A-LEVEL
Mathematics

MFP3 – Further Pure 3
Mark scheme

6360
June 2016

Version: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q1	Solution	Mark	Total	Comment
(a)	$2a-5(ax+b) \quad (=10x)$	B1		$2a-5(ax+b)$ OE PI by two correct equations or correct values for *a* and *b*.
	$-5a=10$, $2a-5b=0$	M1		Equating coefficients to form two equations, at least one correct. PI by next line.
	$a=-2$, $b=-\frac{4}{5}$	A1	3	Correct values for both *a* and *b*.
(b)	Aux eqn $2m-5=0$	M1		PI Or solving $2y'(x)-5y=0$ as far as $y=Ae^{\pm 2.5x}$ OE.
	$(y_{CF}=)\ Ae^{2.5x}$	A1		OE
	$(y_{GS}=)\ Ae^{2.5x}-2x-0.8$	B1F	3	$(y_{GS}=)$ c's CF $-2x-0.8$, must have exactly one arbitrary constant; ft c's non zero values for *a* and *b* from part (a).
	Total		6	

Q2	Solution	Mark	Total	Comment
(a)	$\sin 2x = 2x - \frac{(2x)^3}{3!} + \frac{(2x)^5}{5!} \ldots\ldots$ $= 2x - \frac{4}{3}x^3 + \frac{4}{15}x^5 \ldots$	B1	1	Correct expansion. ISW in higher powers.
(b)	$(1-x^2)^{-1} = 1+x^2+x^4 \ldots..$	B1		Correct simplified expansion seen or <u>**used**</u>. ISW in higher powers.
	$\sin 2x - 2x(1-px^2)(1-x^2)^{-1} =$ $2x - \frac{4}{3}x^3 + \frac{4}{15}x^5 \ldots$ $-(2x-2px^3)(1+x^2+x^4 \ldots)$	M1		Series expansions for sin2*x* and $(1-x^2)^{-1}$ attempted and used in the given function
	$=(-\frac{4}{3}-2+2p)x^3 + (\frac{4}{15}-2+2p)x^5 \ldots$			
	$(-\frac{4}{3}-2+2p)=0$; $(\frac{4}{15}-2+2p)=q$	m1		c's coefficient of x^3 equated to 0 <u>**and**</u> c's coeff of x^5 equated to *q* (PI) and an attempt to solve as far as reaching a value for *p* and a value for *q*.
	$p=\frac{5}{3}$; $q=\frac{8}{5}$	A1	4	ACF of both exact values.
	Total		5	

Q3	Solution	Mark	Total	Comment
(a)	**DO NOT ALLOW ANY MISREADS IN THIS QUESTION 3(a)**			
	$k_1 = 0.1[(1)\ln(2)] = 0.1\ln 2$ (=0.069…)	M1		$k_1 = 0.1[(2(0)+1)\ln(0+2)]$ OE seen or used
	$k_2 = 0.1\times \text{f}(0.1,\ 2+k_1)$ $= 0.1(0.2+1)\ln(0.1+2.0693...)$	M1		$k_2 = 0.1(0.2+1)\ln(0.1+2+\text{c's } k_1)$ OE seen or used
	$= 0.12\ln(2.1693…)$ $= 0.092(9293$	A1		AWFW 0.092 to 0.093 inclusive. PI by final answer 2.081 or 2.0811….
	$y(0.1) = 2+\frac{1}{2}(0.069(3..)+0.092(9..))$	m1		$2+\frac{1}{2}(\text{c's } k_1 + \text{c's } k_2)$ but dep on previous two Ms scored. PI by 2.081 or 2.0811….
	= 2.081 (to 3dp)	A1	5	CAO Must be 2.081
(b)(i)	$\ldots = 2\ln(x+y)+(2x+1)\times\frac{1+\frac{dy}{dx}}{x+y}$	B1		$\frac{d}{dx}(\ln(x+y)) = \frac{1+\frac{dy}{dx}}{x+y}$ seen or used.
		M1		Product rule used
	$2\ln(x+y)+$ $(2x+1)\times\frac{1+(2x+1)\ln(x+y)}{x+y}$	A1	3	ACF for $\frac{d^2y}{dx^2}$ in terms of x and y only.
(ii)	$y(0)=2$, $y'(0)=\ln 2$,	B1		$y(0)=2$ and $y'(0)=\ln 2$ seen or used.
	$y''(0) = 2\ln 2+\frac{1+\ln 2}{2}$	B1F		Seen or used, ft on c's $\frac{d^2y}{dx^2}$, an exact value at some stage which may be left unsimplified.
	$(y(x)=)\ 2+x\ln 2+x^2\left(\frac{1+5\ln 2}{4}\right)\ldots$	B1F	3	$2+x(\text{c's } y'(0))+\frac{x^2}{2}(\text{c's } y''(0))$ values must be exact but may be unsimplified
(iii)	$y(0.1) = 2+0.0693..+0.01116..$ = 2.080(479..) = 2.080 to 3dp	B1	1	Must be 2.080 and dep on values in (b)(ii) being correctly found
	Total		**12**	

