# A-level
# Mathematics

MS03 – Statistics 3
Mark scheme

6360
June 2016

Version 1.0: Final Mark Scheme

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers.  This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination.  The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way.  As preparation for standardisation each associate analyses a number of students' scripts.  Alternative answers not already covered by the mark scheme are discussed and legislated for.  If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper.  Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

**Key to mark scheme abbreviations**

| | |
|---|---|
| M | mark is for method |
| m or dM | mark is dependent on one or more M marks and is for method |
| A | mark is dependent on M or m marks and is for accuracy |
| B | mark is independent of M or m marks and is for method and accuracy |
| E | mark is for explanation |
| √or ft or F | follow through from previous incorrect result |
| CAO | correct answer only |
| CSO | correct solution only |
| AWFW | anything which falls within |
| AWRT | anything which rounds to |
| ACF | any correct form |
| AG | answer given |
| SC | special case |
| OE | or equivalent |
| A2,1 | 2 or 1 (or 0) accuracy marks |
| –*x* EE | deduct *x* marks for each error |
| NMS | no method shown |
| PI | possibly implied |
| SCA | substantially correct approach |
| c | candidate |
| sf | significant figure(s) |
| dp | decimal place(s) |

**No Method Shown**

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

## General Notes for MS03

**GN1** There is no allowance for misreads (MR) or miscopies (MC) unless specifically stated in a question

**GN2** In general, a correct answer (to accuracy required) without working scores full marks but an incorrect answer (or an answer not to required accuracy) scores no marks

**GN3** In general, a correct answer (to accuracy required) without units scores full marks

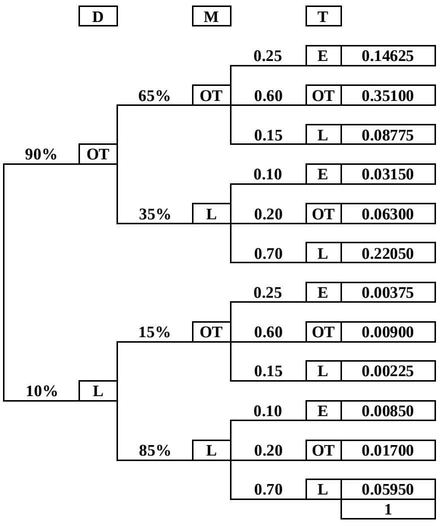
**GN4** When applying AWFW, a slightly inaccurate numerical answer that is subsequently rounded to fall within the accepted range cannot be awarded full marks

**GN5** Where percentage equivalent answers are permitted in a question, then penalise by **one accuracy mark** at the first **correct** answer but only if no indication of percentage (eg %) is shown

**GN6** In questions involving probabilities, do **not** award **accuracy** marks for answers given in the form of a ratio or odds such as 13/47 given as 13:47 or 13:34

**GN7** Accept decimal answers, providing that they have **at least two** leading zeros, in the form $c \times 10^{-n}$ (eg 0.00321 as $3.21 \times 10^{-3}$)

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| 1 (a) | $\hat{p}_M = \frac{264}{480} = \frac{11}{20}$ **or** <u>**0.55**</u><br>and $\hat{p}_W = \frac{220}{500} = \frac{11}{25}$ **or** <u>**0.44**</u> | B1 | | Both CAO $\left(\hat{p}_p = 0.49388\right)$ |
| | 95% $\Rightarrow$ $z$ = <u>**1.96**</u> | B1 | | AWRT (1.95996) |
| | CI for $p_M - p_W$ is | | | |
| | $(0.55 - 0.44) \pm 1.96\sqrt{\frac{0.55\times0.45}{480} + \frac{0.44\times0.56}{500}}$ | M1<br>M1<br>AF1 | | $\left(\hat{p}_M - \hat{p}_W\right) \pm (1.96 \text{ or } 1.64 \text{ to } 1.65)\sqrt{a}$<br>Expression for $\sqrt{a}$<br>F on $\hat{p}_M$ and $\hat{p}_W$ and $z$ |
| | ie <u>**0.11 ± 0.06**</u><br>or<br><u>**(0.05, 0.17)**</u> | A1 | 6 | CAO/AWRT (0.06224)<br>AWRT |
| **Note** | **1** A pooled estimate of variance (0.11 ± 0.06062) $\Rightarrow$ B1 B1 M1 M0 AF0 A1 (a maximum of 4 marks) | | | |
| **(b)** | CI > 0.025 or LCL > 0.025 | BF1 | | F on CI providing CI > 0.025 |
| | Evidence to **support the claim** | Bdep1 | 2 | Dep on BF1 |
| **Notes** | **1** There must be a reference to 0.025 (OE) **and** a clear comparison with the answer to (a)<br>**2** Accept answers suggesting that selections may not be random and/or independent **or** that based on 480 & 500 may not be representative **or** changes of opinions between opinion poll and referendum | | | |
| | | | | |
| | | **Total** | **8** | |

