AS

Further Mathematics

Discrete

Mark scheme

Specimen

Version 1.1

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
  the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
dM	mark is dependent on one or more M marks and is for method
R	mark is for reasoning
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
‘their’	Indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
sf	significant figure(s)
dp	decimal place(s)

Examiners should consistently apply the following general marking principles

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

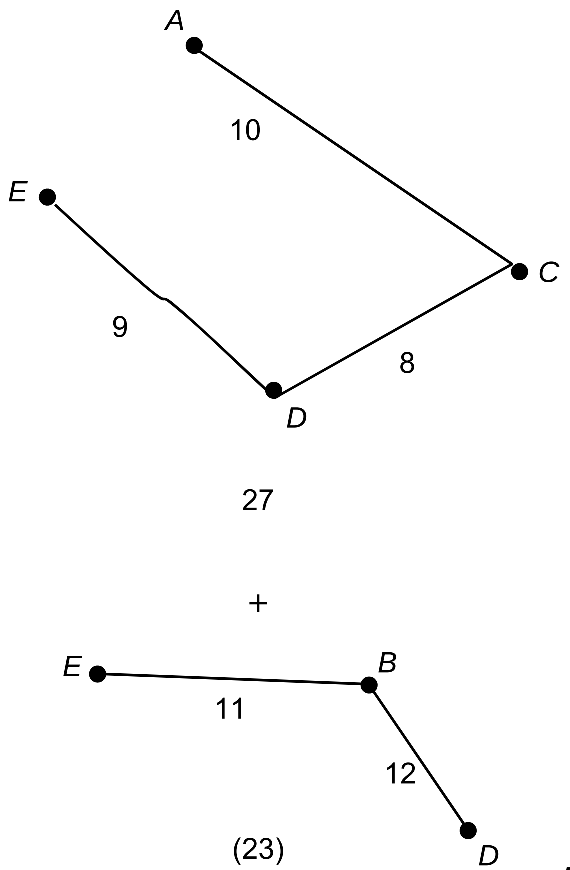
Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, only the last complete attempt should be awarded marks.

Q	Marking Instructions	AO	Mark	Typical Solution
1	Circles correct answer	AO1.1b	B1	12
	Total		**1**	
2	Circles correct answer	AO1.1b	B1	$x-2$
	Total		**1**	
3	Searches for and finds correct counter-example to the associativity condition	AO1.1b	M1	$(3\,\Delta\,1)\,\Delta\,2=-2$ $3\,\Delta\,(1\,\Delta\,2)=-4$
	Correctly argues that the binary operation is not associative, disproving Gary's claim	AO2.3	R1	$(3\,\Delta\,1)\,\Delta\,2\neq 3\,\Delta\,(1\,\Delta\,2)$ Therefore Δ is not an associative binary operation, which disproves Gary's claim.
	Total		**2**	

