UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

MARK SCHEME for the May/June 2008 question paper

9231 FURTHER MATHEMATICS

9231/02 Paper 2, maximum raw mark 100

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the May/June 2008 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Qu No	Mark Scheme Details			Part Mark	Total
1 (i)	Find impulse I from change in momentum:	$\pm I = 0.02\,(240 - 10) = 4.6$ [N s]	M1 A1	2	
(ii)	Find force F from I/t:	$\pm F = I / 0.004 = 1150$ [N]	M1 A1	2	**[4]**
2 (i)	State or use relation between T and ω:	$T = 2\pi / \omega$	B1		
	Use trigonometric form for SHM, e.g.:	$x = a \sin \omega t$ *or* $a \cos \omega t$	M1		
	Find time to move BO or OB', e.g.:	$(1/\omega) \sin^{-1} \frac{1}{2}$			
		or $(1/\omega)(\cos^{-1} 0 - \cos^{-1} \frac{1}{2})$	A1		
	Replace $\sin^{-1}$ or $\cos^{-1}$ and relate to T:	$(T/2\pi)(\pi/6)$ *or* $(T/2\pi)(\pi/2 - \pi/3)$	A1		
	Find time to move BB':	$2 \times T/12 = T/6$	A1	5	
(ii)	Use SHM formula for v or v^2, e.g.:	$v = a\omega \cos \omega t$ *or* $v^2 = \omega^2 (a^2 - x^2)$	M1		
	Form ratio v_B / v_O or v_B^2 / v_O^2:	$a\omega \cos (\sin^{-1} \frac{1}{2}) / a\omega \cos (\sin^{-1} 0)$			
		or $(a^2 - a^2/4) / (a^2 - 0)$	A1		
	Evaluate ratio:	$\cos (\pi/6) / \cos 0 = \sqrt{3}/2$ *or* 0.866	A1	3	**[8]**
3	Take moments for rod about A:	$R_B \sin 30° \times 0.8 = 10x$	M1		
	Evaluate force R_B at B:	$R_B = 25x$	A1	2	
	Resolve horizontally and vertically for rod:	$F_A = 10 - R_B \sin 30°$			
	and	$R_A = R_B \cos 30°$	M1 A1		
	Use $F_A \leqslant \mu R_A$ (or <; A.E.F.):	$\mu \geqslant (10 - 12.5x) / 25x \sqrt{3}/2$	M1 A1		
	Find μ_{min} by putting $x = 0.4$:	$\mu_{min} = 5/10\sqrt{3}/2 = 1/\sqrt{3}$ *or* 0.577	M1 A1	6	
	S.R. Using $F_A = \mu R_A$ without justification:	$\mu = (10 - 12.5x) / 25x \sqrt{3}/2$	(B1)		
	Using $x \geqslant 0.4$ to give:	$\mu \leqslant 1/\sqrt{3}$	(B1)		**[8]**
4	Equate vertical speeds to zero (here $\tan \alpha = 4/3$):	$u \sin \alpha - gt = 0 = ku \cos \alpha - gt$			
	or:	$(u \sin \alpha)^2 - 2gs = (ku \cos \alpha)^2 - 2gs$			
	or equate vertical distances at collision:	$ut \sin \alpha - \frac{1}{2}gt^2 = kut \cos \alpha - \frac{1}{2}gt^2$	M1 A1		
	Simplify:	$u \sin \alpha = ku \cos \alpha$	A1		
	Evaluate k:	$k = 4/3$	M1 A1	5	
	Find time t to reach ground:	$t = (u \sin \alpha) / g = 4u/5g$	B1		
	Find speed of separation (ignore sign):	$v_P - v_Q = -e\,(u_P - u_Q)$	M1		
	Substitute for u_P, u_Q:	$v_P - v_Q = -e\,(u \cos \alpha + ku \sin \alpha)$			
	(ignore sign)	$= -eu\,(3 + 4k)/5$ $[= -5eu/3]$	A1		
	Find distance apart:	$\lvert v_P - v_Q \rvert\, t = 4eu^2/3g$	M1 A1	5	**[10]**