Q4	Solution	Mark	Total	Comment
(a)	$r=\dfrac{10}{3-2\cos\theta}$; $3r-2r\cos\theta=10$			
	$3r-2x=10$	M1		$r\cos\theta=x$ **used** at any stage.
	$3r=2x+10$	A1		$3r=2x+10$ or a more suitable correct form to allow elimination of r; or a correct Cartesian equation in ACF
	$9r^2=(2x+10)^2$ $9(x^2+y^2)=(2x+10)^2$	M1		$r^2=x^2+y^2$ **used** to form a Cartesian equation.
	$5x^2+9y^2-40x=100$ $5(x^2-8x+16)+9y^2=180$ $5(x-4)^2+9y^2=180$	m1		Completing the square $5x^2+9y^2\pm40x$ = $5[(x\pm4)^2-4^2]+9y^2$ OE; this m1 cannot be awarded retrospectively
	$\dfrac{(x-4)^2}{36}+\dfrac{y^2}{20}=1$			
	$c=36$, $d=20$	A1	5	CSO; $c=36$, $d=20$ stated
(b)	Translation $\begin{bmatrix}4\\0\end{bmatrix}$	B1F	1	ft is only applied if m1 scored in **(a)**. Must be 'translat…' and vector form. **B0F** if more than one transformation
	Total		**6**	
	(a) $3\sqrt{x^2+y^2}-2x=10$ scores the first 3 marks			

Q5	Solution	Mark	Total	Comment
(a)	$\frac{1}{1+x}-\frac{1}{2+x}$	B1	1	Condone $A=1$, $B=-1$
(b)	$\frac{\mathrm{d}u}{\mathrm{d}x}+\frac{1}{(1+x)(2+x)}u=\ldots$	M1		Replacing $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ by $\frac{\mathrm{d}u}{\mathrm{d}x}$ and $\frac{\mathrm{d}y}{\mathrm{d}x}$ by u
	I.F. $\mathrm{e}^{\int\frac{1}{(1+x)(2+x)}(\mathrm{d}x)}$	M1		$\mathrm{e}^{\int\frac{1}{(1+x)(2+x)}(\mathrm{d}x)}$ OE PI
	$=\mathrm{e}^{\ln(1+x)-\ln(2+x)}$	A1F		$\mathrm{e}^{A\ln(1+x)+B\ln(2+x)}$ OE ft c's non-zero A and B values.
	$=\frac{1+x}{2+x}$	A1		A correct IF in form $\frac{\lambda(1+x)}{2+x}$
	$\frac{1+x}{2+x}\frac{\mathrm{d}u}{\mathrm{d}x}+\frac{u}{(2+x)^2}=1$ $\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{(1+x)u}{2+x}\right]=1$	m1		Dep on both previous M1 s LHS as $\frac{\mathrm{d}}{\mathrm{d}x}[u\times\text{candidate's IF}]$ PI
	$\frac{1+x}{2+x}u=x\quad(+c)$	A1		
	$x=0,\ u=4\Rightarrow c=2$ so $\frac{1+x}{2+x}u=x+2$	A1		$\frac{1+x}{2+x}u=x+2$ OE
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(2+x)^2}{1+x}$	m1		Dep on previous MMm, replacing u to form $\frac{\mathrm{d}y}{\mathrm{d}x}=\mathrm{g}(x)$ seen or used (c could still be present)
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{x(1+x)+3(1+x)+1}{1+x}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=x+3+\frac{1}{1+x}$	m1		$\frac{(2+x)^2}{1+x}$ in form $x+p+\frac{q}{1+x}$ or other valid method to integrate $(2+x)^2/(1+x)$ or to integrate $\frac{(2+x)(x+c)}{1+x}$, where c is a general const of integration or a non-zero value other than 1
	$y=\frac{1}{2}x^2+3x+\ln(1+x)\quad(+d)$	A1		ACF
	$x=0,\ y=1\Rightarrow d=1$ $y=\frac{1}{2}x^2+3x+\ln(1+x)+1$	A1	11	ACF eg $y=\ln(1+x)+2(1+x)+\frac{(1+x)^2}{2}-\frac{3}{2}$
	Total		**12**	

