j} - 9\mathbf{k}$. (This follows immediately from the working in part **(ii)** and, in fact, most candidates had already obtained this position vector.) Thus $p = \sqrt{3^2 + 3^2 + 6^2} = 3\sqrt{6}$.

Very few candidates argued in this simple way, but preferred to use the standard formula for the length of the common perpendicular between 2 skew lines. This strategy was not always implemented accurately so that arguments such as $p = \left|(\mathbf{i} - \mathbf{j} - 2\mathbf{k}).(3\mathbf{i} + 7\mathbf{j} + 7\mathbf{k})\right| / \sqrt{6} = \ldots = 3\sqrt{6}$, often appeared in an erroneous form.

Answers: **(i)** $7x - y + 4z = -7$; **(iii)** $3\sqrt{6}$.

Question 12 EITHER

A minority of candidates began by attempting to resolve the given rational function of x into partial fractions without any particular objective in view. Moreover, a number of such resolutions began with the form $B/(x - 2a) + C/(x + 2a)$, and not with $A + B/(x - 2a) + C/(x + 2a)$. In fact, the derivation of the correct partial fraction form of y does not enhance prospects in parts **(i)** or in **(iii)**, though it can be helpful in part **(ii)** if the sign of $\frac{d^2y}{dx^2}$ is used to determine the nature of the stationary points.

In part **(i)** most candidates obtained, or simply wrote down, the equations of the vertical asymptotes though some missed out the horizontal asymptote altogether or gave an incorrect result, e.g., the x- axis.

In part **(ii)** most candidates got as far as showing $\frac{dy}{dx} = 0 \Rightarrow x = a, 4a$ and went on to obtain the ordinates of the stationary points. Beyond this, however, responses generally ran into difficulties, mainly on account of inaccuracies in the working. Candidates, generally, appeared not to have the technical expertise necessary either to obtain a correct result for $\frac{d^2y}{dx^2}$, in any form, or to establish its sign at the stationary points of C in a convincing way. A simple argument in this context is to observe that $\frac{dy}{dx}$ can be written as $(x - a)\mathrm{F}(x)$ where $\mathrm{F}(x)$ is an easily identifiable rational function, actually it is $2a^2(x - 4a)/(x^2 - 4a^2)^2$, and thus as $\frac{d^2y}{dx^2} = \mathrm{F}(x) + (x - a)\mathrm{F}'(x)$, then at $x = a$, $\frac{d^2y}{dx^2} = \mathrm{F}(a) = \frac{-2}{3a} < 0$. The other stationary point can be considered similarly by writing $y = (x - 4a)\mathrm{G}(x)$, where $\mathrm{G}(x) = 2a^2(x - a)/(x^2 - 4a^2)^2$. It then follows that at $x = 4a$, $\frac{d^2y}{dx^2} = \mathrm{G}(4a) = \frac{1}{24a} > 0$. However, very few candidates argued in this way.

In part **(iii)** few sketch graphs were without error and, in fact, some did not even show 3 branches. Undoubtedly failure here was due to erroneous or incomplete results obtained earlier on. No doubt, on this account, some candidates must have been baffled by the clear inconsistency between the number of asymptotes obtained in part **(i)** and the display on their graphic calculator. This is especially a type of question for which the intelligent use of such a calculator can materially enhance the quality of responses, but in this instance there was very little evidence of such a causal relationship. Less than half of all candidates obtained full credit here.

Answers: **(i)** $x = 2a$, $x = -2a$, $y = a$; **(ii)** maximum at $(a, 0)$, minimum at $(4a, \frac{3a}{4})$.

Question 12 OR

In part **(i)** although the outline of *C* was usually correct, there was a persistent failure to indicate the scale in terms of *a*. Responses to this question were expected to include a clear indication of the location of the origin, the line $\theta = 0$ and the labelling of the extreme point $(2a, 0)$, yet in this respect there were many deficiencies.

In part **(ii)** most responses began with the integral $\frac{k}{2}\int_0^{\pi} a^2(1+\cos\theta)^2 \,\mathrm{d}\theta$ where, in most cases, *k* was either 1 (the most popular erroneous value) or 2, which is correct. Some candidates started with $k = 1$, but then introduced a factor of 2, without explanation, later on in the working. A few started with other correct forms such as $\frac{1}{2}\int_{-\pi}^{\pi} a^2(1+\cos\theta)^2 \,\mathrm{d}\theta$.

For the integration, the working was generally accurate and complete.

