A-LEVEL

Mathematics

Statistics 3 – MS03
Mark scheme

6360
June 2015

Version/Stage: 1.0: Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

General Notes for MS03

GN1 There is no allowance for misreads (MR) or miscopies (MC) unless specifically stated in a question

GN2 In general, a correct answer (to accuracy required) without working scores full marks but an incorrect answer (or an answer not to required accuracy) scores no marks

GN3 When applying AWFW, a slightly inaccurate numerical answer that is subsequently rounded to fall within the accepted range cannot be awarded full marks

GN4 Where percentage equivalent answers are permitted in a question, then penalise by **one accuracy mark** at the first **correct** answer but only if no indication of percentage is shown

GN5 In questions involving probabilities, do **not** award **accuracy** marks for answers given in the form of a ratio or odds

GN6 Accept decimal answers, providing that they have **at least two** leading zeros, in the form $c \times 10^{-n}$

Q	Solution	Marks	Total	Comments
1(a)	$r = \dfrac{3095}{\sqrt{7410 \times 1642}} = \underline{\mathbf{0.887}}$	M1 A1		Numerical expression AWRT (0.88729)
	or $r = \underline{\mathbf{0.887}}$ $r = \underline{\mathbf{0.88\ \ to\ \ 0.89}}$	(B2) (B1)	**2**	AWRT AWFW
Note	**1** $\sum x = 3036$ $\sum x^2 = 775518$ $\sum xy = 561719$ $\sum y = 2208$ $\sum y^2 = 407914$ $\bar{x} = 253$ $\bar{y} = 184$			
(b)	H_0: $\rho = 0$ H_1: $\rho > 0$	B1		Both; do not accept in terms or r but accept in words providing clear indication of population pmcc
	SL $\alpha = 0.01$ (1%)			
	CV $r = \underline{\mathbf{(+)0.658\ \ to\ \ (+)0.6581}}$	B1		AWFW (0.6581)
	Calculated r > Tabulated r	M1		Comparison; can be implied by conclusion
	Evidence, at 1% level, of **a positive correlation** between the right foot length and right hand length of males aged between 19 years and 25 years	AF1	**4**	F on r and CV OE in context
Note	**1** For H_1: $\rho \neq 0$ then CV $r = (\pm)0.7079$ so same conclusion $\Rightarrow$ B0 B0 M1 AF1			
		Total	**6**	

Q	Solution	Marks	Total	Comments
2(a)	$99\% \Rightarrow z = \underline{\mathbf{2.57 \text{ to } 2.58}}$	B1		AWFW (2.5758)
	CI for 26 weeks is:			
		M1		$(507-416) \pm z\sqrt{a}$
	$(507-416) \pm 2.5758\sqrt{507+416}$	m1		$z\sqrt{507+416}$
		A1		Correct expression; $2.32 \le z \le 2.58$
	ie 91 ± (78 to 78.5) or (12.5 to 13, 169 to 169.5)			
	Dividing by 26 gives:	M1		
	<u>3.5 ± 3.0 or (0.5, 6.5)</u>	A1		CAO/AWRT or AWRT
	OR			
	$99\% \Rightarrow z = \underline{\mathbf{2.57 \text{ to } 2.58}}$	(B1)		AWFW (2.5758)
	CI for 1 week is:	(B1)		19.5 & 16
	$\left(\frac{507}{26}-\frac{416}{26}\right) \pm 2.5758\sqrt{\frac{507}{26^2}+\frac{416}{26^2}} =$	(M1)		$(19.5-16.0) \pm z\sqrt{b}$
	$(19.5-16.0) \pm 2.5758\sqrt{\frac{19.5}{26}+\frac{16.0}{26}}$	(m1)		$z\sqrt{\frac{35.5}{26}}$ or $z\sqrt{35.5}$
		(A1)		Correct expression; $2.32 \le z \le 2.58$
	ie **<u>3.5 ± 3.0 or (0.5, 6.5)</u>**	(A1)	6	CAO/AWRT or AWRT
(b)	Since CI **is above 0**	B1		OE; providing CI > 0
	Emilia's belief is **justified**	Bdep1	2	Dependent on B1; OE in context
		Total	**8**	