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| 2 (a) |  | M1<br><br>M1<br><br>M1 | 3 | Shape; $2 \times 2 \times 3 = 12$ branches<br><br>Labels; OT & L **and** E & OT & L<br><br>Attempt at percentages or probabilities for D **and** M **and** T |
| (b)(i) | $P(T_{OT}) = 0.351 + 0.063 + 0.009 + 0.017 = \underline{\mathbf{0.44}}$ | B1 | (1) | CAO |
| (ii) | $P(T_{OT} \mid D_{OT}) = \dfrac{0.351 + 0.063}{0.9} = \dfrac{0.414}{0.9}$<br>$= \underline{\mathbf{0.46}}$ | M1<br>A1 | (2) | Correct numerator; PI<br>CAO |
| (iii) | $P(T_{E\text{ or }OT} \mid D_{OT}) = 0.46 + \dfrac{0.14625 + 0.0315}{0.9} =$<br>$0.46 + \dfrac{0.17775}{0.9} = 0.46 + \underline{\mathbf{0.197 \text{ to } 0.20}}$<br>$= \underline{\mathbf{0.657 \text{ to } 0.66}}$ | M1<br>A1<br>A1 | (3) | (ii) + $p$<br>AWFW; PI (0.1975)<br>AWFW (0.6575) |
| (iv) | $P(T_{E\text{ or }OT} \mid M_{OT}) =$<br>$\dfrac{0.14625 + 0.351 + 0.00375 + 0.009}{0.9 \times 0.65 + 0.1 \times 0.15} = \dfrac{0.51}{0.6}$<br>$= \underline{\mathbf{0.85}}$ | M1<br>A1 | (2) | Correct numerator; PI<br>CAO |
| SCs | **1** $0.25 + 0.60 = 0.85 \Rightarrow$ B2 **2** $1 - 0.15 = 0.85 \Rightarrow$ B2 | | | |
| | | | 8 | |
| (c) | $P(T_{OT} \mid D_{OT}) = 0.46$<br>$P(T_E \mid D_{OT}) = 0.6575 - 0.46 = \underline{\mathbf{0.197 \text{ to } 0.20}}$<br>$P(T_{OT} \cap T_{OT} \cap T_E) = 0.46^2 \times 0.1975$<br>$\times 3$<br>$= \underline{\mathbf{0.125 \text{ to } 0.126}}$ | B1<br>M1<br>m1<br>A1 | 4 | AWFW; PI (0.1975)<br>$p_1^2 \times p_2$<br>CAO<br>AWFW (0.12537) |
| | | **Total** | **15** | |

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **3** | $H_0$: $\lambda_B = \lambda_A$<br>$H_1$: $\lambda_B > \lambda_A$ | B1 | | Both |
| | CV(1%) $\Rightarrow$ $z$ = **<u>2.32 to 2.33</u>** | B1 | | AWFW (2.3263) |
| | $\hat{\lambda}_A = \frac{315}{30}$ = **<u>10.5</u>** and $\hat{\lambda}_B = \frac{747}{60}$ = **<u>12.45</u>** | B1 | | Both CAO $\hat{\lambda} = \frac{1062}{90}$ = **11.8** |
| | $z = \frac{12.45 - 10.5}{\sqrt{\frac{12.45}{60} + \frac{10.5}{30}}}$ = **<u>2.61</u>** | M1<br>M1<br>Adep1 | | Correct numerator<br>Correct denominator<br>AWRT; dep on M1 M1 (2.61163) |
| | **or** | | | |
| | $z = \frac{12.45 - 10.5}{\sqrt{11.8\left(\frac{1}{60} + \frac{1}{30}\right)}}$ = **<u>2.54</u>** | (M1)<br>(M1)<br>(A1) | | Correct numerator<br>Correct denominator<br>AWRT; dep on M1 M1 (2.53868) |
| | Thus evidence, at 1% level, to **support the claim** that $\lambda_B > \lambda_A$ | Adep1 | **7** | Dep on $z$-value and CV |
| | | | | |
| | | **Total** | **7** | |