Qu No	Mark Scheme Details			Part Mark	Total
5	Find moment of inertia of disc:	$I_o = \frac{1}{2}(\pi \times 0.5^2 \times 6.2)\ 0.5^2$	M1 A1		
	Find moment of inertia of rectangle:	$I_\square = \frac{1}{3}(0.4 \times 0.3 \times 6.2)(0.2^2 + 0.15^2)$	M1 A1		
	Combine to give M.I. of lamina about O:	$I = I_o - I_\square$ [= 0.6087 – 0.0155]	M1		
	Apply parallel axes theorem for M.I. about A:	$I' = I + M'\ AO^2$ [$= I + M'\ 0.25^2$],	M1		
		and $M' = (\pi\ 0.5^2 - 0.4 \times 0.3)\ 6.2$	A1		
		[= 4.869 – 0.744 = 4.1255]			
	Evaluate I':	0.5932 + 0.2578 *or* 0.913 – 0.062			
		= 0.851 **A.G.**	A1	8	
	Consider energy E when O rises by some h:	$E = \frac{1}{2} I' \omega^2 - M'gh$	M1		
	Find ω_{min} from $E = 0$ *or* $E \geqslant 0$ when AO vertical		M1		
	Find h when AO vertical:	$h = \frac{1}{2}AD + AO = 0.4$	A1		
	Evaluate ω_{min}:	$\omega_{min} = \sqrt{(4.126gh / \frac{1}{2}\ 0.851)}$	A1		
		= 6.23	A1	5	**[13]**
6	Consider distn. Y of differences in diameters (or $-Y$):	$Y = 1.02\ T - 1.04\ R$	M1		
	Find E(Y):	E(Y) = 1.3005 – 1.2948 [= 0.0057]	A1		
	Find Var(Y):	Var(Y) = $1.02^2 \times 0.015^2$			
		$+ 1.04^2 \times 0.028^2$ [=0.001082]	A1		
	Find P($Y < 0$):	$P(Y < 0) = \Phi(-E(Y) / \sqrt{Var(Y)})$	M1		
	(Condone incorrect sign here)	$= \Phi(-0.0057 / \sqrt{0.001082})$			
		or $\Phi(-0.0057 / 0.03289)$	A1		
	(0.569 earns 5/6)	$= \Phi(-0.1733) = 0.431$	A1	6	**[6]**
7	Use valid formula for C.I. with any t (or z) value:	$\bar{x} \pm t\sqrt{(s^2/n)}$ with n = 10 (*or* 9)	M1		
	Find mean of sample:	$\bar{x}$ = 28.54	A1		
	Find population variance consistent with n (to 2 dp):	s^2 = 49.85 *or* 7.06^2 (n = 10)			
		or 44.865 *or* 6.698^2 (n = 9)	A1		
	Use correct tabular t value:	$t_{9,\ 0.975}$ = 2.262 (to 3 sf)	*A1		
	Evaluate C.I. (dep *A1):	28.54 ± 5.05 *or* [23.5, 33.6]	A1	5	
	Use valid formula for width (or CI) with any t value:	$2\ t\sqrt{((448.65 / 9) / 20)}$	M1		
	Use correct tabular t value:	$t_{19,\ 0.975}$ = 2.093 (to 3 sf)	*A1		
	Evaluate width of C.I. (dep *A1):	6.61	A1	3	**[8]**

Qu No	Mark Scheme Details			Part Mark	Total
8 (i)	Relate $P(X > x)$ to number of faults:	Equivalent to 0 faults in x km	M1		
	State or use distribution N of faults in x km:	Poisson with mean $2.1x$	A1		
	Find $P(X > x)$:	$P(X > x) = P(N = 0) = e^{-2.1x}$ **A.G.**	A1	3	
(ii)	Find distribution function of X:	$F(x) = 1 - P(X > x) = 1 - e^{-2.1x}$	B1		
	Find probability density function of X:	$f(x) = F'(x) = 2.1\ e^{-2.1x}$	M1 A1	3	
(iii)	Find mean distance:	$1/2.1 = 10/21$ *or* 0.476 [km]	B1	1	
(iv)	Find median m from $F(m)$ *or* $\int_0^m f(x) = ½$:	$1 - e^{-2.1m} = ½$, $m = 0.330$ [km]	M1 A1	2	**[9]**
9 (i)	State hypotheses:	H_0: $\mu_B - \mu_A = 0$, H_1: $\mu_B - \mu_A > 0$	B1		
	State assumption (AEF):	Population of differences is normal	B1		
	Consider differences $B - A$ (*or* $A - B$):	–2 8 –8 11 29 15 38 23	M1		
	Estimate mean and population variance:	$\bar{x} = 114/8$ [= 14.25] *and*			
	(allow biased: 208.44 *or* 14.437^2)	$s^2 = (3292 - 8 \times 14.25^2) / 7$	M1		
		[= 238.21 *or* 15.434^2]			
	Calculate value of t:	$t = \bar{x} /(s/\sqrt{8}) = 2.61[1]$	*B1		
	Use correct tabular t value:	$t_{7,\ 0.9} = 1.41[5]$	*B1		
	Correct conclusion (AEF, dep both *B1):	There is a reduction	B1	7	
(ii)	Calculate t (M0 if inconsistent with s):	$t = (\bar{x} - 5) /(s/\sqrt{8}) = 1.69[5]$	M1*A1		
	Correct conclusion by comparing with				
	$t_{7,\ 0.9} = 1.41[5]$ (AEF, dep *A1):	There is a reduction > 5 units	B1	3	**[10]**