Q6	Solution	Mark	Total	Comment
(a)	$\lim_{p\to\infty}\frac{\ln p}{p^k} = \lim_{a\to 0^{(+)}}\frac{\ln\left(\frac{1}{a}\right)}{\left(\frac{1}{a}\right)^k}$	M1		Changing $\lim_{p\to\infty}$ to $\lim_{a\to 0^{(+)}}$
	$= \lim_{a\to 0^{(+)}} -a^k \ln a$	M1		Changing $\frac{\ln p}{p^k}$ to $-a^k \ln a$
	$= 0$	A1	3	
(b)	$\int \frac{\ln x}{x^7}\,dx$			
	$u = \ln x,\quad \frac{dv}{dx} = x^{-7},\quad \frac{du}{dx} = \frac{1}{x},\quad v = \frac{x^{-6}}{-6}$	M1		$\frac{du}{dx} = \frac{1}{x},\quad v = \frac{x^{-6}}{k}$ with $k = -6$ or 6
	$\int \frac{\ln x}{x^7}\,dx = -\frac{1}{6}x^{-6}\ln x + \frac{1}{6}\int x^{-6}\frac{1}{x}\,dx$			
	$= -\frac{1}{6}x^{-6}\ln x - \frac{1}{36}x^{-6}\quad (+c)$	A1		ACF
	$\int_1^\infty \frac{\ln x}{x^7}\,dx = \lim_{p\to\infty}\int_1^p \frac{\ln x}{x^7}\,dx$			
	$= \lim_{p\to\infty}\left(-\frac{1}{6}\frac{\ln p}{p^6} - \frac{1}{36p^6}\right) - \left(0 - \frac{1}{36}\right)$	M1		$\lim_{p\to\infty}\left[F(p) - F(1)\right]$ OE provided consistent use of 'same' letter and F follows from an attempt at integration by parts
	$= \lim_{p\to\infty}\left(-\frac{1}{6}\frac{\ln p}{p^6}\right) - \lim_{p\to\infty}\left(\frac{1}{36p^6}\right) - \left(0 - \frac{1}{36}\right)$			
	$\int_1^\infty \frac{\ln x}{x^7}\,dx = 0 - 0 - 0 + \frac{1}{36} = \frac{1}{36}$	A1	4	$\frac{1}{36}$ together with either the split into two relevant limit expressions (eg see previous line in solutions) with indication of 0 evaluations or reference to **(a)** with general k or k=6 or an explicit statement $\lim_{p\to\infty}\frac{\ln p}{p^k}$ for $k > 0$ $= 0$ OE with general k or k=6
	Total		7	

Q7	Solution	Mark	Total	Comment
	Aux eqn $m^2+4=0$	**M1**		PI by correct values of '*m*' seen/used.
	$(y_{CF}=)\ A\sin 2x+B\cos 2x$	**A1**		$A\sin 2x+B\cos 2x$ OE
	Try $(y_{PI}=)\ ae^{4x}$	**M1**		
	$+bx\sin 2x+cx\cos 2x$	**M1**		
	$(y''_{PI}=)$ $16ae^{4x}+(4b-4cx)\cos 2x-(4c+4bx)\sin 2x$	**A1**		Correct $(y''_{PI}=)$
	$16ae^{4x}+(4b-4cx)\cos 2x-(4c+4bx)\sin 2x$ $+\ 4(ae^{4x}+bx\sin 2x+cx\cos 2x)=$ $=10e^{4x}+8\sin 2x+4\cos 2x$	**m1**		Substitution into $y''+4y$, seen or used, dep on 2nd and 3rd **M** and at least one set of differentiations being in the form $ke^{4x}+(p+qx)\cos 2x+(r+sx)\sin 2x$ for non-zero constants *k*, *p*, *q*, *r* and *s*.
	$20a=10\ \Rightarrow a=0.5$ $-4c=8\ \Rightarrow c=-2$; $x\sin 2x$ terms cancel $4b=4\ \Rightarrow b=1$; $x\cos 2x$ terms cancel			
	$(y_{PI}=)\ 0.5e^{4x}$	**B1**		$0.5e^{4x}$ term in *PI*
	$+x\sin 2x-2x\cos 2x$	**B1**		$+x\sin 2x-2x\cos 2x$ term in *PI* with correct $x\sin 2x$ and $x\cos 2x$ terms in **m1** line
	$(y_{GS}=)\ A\sin 2x+B\cos 2x+$ $+0.5e^{4x}+x\sin 2x-2x\cos 2x$ $y=2.5,\ x=0;\ B+0.5=2.5\Rightarrow B=2$			
	$y=\frac{\pi}{4}, x=\frac{\pi}{4}; A+\frac{e^{\pi}}{2}+\frac{\pi}{4}=\frac{\pi}{4};\ A=-\frac{e^{\pi}}{2}$	**A1F**		Correct ft value of either *A* or *B*, the coeffs of sin2*x* and cos2*x* respectively, ft only on wrong non-zero coefficients in the GS; m1 must have been scored.
	$y=2(1-x)\cos 2x+x\sin 2x-\frac{e^{\pi}}{2}\sin 2x+\frac{e^{4x}}{2}$	**A1**	**10**	ACF. Must be a correct **eqn**. and ALL previous 9 marks must be scored
	Total		**10**	

