

A-LEVEL MATHEMATICS

MS2B – Statistics 2B
Mark scheme

6360
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Version: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√ or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
-x EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q1	Solution	Marks	Total	Comments
	Throughout parts (a) to (d)			Unsupported correct answers score full marks
(a)	0.703	B1	1	AWRT
(b)	$e^{-2.3} + e^{-2.3} \times 2.3$ (= 0.100... + 0.231...) (= 0.331) $1 - (e^{-2.3} + e^{-2.3} \times 2.3) = 0.669$	M1 A1 A1	3	PI For either P(X=0) or P(X=1) For both P(X=0) and P(X=1) or sum AWRT 0.669
(c)	<u>Use</u> of Po(6) 0.9799 for top value - 0.8472 for bottom value Or $e^{-6} \left(\frac{6^9}{9!} + \frac{6^{10}}{10!} + \frac{6^{11}}{11!} \right)$ = 0.1327 = 0.133	M1 A1 A1	3	0.7440, 0.8472, 0.9161, 0.9799, 0.9912 or 0.9964 seen AFWW 0.9798 to 0.9800 AFWW 0.8470 to 0.8473 A1 for either of these values or this expression OE AWRT 0.133
(d)	<u>Use</u> of Po(0.8) $e^{-0.8} \times 0.8^2/2$ or 0.9526 – 0.8088 = 0.1438 = 0.144	M1 m1 A1	3	Correct formula or 0.9526 or 0.8088 seen AWRT 0.144
(e)	$(1 - 0.0111) \times (1 - e^{-2.3}) \times (1 - 0.3012)$ $0.9889 \times 0.8997 \times 0.6988 = 0.622$	B1 M1 A1	3	For any one of these seen or PI by the correct value seen (3 sf or better) For all three correct and multiplied PI AWRT
			13	

Q2	Solution	Marks	Total	Comments
	\bar{x} or $\mu = (32.93 + 30.47) \div 2 = \underline{31.7}$	B1		CAO \bar{x} or μ not necessary but do not ISW. If contradictory value for μ seen then B0.
	$t_9 = 2.262$	M1		AWRT 2.26
	$(32.93 - 30.47) = (2 \times 2.262 \times s) \div \sqrt{10}$	m1		OE single correct equation with only s or σ unknown
	$s = 1.72$ so unbiased estimate for σ^2 is 2.96	A1	4	AWFW 2.95 to 2.96 Final answer 1.72 earns M1 m1 A0
			4	

Q3	Solution	Marks	Total	Comments
(a)	0.35	B1	1	CAO or equivalent fraction or %
(b)	$P(< 3) = 0.45$ '0.35' \times '0.45' (= 0.1575) $\times 2 = 0.315$	B1 M1 A1	3	Their 0.35 and 0.45 CAO or equivalent fraction or %
(c)	$\text{Mean} = 1 \times 0.19 + 2 \times 0.26 + 3 \times 0.20 + 4 \times 0.13 + 5 \times 0.07 + 6 \times 0.15$ $= 0.19 + 0.52 + 0.60 + 0.52 + 0.35 + 0.90$ $= 3.08$ $\text{Variance} = 1^2 \times 0.19 + 2^2 \times 0.26 + 3^2 \times 0.20 + 4^2 \times 0.13 + 5^2 \times 0.07 + 6^2 \times 0.15 - 3.08^2$ $= 2.77(36)$	M1 A1 M1 A1	4	This working, or the next line, must be seen (at least 3 products) CAO. AG. PI AWFW 2.77 to 2.78
(d)	No probability of 0 books borrowed.. ..and no probability of more than 6 books. Wrong shape or probability increases at top end.	E2,1,0	2	Cannot borrow no books Cannot borrow more than 6 books E1 for each of these 3 distinct points up to a maximum of 2 marks
(e)(i)	$10 \times 3.08 = 30.8$ (pence)	B1	3	CAO 31, without 30.8 seen, scores B0
(ii)	'2.7736' $\times 100 = '277.36'$ then $\sqrt{'277.36'}$ $= 16.7$ (pence) or $\sqrt{'2.7736'} = '1.66\dots'$, '1.67' $\times 10$ $= 16.7$ (pence)	M1 A1 (M1) (A1)		For their variance $\times 100$ and $\sqrt{\quad}$ AWFW 16 to 17 For $\sqrt{\quad}$ their variance and $\times 10$ AWFW 16 to 17
	Where working is in £ in (i) or (ii) or both (i) £0.308 (ii) '2.7736' $\times 0.01 = 0.027736$ $\sqrt{'0.027736'}$ $= £0.167$	(B1) (M1) (A1)		Must show £ sign For their variance $\times 0.01$ and $\sqrt{\quad}$ AWFW 0.16 to 0.17. Must show £ sign
	Or $\sqrt{'2.7736'} = '1.66\dots'$, '1.67' $\times 0.1$ $= £0.167$	(M1) (A1)		For $\sqrt{\quad}$ their variance and $\times 0.1$ AWFW 0.16 to 0.17. Must show £ sign
				3
			13	