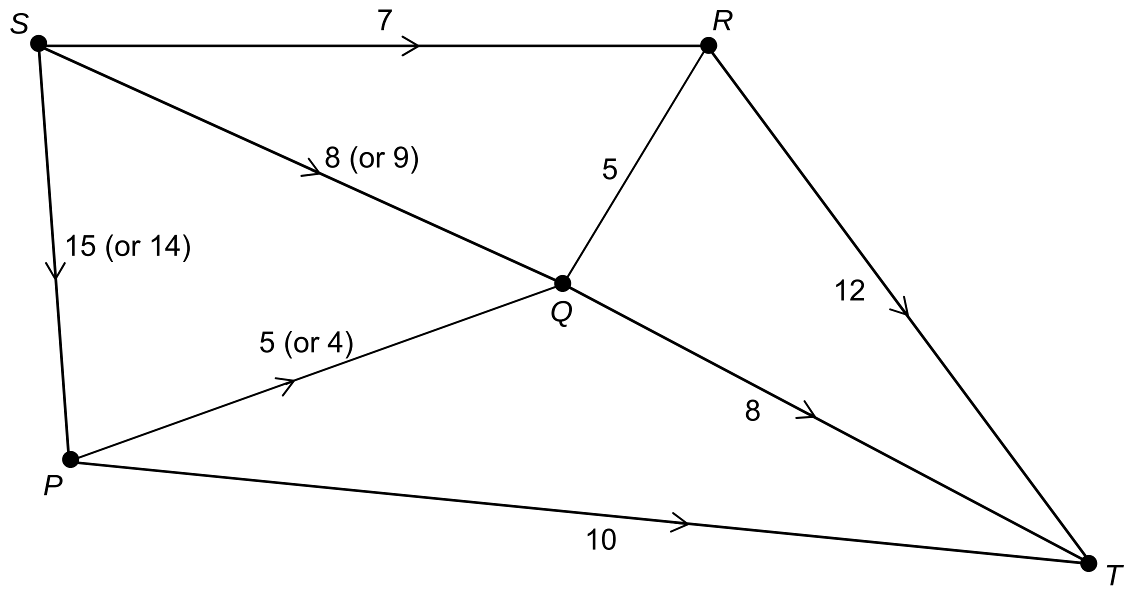
Q	Marking Instructions	AO	Mark	Typical Solution
4(a)	Gives a plausible reason, in the context of the question, as to what *** could mean, such as no cycle path exists.	AO2.4	E1	A cable cannot be laid between these two districts as there may be a river or housing estate in the way
(b)	Identifies the problem as a route inspection problem by noting that *B*, *G*, *H* and *O* are odd-degree nodes. (PI)	AO3.1b	M1	odd nodes: *B*, *G*, *H* and *O*
	Finds the shortest distance between each pair of odd nodes (at least four correct) OR determines correctly values for the *** entries in the table, with clear reasoning.	AO1.1b	M1	*B-G*: 2.5; *B-H*: 5.6; *B-O*: 4.3; *G-H*: 3.1; *G-O*: 6.8; *H-O*: 6.7. OR *B-H*: 5.6 (via *G*) and *G-O*: 6.8 (via *B*)
	Determines correctly the minimum total distance the engineer will cover during the journey. CAO	AO1.1a	A1	Minimum pair of shortest distances is *B-O* and *G-H*, with a minimum extra distance of 7.4 miles 37.2 + 7.4 = 44.6 miles
	Determines the minimum time in minutes that the engineer could complete the journey in by using ‘their’ minimum total distance and the average speed of 12 miles per hour.	AO3.2a	A1F	(44.6 / 12) × 60 = 223 minutes.
	Total		**5**	

Q	Marking Instructions	AO	Mark	Typical Solution
5(a)	Finds the correct Hamiltonian cycle starting at *A* using the nearest neighbour algorithm	AO1.1a	M1	*A* *C* *D* *E* *B* *A* (10 + 8 + 9 + 11 + 15)
	Correctly determines and states the upper bound	AO1.1b	A1	= 53
(b)	Correctly determines a minimum spanning tree connecting *A*, *D*, *E* and *C* (or 27 seen)	AO1.1b	B1	
	Correctly identifies the two shortest arcs connected to *B*	AO1.1a	M1	
	Calculates the sum of 'their' minimum spanning tree and the two shortest arcs	AO1.1b	A1	= 50

Q	Marking Instructions	AO	Mark	Typical Solution
(c)	Infers correctly that the optimal Hamiltonian cycle has a weight between 50 and 53 minutes, and that there is currently no further information to suggest a route of less than 53 minutes exists as the lower bound is not a Hamiltonian cycle.	AO2.2b	R1	The optimal time for a tour lies between 50 and 53 minutes, and the lower bound is not a Hamiltonian cycle, so we cannot be sure that a Hamiltonian cycle of less than 53 minutes exists.
(d)	Evaluates answers in parts **(a)** and **(b)** with reference to a plausible reason why Charlotte has taken longer than the typical tourist.	AO3.5a	E1	The model only applies to a typical tourist. Charlotte takes longer than the typical tourist because she walks more slowly than the typical tourist.
(e)	Gives a plausible reason for why the times taken to walk between the monuments would likely to be different during the evening.	AO3.5b	E1	It is likely there will be less traffic at night, meaning that tourists travel time between each monument is likely to be less than the times during the morning. For example, they would not have to wait as long to walk across roads. OR At night there may be an increase in the number of pedestrians and it will therefore take longer to walk between the monuments.
	Total		**8**	