Q	Solution	Marks	Total	Comments
3	S: 0.55 L: 30 VL: 0.15			**In (a)(i) & (iv), accept any equivalent fractional answer with den ≤ 100 or the equivalent percentage answer with %- sign (see GN4)**
(a) (i)	P(S ∩ £1) = 0.55 × 0.20 = **0.11**	B1	**(1)**	CAO
(ii)	P(£0) = (0.55 × 0.70) + (0.30 × 0.65) + (0.15 × 0.55) = 0.385 + 0.195 + 0.0825 = **0.662 to 0.663**	M1 A1	**(2)**	>1 term correct; may be implied AWFW (0.6625)
(iii)	$P(L \mid £0) = \dfrac{P(L \cap £0)}{P(£0)} = \dfrac{0.30 \times 0.65}{(ii)}$ $= \dfrac{0.195}{0.6625} =$ **0.294 to 0.295**	M1 A1	**(2)**	May be implied AWFW (0.29434)
(iv)	$P(VL \mid >£0) = \dfrac{P(VL \cap >£0)}{P(>£0)} = \dfrac{0.15 \times 0.45}{1-(ii)}$ $= \dfrac{0.0675}{0.3375} =$ **0.2**	M1 M1 A1	**(3)**	Numerator Denominator CAO
			8	
(b)	P((S ∩ L ∩ VL) | >£0) = $\dfrac{0.55 \times 0.30}{0.3375} \times \dfrac{0.30 \times 0.35}{0.3375} \times \dfrac{0.15 \times 0.45}{0.3375} \times 6 =$ $\dfrac{0.165 \times 0.105 \times 0.0675 \times 6}{0.3375^3} = \dfrac{0.0011694375 \times 6}{0.3375^3}$ **or** $= \dfrac{22}{45} \times \dfrac{14}{45} \times \dfrac{9}{45} \times 6 = \dfrac{16632}{91125} = \dfrac{616}{3375}$ = **0.182 to 0.183**	M1 M1 m1 A1	**4**	>1 term correct in numerator (1 – (ii)) in denominator 6 or 3!; must have at least one M1 AWRT (0.18252)
		Total	**12**	