Q4	Solution	Marks	Total	Comments
(a)	10	B1	1	CAO
(b)	$0.07 \times 10 = 0.7$	B1	1	OE
(c)(i)	$E(X) = \frac{1}{20}$	B1		Decimal or fraction
(ii)	$E(X^2) = \int_0^{0.1} 10x^2 dx$ $= \left[\frac{10}{3} x^3 \right]_0^{0.1} = \frac{1}{300}$	M1 A1		Integration, correct limits, their k Or at least 3sf equivalent
(iii)	$\text{Var}(X) = E(X^2) - E(X)^2 = \frac{1}{300} - \left(\frac{1}{20}\right)^2$ $= \frac{1}{1200}$ So sd = $\sqrt{\frac{1}{1200}} = \frac{\sqrt{3}}{60} = 0.0289$	M1 A1		Correct expression – allow use of 0.003 or 0.0033 or better for M1 Equivalent surd or AFWW 0.0288 to 0.0289 Correct answer but not derived from (ii) scores M0 B1.
			5 7	

Q5	Solution	Marks	Total	Comments				
(a)	Expected values							
		<3	<12	None	Total			
	Petrol	12.0 4	36.98	165.98	215	M1		For attempt to find expected values (at least 2 correct) either before or after combining. Allow rounding to nearest integer or 1 d.p.
	Diesel	1.96	6.02	27.02	35			
	Total	14	43	193	250			
	Combine first two columns Observed					M1		For combining (at least 1 correct)
		Problems	None	Total				
	Petrol	45	170	215				
	Diesel	12	23	35		A1		Combined Observed CAO
	Total	57	193	250				
Expected					A1		Combined expected (both correct). Allow rounding to nearest integer or 1 d.p.	
	Problems	None	Total					
Petrol	49.02	165.98	215					
Diesel	7.98	27.02	35					
Total	57	193	250					
	$ (O - E) - 0.5 = 3.52$ $\frac{3.52^2}{49.02} + \frac{3.52^2}{7.98} + \frac{3.52^2}{165.98} + \frac{3.52^2}{27.02}$ 49.02 7.98 165.98 27.02 (0.2527 + 1.5526 + 0.0746 + 0.4585) = 2.338...	M1 A1 B1 B1		Attempt including Yates (at least 1 correct) AWRT 2.34 (Implies first 6 marks) PI by CV AWRT 2.71				
1 degree of freedom CV = 2.706		B1		Allow "problems independent of engine type" but not reverse or "they are independent" or simply "no association"				
H ₀ : No association between problems and engine type		A1 dep		Comparison or a diagram equivalent Dep on A1 for ts and B1 for CV				
2.34 < 2.706 (so accept H ₀)		E1 dep		OE Conclusion in context. Dep on previous A1				
There is no significant evidence of an association between mechanical problems and type of engine.								
(b) Dependent on a conclusion of no association . No evidence that diesels have more problems		E1		Even from incorrect working Use of result, however any additional suggestion that diesels may have more problems than E0 Or similar dep on previous E1				
Go with her preference or buy either.		E1 dep						
			11					
			2					
			13					

No Yates: Can score M1, M1, A1, A1, M0, A0, B1, B1, B1, A0, E0 = 7 out of 11 ($\chi^2 = 3.05$)

No combining: Can score M1, M0, A0, A0, M0, A0, B1, B1 (for 2 d.o.f. giving 4.605), B1, A0, E0 = 4 out of 11 ($\chi^2 = 3.92$)

