CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

MARK SCHEME for the May/June 2014 series

9231 FURTHER MATHEMATICS

9231/21 Paper 2, maximum raw mark 100

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International Examinations

Question Number	Mark Scheme Details		Part Mark	Total
1	Equate impulse to momentum to find initial speed v and Newton's law of restitution to find new speed:	$v = 4u,\ v' = ev = [-]\ 3u$ M1 A1	2	**2**
2	Find v^2 at both A and B:	$v_A{}^2 = \omega^2(a^2 - 0{\cdot}5^2)$ *and* $v_B{}^2 = \omega^2(a^2 - 0{\cdot}75^2)$ B1		
	Find amplitude a m from given K.E. ratio:	$\frac{1}{2} mv_A{}^2 = (12/11)\ \frac{1}{2} mv_B{}^2$		
		$11\ (a^2 - 0{\cdot}5^2) = 12\ (a^2 - 0{\cdot}75^2)$		
		$a^2 = \frac{1}{4}(27 - 11) = 4,\ \ a = 2$ M1 A1	3	
	Find ω from $v_{max} = a\omega$:	$0{\cdot}6 = 2\omega,\ \ \omega = 0{\cdot}3$ B1		
	Find time (√ on a) at A	$\omega^{-1} \sin^{-1} (0{\cdot}5/2)$ *or* $\omega^{-1} \cos^{-1} (0{\cdot}5/2)$		
	or at B, e.g.:	$\omega^{-1} \sin^{-1} (0{\cdot}75/2)$ *or* $\omega^{-1} \cos^{-1} (0{\cdot}75/2)$ M1 A1√		
	Combine correctly to find time from A to B:	$\omega^{-1} \sin^{-1} (0{\cdot}75/2) - \omega^{-1} \sin^{-1} (0{\cdot}5/2)$		
		or $\omega^{-1} \cos^{-1} (0{\cdot}5/2) - \omega^{-1} \cos^{-1} (0{\cdot}75/2)$ M1		
	Evaluate to 3 d.p.:	$= \omega^{-1} (0{\cdot}3844 - 0{\cdot}2527)$ or $\omega^{-1} (1{\cdot}318 - 1{\cdot}186)$		
		$= 1{\cdot}2813 - 0{\cdot}8423$		
		$4{\cdot}3937 - 3{\cdot}9547 = 0{\cdot}439$ [s] A1	5	**8**

3	Use conservation of momentum, e.g.:	$mv_A + 9mv_B = mu$ M1		
	Use Newton's law of restitution (consistent signs):	$v_B - v_A = eu$ M1		
	Relate v_A to v_B using K.E. (A.E.F.):	$\frac{1}{2}mv_A^2 + \frac{1}{2}9mv_B^2 = \frac{1}{4}mu^2$ M1		
	Combine two eqns to find v_A and v_B e.g.:	$v_A = (1 - 9e)u/10$, $v_B = (1 + e)u/10$		
		or v_A, $v_B = -u/2$, $u/6$ [or $7u/10$, $u/30$] M1 A1		
	Use in 3rd eqn to find e, e.g.:	$(1 - 9e)^2 + 9(1 + e)^2 = 50$		
	(A0 if finally $\pm\frac{2}{3}$)	$90e^2 = 40$, $e = \frac{2}{3}$ M1 A1	7	
	Use Newton's law of restitution with	$v_C = 2v_B'$, e.g.: $v_C - v_B' = ev_B$, $v_B' = \frac{2}{3}v_B$ B1		
		[$v_B = u/6$, $v_B = u/9$, $v_C = 2u/9$]		
	Use conservation of momentum to find k:	$9mv_B' + kmv_C = 9mv_B$		
		$9v_B' + 2kv_B' = 13{\cdot}5v_B'$, $k = 9/4$ M1 A1	3	**10**