Q	Solution	Marks	Total	Comments
4(a)	H_0: p = 0.60 (60%) H_1: $p \neq$ 0.60 (60%)	B1		Both
	5% $\Rightarrow$ z = **1.96**	B1		AWRT
	$\hat{p} = \dfrac{164}{250}$ = **0.656**	B1		CAO
	$z = \dfrac{0.656 - 0.6}{\sqrt{\dfrac{0.6 \times 0.4}{250}}}$	M1		Allow use of 0.656 in denominator
		m1		Correct denominator
	= **1.8 to 1.81**	A1		AWFW (1.80739) (p-value = 0.07070 > 0.05)
	No evidence, at 5% level, to suggest percentage **is not 60%** or is different	AF1	**7**	F on z and CV OE in context
Notes	1 (0.656 – 0.6)/√((0.656 × 0.344)/250) = 1.86392 $\Rightarrow$ (B1) (B1) (B1) M1 m0 A0 (AF1) **2** ((163.5 or 164) – 150)/√60 = 1.74284 or 1.80739 $\Rightarrow$ (B1) (B1) (B1) M1 m1 A1 (AF1) **3** (164.5 – 150)/√60 = 1.87194 $\Rightarrow$ (B1) (B1) (B1) M1 m0 A0 (AF1) **4** ((163.5 or 164 or 164.5) – 150)/√56.416 = 1.79735 or 1.86392 or 1.93049 $\Rightarrow$ (B1) (B1) (B1) M1 m0 A0 (AF1) **5** P($X \geq$ 164 | B(250, 0.6)) = 0.039794 > 0.025 $\Rightarrow$ (B1) B4 M1 (AF1)			
(b)	H_0: p = 0.25 (25%) H_1: p < 0.25 (25%)	B1		Both
	Use of B(40, 0.25)	M1		May be implied
	P($X \leq$ 5) = **0.043**	A1		AWRT (0.0433)
	Calculated p-value < 0.05 (5%)	M1		Comparison of p-value and 0.05
	Evidence, at 5% level, to suggest percentage **is less than 25%**	AF1	**5**	F on p-value and 0.05 OE in context
Notes	**1** P($X \leq 4$) = 0.0160 and P($X \leq 6$) = 0.0962 **2** Use of normal approximation $\Rightarrow$ B1 max			
(c)	98% $\Rightarrow$ z = **2.32 to 2.33**	B1		AWFW (2.3263)
		M1		Use of $z \times \mathrm{SD}(\hat{p})$
	$z\sqrt{\dfrac{p(1-p)}{n}} = 2.3263\sqrt{\dfrac{0.3 \times 0.7}{n}} < 0.05$	A1		Allow use of p = 0.5, (×2) & z = 2.05 to 2.33
	$n > \dfrac{2.3263^2 \times 0.21}{0.05^2}$ = **450 to 460**	m1 A1	**5**	Attempt at solution for n AWFW; must be an integer (455)
Note	**1** Use of p = 0.5 gives n = 541.2 so 535 to 545 (AWRT) $\Rightarrow$ B1 M1 AF1 M1 A0			
		Total	**17**	

Q	Solution	Marks	Total	Comments
5 (a)(i)	$\mathrm{E}(X)=\sum_{x=0}^{n} x\binom{n}{x}p^x(1-p)^{n-x}\ =$	M1		Used; ignore limits until A1
	$np\sum_{x=1}^{n-1}\frac{(n-1)!}{(x-1)!(n-x)!}p^{x-1}(1-p)^{n-x}\ =$	M1		**≥ 2** of: factor of np plus p^x to p^{x-1}, $n!$ to $(n-1)!$ and $x!$ to $(x-1)!$
	$np\sum_{x=1}^{n-1}\mathrm{B}(n-1,\ p)\ =\ np$	A1	3	Fully complete and correct derivation **AG**
(ii)	$\mathrm{Var}(X)=\mathrm{E}(X^2)-n^2p^2$ $\mathrm{E}(X(X-1))=\mathrm{E}(X^2)-np\ =\ n(n-1)p^2$	M1		Both used; OE
	so $\mathrm{Var}(X)=n(n-1)p^2+np-n^2p^2\ =\ \underline{\boldsymbol{np(1-p)}}$	A1	2	Fully complete and correct derivation
Notes	**1** $\mathrm{E}(X(X-1))\ =\ \mathrm{E}(X^2)-np\ =\ \mathrm{V}(X)+n^2p^2-np\ =\ n(n-1)p^2\ \Rightarrow\ \mathrm{V}(X)\ =\ np(1-p)\ \Rightarrow$ M1 A1 **2** $\mathrm{E}(X^2)\ =\ n^2p^2-np^2\ +np\ \Rightarrow\ \mathrm{V}(X)\ =\ n^2p^2-np^2\ +np\ -n^2p^2\ =\ np(1-p)\ \Rightarrow$ M1 A1			
(b)(i)	$\frac{\mathrm{Var}(Y)}{\mathrm{E}(Y)}=\frac{np(1-p)}{np}=1-p=\frac{2.985}{3}=0.995$	M1		OE
	so $p\ =\ \underline{\mathbf{0.005}}$ and so $n\ =\ \frac{3}{0.005}\ =\ \underline{\mathbf{600}}$	A1 A1	3	CAO both
(ii)	$\frac{\mathrm{Var}(U)}{\mathrm{E}(U)}=\frac{np(1-p)}{np}=1-p=\frac{6.25}{5}=1.25$	M1		OE
	$\Rightarrow$ $\boldsymbol{p<0}$ **or** $\boldsymbol{(1-p)>1}$ which is impossible	A1	2	Indication that $p<0$ or $(1-p)>1$
(c)	$\mathrm{E}(W)=2\times5+10\ =\ \underline{\mathbf{20}}$	B1		CAO; must be justified
	$\mathrm{Var}(W)=2^2\times5\ =\ \underline{\mathbf{20}}$	B1		CAO; must be justified
	No odd values **or** no values < 10	B1	3	Either
(d)	$n\ =\ 5000\ \&\ p\ =\ 0.002\ \Rightarrow$ $\underline{\mathbf{Po(10)}}$	B1		
	P(6 ≤ *AB*– ≤ 12) = **0.7916** **– (0.0671 or 0.1301)**	M1		
	= $\underline{\mathbf{0.724\ to\ 0.725}}$	A1	3	AWFW (0.7245)
Note	**1** Use of normal approximation ⇒ B0 M0 A0			
		Total	**16**	