Qu No	Mark Scheme Details			Part Mark	Total
10	Find coefficient b in regression line for y:				
		$b = (37.338 - 5.22 \times 85.9/12) / (2.2774 - 5.22^2/12)$			
	SR: Penalise rounding error in b here only	$= -0.002375 / 0.00055833$			
		or $-0.0285 / 0.0067 = -4.25[4]$	B1		
	Find equation of regression line:	$y = b(x - 0.435) + 7.158$			
	(√ on rounding error in b)	$= 9.01 - 4.25\,x$	M1 A1√		
	SR: B1 for x on y:	$x = 0.499 - 0.008881\,y$			
	Estimate y when $x = 0.4$ (√ on rounding error in b):	$y = 7.31$	B1√	4	
	Find r: $r = (37.338 - 5.22 \times 85.9/12) / \sqrt{\{(2.2774 - 5.22^2/12)(618.11 - 85.9^2/12)\}}$		M1		
		$= -0.194 \pm 0.001$	*A1	2	
	State hypotheses:	H_0: $\rho = 0$, H_1: $\rho \neq 0$	B1		
	Reject H_0 if $\lvert r \rvert$ exceeds critical value:	$r_{12,\,2.5\%} = 0.576$	M1 *A1		
	Correct conclusion (AEF, dep both *A1):	Variables are not correlated	A1	4	**[10]**
11 EITHER	Reasonable attempt to differentiate θ twice:		M1		
		$\mathrm{d}\theta/\mathrm{d}t = 2k \sin kt \cos kt$			
		or $k \sin 2kt$	A1		
		$\mathrm{d}^2\theta/\mathrm{d}t^2 = 2k^2 (\cos^2 kt - \sin^2 kt)$			
		or $2k^2 \cos 2kt$	A1		
	Rearrange in the form $b + c\theta$:	$\mathrm{d}^2\theta/\mathrm{d}t^2 = 2k^2 (1 - 2 \sin^2 kt)$			
		$= 2k^2 - 4k^2\,\theta$	M1 A1		
	Show SHM by e.g. change of variable:	$\phi = \theta - ½$, $\mathrm{d}^2\phi/\mathrm{d}t^2 = -4k^2\,\phi$	M1 A1		
	State centre and amplitude:	Centre $\theta = ½$; amplitude ½	B1; B1	9	
	Find tangential force (AEF in terms of sin or cos):	$ma\,\mathrm{d}^2\theta/\mathrm{d}t^2 = 2mak^2 \cos 2kt$	B1		
	Find radial force (AEF in terms of sin or cos):	$ma\,(\mathrm{d}\theta/\mathrm{d}t)^2 = mak^2 \sin^2 2kt$	B1		
	Find magnitude of resultant force:	$mak^2 \sqrt{(4 \cos^2 2kt + \sin^4 2kt)}$	M1		
	Simplify:	$mak^2 \sqrt{(4 \cos^2 2kt + (1 - \cos^2 2kt)^2)}$			
	or	$mak^2 \sqrt{(4 - 4 \sin^2 2kt + \sin^4 2kt)}$	M1		
	Hence result:	$mak^2 (1 + \cos^2 2kt)$ **A.G.**	A1	5	**[14]**

Qu No	Mark Scheme Details			Part Mark	Total
OR	State distribution:	Geometric	B1		
	Find sample mean:	$\bar{x} = 1.84$	B1		
	Estimate p from sample mean:	$1 / \bar{x} = 0.5435$ **A.G.**	M1 A1	4	
	Calculate expected values (correct to ± 0.01):	54.35 24.81 11.33 5.17 2.36 1.98	*M1 A1		
	Combine values as appropriate:	Last 3 to give 9.51 (dep *M1)	M1√		
	Calculate value of χ^2 :	$\chi^2 \approx 2.27$ [± 0.01]	M1 *A1		
	Compare with consistent tab. value (to 2 dp):	$\chi_{2,\,0.975}{}^2 = 7.378$ [$\chi_4{}^2 = 11.14$]	*B1√		
	Consistent conclusion (A1 dep *A1, *B1):	Distribution fits data (A.E.F.)	M1√ A1	8	
	State 2 changes (A.E.F.):	Change estimated p to 0.5	B1		
		Increase degrees of freedom by 1	B1	2	**[14]**