4	**(i)**	Use conservation of energy at lowest point: Use $F = ma$ radially at lowest point: Eliminate v^2 to find R [$v^2 = 2{\cdot}3\,ga$]:	$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mga$ B1 $R - mg = mv^2/a$ B1 $R = mu^2/a + 3mg = 3{\cdot}3\,mg$ B1	3	
	(ii)	Use conservation of energy at B to find V_B : (A.E.F.)	$\frac{1}{2}mV_B{}^2 = \frac{1}{2}mu^2 + mga\sin\theta$ M1A1 $V_B{}^2 = (0{\cdot}3 + 0{\cdot}5)\,ga,\ V_B = \sqrt{(0{\cdot}8\,ga)}$ *or* $2\sqrt{(ga/5)}$ *or* $0{\cdot}894\sqrt{(ga)}$ A1	3	
	(iii)	Use vertical component v_B of speed V_B at B: Find height h reached above B: Find height h reached above level of O:	$v_B = V_B\cos\theta$ [$= \frac{1}{4}\sqrt{15}\,V_B = \sqrt{(\frac{3}{4}ga)}$] M1 $h = v_B{}^2/2g = 3a/8$ M1 A1 $h - a\sin\theta = 3a/8 - \frac{1}{4}a = a/8$ **A.G.** A1	4	**10**
5		Find MI of components about A: (M1 for BC or CD) Find total MI about A: (OR can first find total MI about centre of mass) State or imply total mass acts at mid-point of AC Use eqn of circular motion to find $d^2\theta/dt^2$: Approximate $\sin\theta$ by θ and substitute for I:	Glass $(3M/5)\{\frac{1}{3}(5a)^2 + 25a^2\} = 20Ma^2$ M1 A1 AB $M\{\frac{1}{3}(4a)^2 + (4a)^2\} = 64Ma^2/3$ B1 AD $\frac{1}{3}M\{\frac{1}{3}(3a)^2 + (3a)^2\} = 4Ma^2$ B1 BC $\frac{1}{3}M\{\frac{1}{3}(3a)^2 + 73a^2\} = 76Ma^{2/3}$ M1 A1 CD $M\{\frac{1}{3}(4a)^2 + 52a^2\} = 172Ma^{2/3}$ A1 $I = 128Ma^2$ A.G. A1 M1 $I\,d^2\theta/dt^2 = [-]\,(49Mg/15)\,5a\sin\theta$ M1 A1 $d^2\theta/dt^2 = -(49g/384a)\,\theta$ A1	8	
		Find period $T = 2\pi/\omega$ with $\omega = \sqrt{(49g/384a)}$:	$T = 2\pi\sqrt{(384a/49g)}$ *or* $(16\pi/7)\sqrt{(6a/g)}$ *or* $17{\cdot}6\sqrt{(a/g)}$ (A.E.F.) B1	5	**13**

6		State or find the expected value of X: using $p = ¼$:	$E(X) = 1/p = 1/¼ = 4$ B1	1	
	(i)	Find $P(X = 4)$:	$P(X = 4) = (¾)^3 ¼ = 27/256$ or $0{\cdot}105$ M1 A1	2	
	(ii)	Find $P(X<6)$:	$P(X < 6) = 1 - (¾)^5$		
			or $\{1 + ¾ + (¾)^2 + (¾)^3 + (¾)^4\}¼$		
		S.R. Using $p = ½$ can earn B0 M1 A0 M0 A0	$= 781/1024$ or $0{\cdot}763$ M1 A1	2	**5**
7	**(i)**	State probability density function of T:	$f(t) = 0{\cdot}001 \exp(-0{\cdot}001t) \quad (t \geq 0)$ B1 $[= 0 \quad$ (otherwise *or* $t < 0$)]	1	
	(ii)	Find $P(T > 2000)$: **S.R.** $1 - e^{-2} = 0{\cdot}865$ earns B1 only (max 1/3)	$P(t > 2000) = 1 - F(2000)$ M1 $= 1 - (1 - e^{-2}) = e^{-2}$ *or* $0{\cdot}135$ M1 A1	3	
		State inequality for t (lose A1 if = or ≤): Solve for t_{max}: (Omitting power 10 earns 0/4; using $1 - (\exp(-0{\cdot}001t))^{10}$ can earn M1 A0 M1 A0 only)	$(\exp(-0{\cdot}001t))^{10} \geq$ [*or* >] $0{\cdot}9$ M1 A1 $t_{max} = (\ln 0{\cdot}9) / (-0{\cdot}01) = 10{\cdot}5$ M1 A1	4	**8**