In part **(iii)** the starting point here is to write $y = r\sin\theta = a\sin\theta\,(1 + \cos\theta)$ and then to set $\frac{\mathrm{d}y}{\mathrm{d}\theta} = 0$. In this respect, it is helpful to write $y = a\sin\theta + (a/2)\sin(2\theta)$ so that $\frac{\mathrm{d}y}{\mathrm{d}\theta} = 0 \Rightarrow \cos\theta + \cos(2\theta) = 0$ follows immediately. The minimum of *y* is then easily found to occur at $\theta = -\pi/3$. However, only a small minority of candidates were able even to formulate *y* in terms of θ, as above, and few of these went on to obtain the correct minimum value of *y*.

Finally there was a small minority of candidates who attempted to obtain the *x* - *y* equation of *C* and then by implicit differentiation go on to obtain the minimum of *y*. However, few of these had the necessary technical expertise to work this complicated strategy through to a successful conclusion.

Answer: **(iii)** Minimum value of $y = \frac{-3\sqrt{3}}{4}$.

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Paper 2

General comments

The standard of the candidates was very variable, some producing excellent work while others had no real grasp of the syllabus. With the exception of the latter, almost all candidates were able to complete all the questions, suggesting that there was no undue time pressure. Although intermediate working was usually shown, some candidates simply wrote down final values of, for example, variances and correlation coefficients. Where such values were incorrect, these candidates may have needlessly lost marks for a correct method, since insufficient working was given to demonstrate their method of calculation. There appeared to be a slight preference for the Mechanics alternative over the Statistics one in the final question, though in some cases different candidates from the same Centre made differing choices.

Comments on specific questions

Question 1

Finding the impulse of the force from the product of the force's magnitude and its period of action rarely presented problems, and the units were usually given correctly. Most candidates then equated the impulse to the product of the bullet's mass and the required speed, again stating the units of the result, while others first calculated the acceleration and hence the speed.

Answers: **(i)** 20 Ns; **(ii)** 250 ms^{-1}.

Question 2

The question states that the components of the velocity after the collision should first be found, but some candidates ignored this and tried unsuccessfully to derive the given equation directly by some invalid method. Instead they should have noted that the component $U \cos \theta$ parallel to the cushion is unchanged, while the perpendicular component changes by a factor e to $eU \sin \theta$. The most convincing way of finding the lost kinetic energy is to consider the difference in the total kinetic energy before and after impact. Some candidates apparently relied on the fact that only the velocity component perpendicular to the cushion changes, and therefore found only the loss in the corresponding component of the kinetic energy. Unfortunately the majority of candidates who derived the given result in this way did not give an explicit justification, leaving open the possibility that they had simply worked backwards from the expression quoted in the question without any real understanding.

Question 3

The simplest approach is to use the expression for the moment of inertia of a rectangular lamina given in the *List of Formulae*, substituting $\frac{r}{2\sqrt{2}}$ in place of a and b. Many candidates failed to do this, with some using the given formula for a thin rod in association with the perpendicular axis theorem, but without adequate justification, and others purporting to achieve the given result without any valid reasoning. The second part, concerning the moment of inertia of the combined lamina, was frequently omitted. Those who attempted it successfully usually related the masses of the square and circular laminas to that of the combined body in terms of their common density, and substituted for the latter in an expression for the required moment of inertia. Although many candidates realised that the final part can be solved using conservation of energy, they often omitted one of the three contributory energy terms. An alternative valid approach which was also seen is to relate the net force and the couple acting on the particle and the lamina to their linear and rotational acceleration respectively.

Answer: 6.52 rad s^{-1}.

Question 4

The first equation of motion was often derived successfully by applying Newton's Law perpendicular to the string, though some candidates wrongly considered the radial direction. Most stated the approximation $\sin \theta \approx \theta$ correctly, and knew the general approach to expressing the right hand side of equation (A) in its alternative form. While most candidates realised that this expression could also be rewritten in terms of ϕ, many overlooked the left hand side of the equation, while a few seemed to believe that the rearrangement is only valid if ϕ is small. The period was often found correctly from $\frac{2\pi}{\omega}$, but common faults were to omit the length of the pendulum, or less seriously not to simplify the expression. The value α about which θ now oscillates defeated the great majority of candidates, most of whom made no attempt to find it.

Answer: **(ii)** $R = \frac{25g}{24}$, $\alpha = \tan^{-1}\left(\frac{7}{24}\right)$.