Q	Solution	Marks	Total	Comments
6 **(a)**	$\text{Var}(\bar{L}-2\bar{S}) = \text{Var}(\bar{L}) + 2^2\,\text{Var}(\bar{S})$	M1		Use of + and 2^2
	but $\text{Var}(S) = \text{Var}(L) = \sigma^2$			
	so $\text{Var}(\bar{S}) = \text{Var}(\bar{L}) = \dfrac{\sigma^2}{n}$	M1		Use of $\dfrac{\sigma^2}{n}$
	giving $\text{Var}(\bar{L}-2\bar{S})$ = $\underline{\mathbf{5\sigma^2/n}}$	A1	3	CAO
Note	**1** Answer of $3\sigma^2/n \Rightarrow$ M0 M1 A0			
(b) **(i)**	H_0: $\mu_L = 2\mu_S$ H_1: $\mu_L > 2\mu_S$	B1 B1		Award B1 B0 for $\mu_L = \mu_S$
	10% $\Rightarrow$ z = $\underline{\mathbf{1.28}}$	B1		AWRT (1.2816)
	$z = \dfrac{522-(2\times 258)}{\sqrt{\dfrac{5\times 8^2}{25}}}$	M1 M1		Numerator; allow (522 – 258) Denominator; allow $\sqrt{2\times 8^2/25}$ OE or $\sqrt{3\times 8^2/25}$ OE
	= $\underline{\mathbf{1.68}}$	A1		AWRT (1.67705)
	Evidence, at 10% level, to suggest that $\mu_L > 2\mu_S$	Adep1	7	Dep on A1 OE in context
(ii)	CV is given by $\dfrac{\bar{l}-2\bar{s}}{\sqrt{\dfrac{5\times 8^2}{25}}}$ or $\dfrac{\bar{l}-2\bar{s}}{\sqrt{12.8}} = 1.28(16)$	M1		Completely correct equality
	ie CV = $\underline{\mathbf{4.585}}$	A1	2	AWRT; **AG** (4.58519)
(iii)	P(Type II error) = P(accept H_0 | H_0 false)	B1		OE; stated or used
	= $P(\bar{L}-2\bar{S} < 4.585 \mid \mu_L - 2\mu_S = 10)$ = $P\left(Z < \dfrac{4.585-10}{\sqrt{\dfrac{5\times 8^2}{25}}}\right)$ = P(Z < $\underline{\mathbf{\pm 1.51}}$)	M1 A1		Must have correct numerator Denominator; allow $\sqrt{2\times 8^2/25}$ OE or $\sqrt{3\times 8^2/25}$ OE AWRT (–1.51354)
	= $\underline{\mathbf{0.064\ \ to\ \ 0.066}}$	A1	4	AWFW (0.06504)
		Total	**16**	