8	State hypotheses (B0 for $\bar{x}$ …):	H_0: $\mu_X = \mu_Y$, H_1: $\mu_X \neq \mu_Y$	B1		
	Estimate both popln. variances using two samples: (allow use of biased: $\sigma_{X,60}{}^2$ = 236 *or* 15·36^2)	$S_x{}^2$ = (626 220 – 6060^2/60) / 59 [= 240 *or* 15·49^2]			
	(allow use of biased: $\sigma_{Y,50}{}^2$ = 265 *or* 16·28^2)	*And* $s_Y{}^2$ = (464 500 – 4750^2/50) / 49 [= 270·4 *or* 16·44^2]	M1 A1		
	Estimate population variance for combined sample: (allow $\sigma_{X,60}{}^2$/60 + $\sigma_{Y,50}{}^2$/50: 9·233 *or* 3·039^2)	s^2 = $s_X{}^2$/60 + $s_Y{}^2$/50 = 9·408 *or* 3·067^2	M1 A1		
	Calculate value of z (to 2 d.p., either sign):	z = (101 – 95) / s = 6/3·067 = 1·96 (*or* 1·97)	M1 A1		
	State or use correct tabular z – value (to 2 d.p.): (or can compare 6 with e.g. 2·326 s = 7·13 *or* 7·07)	$z_{0.99}$ = 2·326 *or* 2·33 (allow 2·36)	B1		
	Correct conclusion (A.E.F, √on z – values):	[Accept H_0] Claims are the same	B1√		
	S.R. Assuming equal population variances: Find pooled estimate of common variance s^2:	Hypotheses; Explicit assumption : s^2 = (626 220 – 6060^2/60 + 464 500 – 4750^2/50) / 108	(B1; B1) (M1 A1)		
	Calculate value of z (to 2 d.p., either sign):	= 253·8 *or* 15·93^2 z = 6 / $s\sqrt{(1/60+1/50)}$ = 1·97	(A1) (M1 A1)		
	Tabular value; conclusion	As above	(B1; B1√)		9

9	Find expected frequency p:	$p = 200 \int_2^3 (1 / x \ln 8)\,dx$ $= (200 / \ln 8)\,[\ln x]_2^{\ 3}$ $= 200 \times 0{\cdot}1950 = 39{\cdot}00$ **A.G.** $q = 21{\cdot}46$ *or* $21{\cdot}45$	 M1A1 M1A1		
	Find q by similar method *or* by using total of 200:			4	
	State (at least) null hypothesis:	H_0: $f(x)$ fits data (A.E.F.)	B1		
	Calculate χ^2 (to 3 s.f.):	$\chi^2 = 0{\cdot}202 + 0{\cdot}923 + 0{\cdot}678 + 0{\cdot}584$ $+ 1{\cdot}134 + 4{\cdot}134 + 3{\cdot}644 = 11{\cdot}3$	 M1A1		
	State or use correct tabular χ^2 value (to 3 s.f.)::	$\chi_{6,\,0.95}^{\ 2} = 12{\cdot}59$	B1		
	Valid method for reaching conclusion:	Accept H_0 if $\chi^2 \leqslant$ tabular value	M1		
	Conclusion consistent with correct values (A.E.F):	Distribution fits observations	A1	6	**10**
10	Find correlation coefficient r: $r = (73\,527 - 866 \times 639 / 10) / \sqrt{\{(121\,276 - 866^2 / 10)(55\,991 - 639^2 / 10)\}}$		M1 A1		
	(A.E.F.; A0 if only 3 s.f. clearly used)	$= 18\,189{\cdot}6 / \sqrt{(46\,280{\cdot}4 \times 15\,158{\cdot}9)}$	A1		
		$= 0{\cdot}687$	*A1	4	
	State both hypotheses (B0 for r …):	H_0: $\rho = 0$, H_1: $\rho \neq 0$	B1		
	State or use correct tabular two-tail r-value:	$r_{10,\,5\%} = 0{\cdot}632$	*B1		
	Valid method for reaching conclusion:	Reject H_0 if $\lvert r \rvert >$ tabular value	M1		
	Correct conclusion (A.E.F, dep *A1, *B1):	There is non-zero correlation	A1		
	Calculate gradient p in $x - \bar{x} = p(y - \bar{y})$:	$p = 18\,189{\cdot}6 / 15\,158{\cdot}9 = 1{\cdot}20$	B1	4	
	Find regression line of x on y:	$x = 86{\cdot}6 + 1{\cdot}20\,(y - 63{\cdot}9)$ $= 1{\cdot}20\,y + 9{\cdot}92$	 M1 A1	3	**11**