Question 5

The tension is found in both parts by taking moments for the stone about A. Although this is fairly simple for T_1, a common fault was to include the moment of the tension in only one of the sections DH or DK of the rope. The coefficient of friction μ is as usual related to the friction F and reaction R by $\mu \geqslant \frac{F}{R}$, with F and R found by horizontal and vertical resolution of forces. Finding the correct moment equation in the second part presented significantly more difficulty, since the most relevant angles are no longer 30° or 60°, thus requiring some trigonometric effort, even though many candidates overlooked this.

Answers: **(i)** 177, 0.5; **(ii)** 131.

Question 6

Although most candidates knew how to find the confidence interval in principle, the majority used an incorrect tabular value instead of the t-value 2.718, with a high proportion opting instead for the z-value 2.326 or 2.054. The necessary assumption that the population is normal was frequently omitted.

Answer: [55.1, 61.2].

Question 7

The usual approach to this question is to use the binomial distribution B(5, 0.5) to calculate the expected values corresponding to the second row of the given table, and then calculate the corresponding χ^2 value, here 5.13. Comparison with the tabular value 11.07 leads to the conclusion that the coins are fair. While several candidates chose instead to attempt to calculate an appropriate z-value, their approach was usually invalid.

Question 8

Most candidates appreciated that the χ^2 test is appropriate here, and found the value of approximately 4.0 correctly. Comparison with the tabular value 9.488 leads to the conclusion of independence. The second part was by contrast very poorly done, with only a handful of candidates both identifying the problem of an expected value being less than 5, and identifying it as roughly 4.85 in the Low/Low cell.

Question 9

Almost all candidates knew how to find the coefficients a and b, usually by first using the given formula for b, and less often by solving the linear least squares equations explicitly. However a common fault was not to retain additional figures in the working, and the resulting rounding errors affected subsequent calculations. Substitution of $x = 80$ gives the corresponding value of y and hence the solution in part **(ii)**. Part **(iii)** was frequently, and wrongly, answered by substituting a value of either 2 or 2000 in the equation of the regression line, instead of noting that the ratio of a change in y to the corresponding change in x is b. Most candidates applied the formula for the product moment correlation coefficient r, and many also commented that their preceding answers are reliable. The correct approach to the final part is to note that the product of the two regression coefficients of y on x and x on y is r^2, since calculating the required coefficient from its formula is not making use of the previous answers as specified in the question.

Answers: **(i)** $a = -1.75$, $b = 1.05$; **(ii)** \$82 400; **(iii)** \$2 100; **(iv)** 0.996; **(v)** 0.944.

Question 10

The correct test to apply in the first part is the two-sample t-test with a common unknown population variance, but many candidates wrongly applied the paired-sample one. The former test yields a t-value of magnitude 1.13, and comparison with the tabular value 1.812 leads to the conclusion that the racing driver's claim is not justified. The corresponding method should be used for the confidence interval, and here the appropriate tabular value of t is 2.228.

Answer: [-32.6, 10.6].

Question 11 EITHER

The first part is readily solved by equating the kinetic and potential energies of the particle at the initial and final points, and simplifying. The following part, in which the required force R is found by summing the other two radial forces on the particle, presented no difficulty for most candidates, and they usually then equated R to zero in order to solve the resulting equation for θ. The final two parts proved much more challenging, however, with some candidates vainly attempting to solve part **(iii)** by considering the vertical component of the particle's motion. The correct approach is to find the constant horizontal component of its velocity and also the distance to the vertical through O, and hence the time. The final part is concerned with the vertical motion under gravity, and is best solved by showing that in the given time the particle falls a distance 0.75 m, which equals the height above D of the point at which contact is lost with the hoop.

Question 11 OR

Most candidates were able to sum the first three terms of the Poisson expansion with parameter 3, and subtract their sum from unity in order to find the first probability. The second part needs only a realisation that the probability of picking up no passengers in a period of *t* hours equals the first term of the Poisson expansion with parameter 12. Convincing explanations of the given equation for $P(T < t)$ were very rare, however, and many candidates made no serious attempt at this. By contrast most found the probability density function $12e^{-12t}$ by differentiation, but while some were able to quote the values of $E(T)$ and $\text{Var}(T)$, others attempted to find them by integration, often unsuccessfully. The median time is found by equating the given expression for $P(T < t)$ to 0.5 and solving for *t*, and the only common fault here was to round the answer to fewer than the 3 significant figures specified in the rubric.

Answers: **(i)** 0.577; $\frac{1}{12}$; $\frac{1}{144}$; 0.0578 hours.