Q8	Solution	Mark	Total	Comment
(a)	(OA=) $1+\tan\frac{\pi}{3}$	M1		$1+\tan\frac{\pi}{3}$ PI by $1+\sqrt{3}$ as final answer
	$OA = 1+\sqrt{3}$	A1	2	Or $1+\sqrt{3}$ as final answer
(b)(i)	Polar eqn of AN: $r\cos\theta = OA\cos\frac{\pi}{3}$	M1		Or $ON = OA\cos\frac{\pi}{3}$ OE
	$r\cos\theta = \frac{1+\sqrt{3}}{2}$	A1F		Ft from **(a).** OE $ON = \frac{1+\sqrt{3}}{2}$
	(At B): $(1+\tan\alpha)\cos\alpha = \frac{1+\sqrt{3}}{2}$	m1		$(1+\tan\alpha)\cos\alpha = \frac{1}{2}$ c's OA. OE
	$(\cos\alpha+\sin\alpha)^2 = \left(\frac{1+\sqrt{3}}{2}\right)^2 = 1+\frac{\sqrt{3}}{2}$	A1	4	AG Printed result convincingly obtained.
(ii)	$1+\sin 2\alpha = 1+\frac{\sqrt{3}}{2} \Rightarrow \sin 2\alpha = \frac{\sqrt{3}}{2}$	M1		$\sin 2\alpha = \frac{\sqrt{3}}{2}$ OE eg $\sin\left(\alpha+\frac{\pi}{4}\right) = \frac{1+\sqrt{3}}{2\sqrt{2}}$
	(Since $0<\alpha(=\theta_B)<\theta_A\ (=\frac{\pi}{3})$)			
	so $\alpha = \frac{\pi}{6}$	A1	2	Condone $\theta_B = \frac{\pi}{6}$ NMS scores 0/2
(c)	(Area of triangle OAB =) $\frac{1}{2}OA(1+\tan\theta_B)\sin(\theta_A-\theta_B)$	M1		Valid method as far as a correct expression in terms of known lengths/angles OE eg $\frac{1}{2}\left(OA\cos\frac{\pi}{3}\right)^2\left(\tan\frac{\pi}{3}-\tan\alpha\right)$ eg $\frac{OA}{4}\left[OA\sin\frac{\pi}{3}-(1+\tan\alpha)\sin\alpha\right]$
	$= \frac{1}{2}(1+\sqrt{3})\left(1+\frac{1}{\sqrt{3}}\right)\sin\frac{\pi}{6} = \frac{3+2\sqrt{3}}{6}$	A1	2	AG

(d)	Area = $\frac{1}{2}\int_{(\theta_B)}^{(\frac{\pi}{3})}(1+\tan\theta)^2\ (\mathrm{d}\theta)$	**M1**		Use of $\frac{1}{2}\int r^2\ (\mathrm{d}\theta)$
	$= \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\left(1+2\tan\theta+\tan^2\theta\right)\ (\mathrm{d}\theta)$	**B1F**		Correct expn of $[1+\tan\theta]^2$ and limits $\frac{\pi}{3}$ and c's α, in terms of π such that $0<\alpha<\frac{\pi}{3}$ used with $k\int r^2\ (\mathrm{d}\theta)$
	$\int\left(\sec^2\theta+2\tan\theta\right)(\mathrm{d}\theta)$	**M1**		$1\pm\tan^2\theta=\pm\sec^2\theta$ with $k\int r^2\ (\mathrm{d}\theta)$
	$=\left[\tan\theta+2\ln\sec\theta\right]$	**A1**		Correct integration of $\sec^2\theta+2\tan\theta$ following use of correct identity
	Shaded area= $\frac{1}{2}\int_{\theta_B}^{\frac{\pi}{3}}(1+\tan\theta)^2\ (\mathrm{d}\theta)-\frac{3+2\sqrt{3}}{6}$	**M1**		Condone difference taken in wrong order for this M mark
	$=\frac{1}{2}\left[\left(\sqrt{3}+2\ln 2\right)-\left(\frac{1}{\sqrt{3}}+2\ln\frac{2}{\sqrt{3}}\right)\right]\ldots$ $-\frac{3+2\sqrt{3}}{6}$	**A1**		Any correct numerical expression for the shaded area, can be unsimplified but must be exact
	$=\frac{\sqrt{3}}{3}+\ln\sqrt{3}-\frac{1}{2}-\frac{\sqrt{3}}{3}=\ln\sqrt{3}-\frac{1}{2}$	**A1**	7	Simplified to the difference of two correct exact terms condoning one of them to be unsimplified; OE factorised form
	Total		**17**	
	TOTAL		**75**	