Just 2 cells combined (1.96 and 6.02) can score M1 M1 and B1 for hypotheses

Q6	Solution	Marks	Total	Comments
(a)	$H_0: \mu = 18.2$ (or $\mu \geq 18.2$) $H_1: \mu < 18.2$ $\bar{x} = 1384.5 \div 78 = 17.75$ test stat = $\frac{'17.75' - 18.2}{(1.8 \div \sqrt{78})}$ = -2.208 $z_{\text{crit}} = \pm 2.0537$ $-2.208 < -2.0537$ or $2.208 > 2.0537$ in critical region, reject H_0 or accept H_1 There is significant evidence at the 2% level of significance to support Gerald's belief.	B1 B1 M1 A1 B1 A1 dep E1 dep	7	For both. μ or "population mean" CAO Condone 18.2 – '17.75' for M1 AWRT -2.21 Must be negative AWRT ± 2.05 Comparison stated or diagram Dep on previous A1 and B1 Context conclusion. Dep on A1 dep. Definitive conclusions (Eg Gerald is correct) score E0. If not referring to belief, must use "mean" or "average"
	Alternative Calculation of critical region boundary value or using confidence interval H_0 and H_1 as above $\bar{x} = 1384.5 \div 78 = 17.75$ $z_{\text{crit}} = \pm 2.0537$ $18.2 - 2.0537 \times \frac{1.8}{\sqrt{78}}$ $17.75 + 2.0537 \times \frac{1.8}{\sqrt{78}}$ = 17.78 = 18.17 $17.75 < 17.78$ $18.2 > 18.17$ so reject H_0 so 18.2 above CI Context conclusion as above	(B1) (B1) (B1) (M1) (A1) (A1dep) (E1dep)		CAO. AWRT ± 2.05 Must be subtracting/adding as appropriate AWRT Comparison stated or diagram. Dep on previous A1 Dep on A1 dep
(b) (i)	$\bar{x} = 16.7, s = 1.94(3)$ $t_6 = 2.447$ $16.7 \pm 2.447 \times '1.94' / \sqrt{7}$ = 14.9, 18.5 or 16.7 ± 1.8	B1 B1 M1 m1 A1		For both, CAO & AWRT 1.94 PI AFWF 2.44 to 2.45 Use of $\sqrt{7}$ Their 1.94 & rest of formula correct AWRT 14.9, 18.5. Correct answer seen, no working shown scores all 5 marks.
(ii)	18.2 (or the mean for mainland lizards) lies within this confidence interval so no evidence to support Gerald's belief.	A1FT E1 dep	7	FT provided both M marks earned and CI includes 18.2 Must be a clear statement of this. Dep on A1 FT
(c)	$\bar{x} = 19.73$ which is > 18.2 so cannot provide evidence to support Gerald's belief.	B1 E1 dep	2	Calculation and this comparison Accept $19.73 > \text{mean on mainland}$ Comment in context. Dep on B1
			16	

Q7	Solution	Marks	Total	Comments
(a)	Candidate's own grid used is acceptable.			Accept wobbly lines as straight if it seems candidate has no ruler
	Nothing drawn beyond 1 and 6 (or line at 0)	B1		Must be at least one line drawn somewhere on the diagram
	Differentiate to give $f(x) = \frac{1}{4}$ for $1 \leq x < 4$	M1		Allow this M1 if seen in (b) PI by correct line
	Straight line joining $(1, \frac{1}{4})$ to $(4, \frac{1}{4})$	A1		Candidate's choice of scale. BOD this horizontal line.
	$f(x) = \frac{1}{8}(6 - x)$ for $4 \leq x \leq 6$	M1		OE Allow this M1 if seen in (b) PI by correct line
	Straight line joining $(4, \frac{1}{4})$ to $(6, 0)$	A1	5	
(b)	$\int_1^4 \frac{1}{4} x dx$ and $\int_4^6 \frac{1}{8} x(6 - x) dx$	M1 A1		For both integrands correct Including correct limits for both
	$E(X) = \left[\frac{x^2}{8} \right]_1^4 + \left[\frac{3x^2}{8} - \frac{x^3}{24} \right]_4^6$	A1		Integrations done correctly and added
	$= 3\frac{1}{24}$ or $73/24$	A1	4	Or AWRT 3.04
			9	