Q	Marking Instructions	AO	Mark	Typical Solution
6(a)	Identifies correctly all three row minima or all three (two) column maxima.	AO1.1a	M1	Row minima: –1, –2, (–1) Column maxima: 4, 0, (1)
	Determines correctly max(row min) and min(col max)	AO1.1b	A1	max (row min) = –1 min (col max) = 0
	Makes correct statement for the play-safe strategies of each player.	AO3.2a	A1 CAO	Victoria plays *R* (or *P*); Albert plays *Y*
(b)	Identifies strategy and provides justification with reference to dominance.	AO2.4	E1	Albert should never play *Z* because strategy *Y* dominates strategy *Z*
(c)(i)	States correctly that strategy *R* dominates strategy *P* (PI)	AO1.1a	B1	Victoria should never play *P* because strategy *R* dominates strategy *P*
	Introduces and defines a probability variable.	AO3.3	M1	Let Victoria play strategy *Q* with probability p and strategy *R* with probability $1 - p$. If Albert plays:
	Finds correctly both expected gain expressions for Victoria.	AO1.1b	A1	*X*: expected gain for Victoria $= -2p + 4(1 - p) = 4 - 6p$ *Y*: expected gain for Victoria $= -(1 - p) = p - 1$
	Clearly states that, as the game has no stable solution, the optimal solution occurs when the two expected gain expressions are set equal to each other, and then solves correctly 'their' equation constructed using 'their' expected gains for Victoria.	AO2.2a	A1F	No stable solution, so optimal value of p occurs when: $4 - 6p = p - 1$, $p = 5/7$
	Interprets correctly the solution to the problem in the context, giving the optimal mixed strategy for Victoria.	AO3.4	E1	Victoria should play strategy *Q* with a probability of 5/7 and *R* with a probability of 2/7

Q	Marking Instructions	AO	Mark	Typical Solution
(c)(ii)	Substitutes 'their' value of p into one of 'their' expected gain expressions, resulting in a value of the game.	AO1.1b	B1F	$4-6\times(5/7) = -2/7$ or $(5/7)-1 = -2/7$
(c)(iii)	Recognises correctly that Victoria can improve on the value of the game if Albert does not play an optimal mixed strategy.	AO3.5b	E1	The expected pay-off for Victoria will only be –2/7 if Albert plays an optimal mixed strategy between *X* and *Y*
	Total		**11**	

Q	Marking Instructions	AO	Mark	Typical Solution
7(a)	Determines correctly the value of both cuts.	AO1.1b	B1	35 and 36
(b)	Deduces the maximum flow in 'their' network by use of the maximum flow-minimum cut theorem and explains their reasoning.	AO2.4	E1	Max flow = 30 as max. flow = min. cut and the value of the minimum cut is 30
				
(c)	Determines correctly the flow along the saturated arcs.	AO1.1b	B1	*SR* = 7, *QR* = 5, *RT* = 12, *QT* = 8, *PT* = 10
	Determines correctly a possible flow for each of *SP*, *SQ* and *PQ*	AO1.1b	B1	*SP* = 15 and *SQ* = 8 *and PQ* = 5 or *SP* = 14 and *SQ* = 9 *and PQ* = 4
	Total		**4**	

Q	Marking Instructions	AO	Mark	Typical Solution
8	Introduces two variables and defines at least one of them as 'number of'.	AO3.3	B1	x = number of pine tables y = number of oak tables
	Finds correctly an inequality for the problem by considering the costs (PI)	AO3.1b	B1	$30x + 70y \le 2100$ OE
	Finds correctly an inequality for the problem by considering the labour (PI)	AO1.1b	B1	$6x + 10y \le 360$ OE
	Finds correctly three inequalities for the problem by considering the wholesaler (PI)	AO1.1b	B1	$y \le x$, $y \ge 10$, $x \ge 0$
	Correctly plots the line $6x + 10y = 360$ OE through (0, 36) and (60, 0) **AND** the line $30x + 70y = 2100$ OE through (0, 30) and (70, 0)	AO1.1b	B1	*See diagram below
	Correctly and accurately plots 'their' $y = x$ **AND** 'their' $y = 10$	AO1.1b	B1	*See diagram below
	Identifies and labels 'their' feasible region.	AO2.2a	B1	*See diagram below
	Uses the optimal vertex of the feasible region and clearly states the solution to the problem in the context of the question.	AO3.2a	A1 CAO	Uses an objective line with a gradient of –8/15 35 Pine Tables and 15 Oak Tables (Profit = £2525)
	Total		**8**	
	TOTAL		**40**	

Q	Marking Instructions	AO	Mark	Typical Solution
8