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| 4(a) (i) | $R$: mean = **35** variance = **125** | B1 | (1) | Both CAO |
| (ii) | $F$: mean = **115** | B1 | | CAO |
| | variance = $15^2 + 20^2 + (2 \times 15 \times 20 \times 0.25)$ | M1 | | Attempt at $a^2 + b^2 \pm (2) \times a \times b \times 0.25$ |
| | = **775** | A1 | (3) | CAO |
| (iii) | $T$: mean = **150**<br>variance = **900** | B1<br>A1 | (2) | CAO<br>CAO |
| (iv) | $D$: mean = **35** | B1 | | CAO |
| | variance = $20^2 + 15^2 - (2 \times 20 \times 15 \times 0.25)$<br>**or**<br>= (ii) $- 4 \times 15 \times 20 \times 0.25$ | (M1) | | Only if M1 not scored in (ii) |
| | = **475** | B1 | (2) | CAO |
| | | | 8 | |
| (b) (i) | $P(T < 180) = P\left(Z < \dfrac{180-150}{\sqrt{900}}\right)$ | M1 | | Standardising 180 with values from (a)(iii) but must involve $\sqrt{\ }$ |
| | $= P(Z < 1)$ = **0.841** | A1 | (2) | AWRT (0.84134) |
| (ii) | $P(W - V > 60)$ =<br>$P(D > 60) = P\left(Z > \dfrac{60-35}{\sqrt{475}}\right)$ | M1 | | Standardising 60 with values from (a)(iv) but must involve $\sqrt{\ }$ |
| | $= P(Z > 1.147) = 1 - P(Z < 1.147)$ | M1 | | Area change; can be implied by **any final answer < 0.5** |
| | $= 1 - (0.873$ to $0.875)$ = **0.125 to 0.127** | A1 | (3) | AWFW (0.12567) |
| | | | 5 | |
| | | | | |
| | | **Total** | **13** | |

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| 5 (a) | $\bar{D}$ has a **normal** distribution with | B1 | | Normal |
| | mean = **0** | B1 | | CAO |
| | and variance $= \frac{\sigma^2}{n} + 1.5^2 \times \frac{\sigma^2}{n}$ | M1 | | Must have (+ sign) & (1.5 or $1.5^2$) but allow no ($\div n$) |
| | $= \mathbf{\frac{3.25\sigma^2}{n}}$ | A1 | 4 | OE single expression |
| (b) | $H_0$: $\mu_{XL} = 1.5\mu_L$<br>$H_1$: $\mu_{XL} \neq 1.5\mu_L$ | B1 | | B1 both; allow any valid notation |
| | 5% $\Rightarrow$ $z =$ **(±)1.96** | B1 | | AWRT (1.95996) |
| | $z = \frac{\lvert 2261 - 1.5\times1509\rvert}{\sqrt{\frac{3.25\times4.5^2}{50}}} = \frac{\pm2.5}{\sqrt{1.31625}}$ | M1 | | Numerator; allow (2261 – 1509) |
| | | M1 | | Denominator; allow $\sqrt{2\times4.5^2/50}$ OE |
| | = **(±)2.18** | A1 | | AWRT (2.17907) |
| | Evidence, at 5% level, that **claim is not supported** | Adep1 | 6 | Dep on $z$-value and CV<br>Must have consistent signs |
| | | | | |
| | | **Total** | **10** | |