11 A (i)	Use Pythagoras to find AB:	$AB = \sqrt{(4a^2 + 12a^2)} = 4a$	**A.G.**	M1 A1		
	Find $\angle SAB$:	$\angle CAB = \sin^{-1} 2a\sqrt{3}/4a$ *or* $\cos^{-1} 2a/4a$				
		or $\tan^{-1} 2a\sqrt{3}/2a$				
		$= 60°$ so $\angle SAB = 30°$	**A.G.**	M1 A1	4	
(ii)	*EITHER*					
	Resolve vertically and horizontally, e.g.:	$\frac{1}{2} N_A + \frac{1}{2}\sqrt{3}\, N_B + \frac{1}{2}\sqrt{3}\, F_A = W$				
	(F_A may be in either direction)	*and* $\frac{1}{2}\sqrt{3}\, N_A = \frac{1}{2} N_B + \frac{1}{2} F_A$		M1 A1		
	Eliminate $N_B + F_A$ to find N_A:	$N_A = \frac{1}{2} W$	**A.G.**	A1		
	OR				3	
(iii)	Resolve in dirn. PQ to find N_A:	$N_A = \frac{1}{2} W$	**A.G.**	(M1 A1)		
	Second resolution, e.g. in dirn. PS:	$N_B + F_A = \frac{1}{2}\sqrt{3}\, W$		(A1)		
	Take moments, e.g. about A:	$\frac{1}{2}\sqrt{3}\, W \times 3a/2 + \frac{1}{2} W \times (2\sqrt{3} - 3)a$				
	(A1 for each side of eqn)	$= N_B \times 2a$		M1 A1 A1		
	Solve to find N_B:	$N_B = \{(7\sqrt{3} - 6)/8\}\, W$		M1 A1		
	Use N_B to find F_A:	$F_A = \sqrt{3}\, N_A - N_B$ *or* $\frac{1}{2}\sqrt{3}\, W - N_B$				
		$= \{3(2 - \sqrt{3})/8\} W$ (A.E.F.)		M1 A1	7	**14**

B	Estimate population variance:	$s_P^{\,2} = (236{\cdot}0 - 42{\cdot}8^2/8)/7$			
	(allow biased here: $0{\cdot}8775$ or $0{\cdot}9367^2$)	$= 351/350$ or $1{\cdot}003$ or $1{\cdot}001^2$	M1		
	Find confidence interval (allow z in place of t) e.g.:	$42{\cdot}8/8 \pm t\sqrt{(s_P^{\,2}/8)}$	M1		
	Use correct tabular t-value:	$t_{7,\,0.975} = 2{\cdot}365$	A1		
	Evaluate C.I. correct to 2 d.p.:	$5{\cdot}35 \pm 0{\cdot}84$ or $[4{\cdot}51, 6{\cdot}19]$	A1	4	
	Formulate inequality for k (or equality for $k_{\max}$):	$(5{\cdot}35 - k)/\sqrt{(s_P^{\,2}/8)} \geqslant [\text{or} >]\ t$	M1		
	Use correct tabular t-value:	$t_{7,\,0.9} = 1{\cdot}415$	A1		
	Solve for $k_{\max}$ (A0 if = or $\leqslant$ was used for k above):	$5{\cdot}35 - k \geqslant 0{\cdot}50$, $k_{\max} = 4{\cdot}85$	A1	3	
	State hypotheses (B0 for $\bar{x}$ …), e.g.:	H_0: $\mu_P = \mu_Q$, H_1: $\mu_P > \mu_Q$	B1		
	State assumption (A.E.F.):	Normal distns. for [P and] Q			
		and equal variances	B1		
	Estimate (pooled) common variance:	$s^2 = (7 \times 1{\cdot}003 + 11 \times 1{\cdot}962)/18$			
		$= 1{\cdot}589$ or $1{\cdot}261^2$	M1 A1		
	Calculate value of t (to 3 s.f.):	$t = (5{\cdot}35 - 4{\cdot}60)/(s\sqrt{(1/8 + 1/12)})$			
		$= 1{\cdot}30$	M1 A1		
	Correct conclusion (A.E.F., $\sqrt{}$ on t):	$t < t_{18,\,0.9} = 1{\cdot}33$ so Q's mean is not less than P's	B1$\sqrt{}$	7	**14**