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| 6 (a) | $\mathrm{E}(X)=\sum_{x=0}^{\infty} x\frac{e^{-\lambda}\lambda^{x}}{x!}$ = | M1 | | Used; ignore limits until A1 |
| | $\lambda\sum_{x=1}^{\infty}\frac{e^{-\lambda}\lambda^{x-1}}{(x-1)!}$ = | M1 | | Factor of $\lambda$ plus $x!$ to $(x-1)!$ |
| | with $y = x - 1$ $\lambda\sum_{y=0}^{\infty}\frac{e^{-\lambda}\lambda^{y}}{y!} = \lambda\times 1 = \lambda$ | A1 | (3) | Fully complete and correct derivation AG |
| | $\mathrm{E}(X(X-1))=\sum_{x=0}^{\infty} x(x-1)\frac{e^{-\lambda}\lambda^{x}}{x!}$ = | M1 | | Used; ignore limits until A1 |
| | $\lambda^2\sum_{x=2}^{\infty}\frac{e^{-\lambda}\lambda^{x-2}}{(x-2)!} = \lambda^2$ | A1 | (2) | Factor of $\lambda^2$ plus $x!$ to $(x-2)!$ and fully complete and correct derivation |
| | $\mathrm{Var}(X)=\mathrm{E}(X^2)-(\mathrm{E}(X))^2$ = | M1 | | Used |
| | $\mathrm{E}(X(X-1))+\lambda-\lambda^2=\lambda$ | A1 | (2) | Fully complete and correct derivation |
| Note | 1 Other derivations are possible throughout (a) | | | |
| | | | 7 | |
| (b)(i) | **Po(0.75)** | B1 | | PI |
| | P(0 faults) = $e^{-0.75}$ = **0.472** | B1 | 2 | AWRT (0.47237) |
| (ii) | Po(37.5) ⇒ **N(37.5, 37.5)** | B1 | | Normal with mean = variance = 37.5 in (A) or (B) |
| (A) | P(F < 30) = $\mathrm{P}\left(Z<\frac{29.5-37.5}{\sqrt{37.5}}\right)$ | M1 | | Standardising (29.5 or 30 or 30.5) with C's mean = variance |
| | = P(Z < –1.30639) = 1 – P(Z < 1.30639) | m1 | | Area change; can be implied by **any final answer < 0.5** |
| | = **0.095 to 0.097** | A1 | (4) | AWFW (0.09571) |
| (B) | P(35 ≤ F ≤ 45) = P(F ≤ 45.5 or 45) – P(F ≤ 34.5 or 35) = | M1 | | Area difference |
| | P(Z < **1.31**) – P(Z < –**0.49**) | A1 | | Both AWRT (1.30639 & 0.48990) |
| | = **0.591 to 0.597** | A1 | (3) | AWFW (0.59219) |
| SC | 1 Use of Poisson: **(A)** 0.092 (AWRT) ⇒ B2 **(B)** 0.582 (AWRT) ⇒ B1 (max of 3 marks) | | | |
| | | | 7 | |
| | | | | |
| | **Total for (a) & (b)** | | 16 | |

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **6** | **Total for (a) & (b)** | | **16** | |
| **(c)** | 98% $\Rightarrow$ $z =$ **2.32 to 2.33** | B1 | | AWFW (2.3263) |
| | CI:<br>$\begin{pmatrix} 49 \\ 4.9 \\ 0.98 \\ 0.098 \end{pmatrix} \pm \begin{pmatrix} 2.32 \text{ to } 2.33 \\ 2.05 \text{ to } 2.06 \end{pmatrix} \begin{pmatrix} \sqrt{49} = 7 \\ \sqrt{4.9/10} = 0.7 \\ \sqrt{0.98/50} = 0.14 \\ \sqrt{0.098/500} = 0.014 \end{pmatrix}$ | M1<br>A1<br>A1 | | $\lambda \pm z\sqrt{a}$<br>Any correct value for $\lambda$<br>Correct expression for $a$ given $\lambda$ |
| | **or**<br>49 ± (16.2 to 16.4) = (32.6 to 32.8, 65.2 to 65.4)<br>4.9 ± (1.62 to 1.64) = (3.26 to 3.28, 6.52 to 6.54)<br>0.98 ± (0.32 to 0.34) = (0.64 to 0.66, 1.30 to 1.32)<br>0.098 ± (0.032 to 0.034) = (0.064 to 0.066, 0.130 to 0.132) | | | |
| | Dividing by 500, 50, 10 or 1 as appropriate | B1 | | CAO |
| | ie **0.098 ± (0.032 to 0.034)**<br>**or**<br>**(0.064 to 0.066, 0.130 to 0.132)** | A1 | 6 | CAO ± AWFW (0.03257)<br>AWFW |
| | | | | |
| | | **Total** | **22** | |