A-LEVEL

Mathematics

Mechanics 4 – MM04
Mark scheme

6360
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Version1 Final Mark Scheme

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q1	Solution	Mark	Total	Comment
a)	Σ components = 0 $2a + 8b + 11 = 0$ and $1 - 2a + 4b = 0$	**M1**		Both equations seen
	Solving gives $b = -1$ and $a = -1.5$	**A1** **A1**	**3**	**A1** each correct value
b)	Moments about O gives 1(1) + 3(3) – 3(1) + 8(4) + 11(2) +4(5)	**M1**		**M1** Use of moments - at least four correct pairings
		A1F **A1F**		**A1** all signs consistent, **A1** fully correct – follow through their values from part a)
	= 81 (Nm)	**A1**	**4**	Magnitude correct - **CAO**
	ALTERNATIVE Use of **r** x **F** three times to get			
	10**k** + 29**k** + 32**k** = 81**k**	**(M1)** **(A1F)**		Correct use of **r** x **F** or **F** x **r** three times At least two determinants correctly evaluated
		(A1F)		All three fully correct - follow through their values from part a)
	Hence magnitude = 81 (Nm)	**(A1)**	**(4)**	Magnitude correct - **CAO**
	Total		**7**	

Q2				
a)	Resolve vertically at *R*, $T_{QR}\cos 60^0 = 250$	**M1**		Forming a correct equation with T_{QR}
	$T_{QR} = 500$ N	**A1**		Obtaining correct value of T_{QR}
	in tension	**E1**	**3**	**CAO**
b)	Vertical component at hinge = 250 N	**B1**		Stated or implied by later calculation
	Let horizontal component at hinge = X Take moments about *P*,			
	$X(2\cos 60^0) = 250\,(4\cos 30^0)$	**M1A1**		**M1 –** set up equation to find horizontal component of the hinge with one side correct. **A1** fully correct
	$X = 500\sqrt{3}$ N	**A1**		Correct value obtained (866.025..)

	Magnitude = $\sqrt{(500\sqrt{3})^2 + 250^2}$ = 901 N (3sf) **ALTERNATIVE for (b)** Vertical component at hinge = 250 N For horizontal component at hinge, resolve forces horizontally X = $T_{SR} + T_{SQ}\cos 30^0$ with $T_{SR} = 250\sqrt{3}$ and $T_{SQ} = 500$ X = $500\sqrt{3}$ N Magnitude = $\sqrt{(500\sqrt{3})^2 + 250^2}$ = 901 N (3sf)	**A1** **(B1)** **(M1)** **(A1)** **(A1)** **(A1)**	**5** **(5)**	Correct method for finding magnitude of reaction – **CAO** Stated or implied by later calculation **M1** – set up a full complete and correct system of equations to find horizontal component of the hinge. **A1** correct tension/compression values obtained for all required rods. Correct value obtained (866.025..) Correct method for finding magnitude of reaction – **CAO**
	Total		**8**	

Q3	Solution	Mark	Total	Comment
a)i)	It is a line of symmetry (and the lamina is uniform)	E1	1	Accept any equivalent statement
ii)	$\int xydx = \int_0^a (a-x)xdx = \int_0^a ax - x^2dx$	M1		Use of $\int xydx$ with evidence of correct integration seen
	$= \left[\frac{ax^2}{2} - \frac{x^3}{3}\right]_0^a$			
	$= \frac{a^3}{6}$	A1		Fully correct integration, limits and evaluation
	Area of triangle = $\frac{1}{2}a^2$	B1		Area of triangle seen
	$\bar{x} = \frac{\int xydx}{\int ydx} = \frac{a^3}{6} \div \frac{a^2}{2} = \frac{a}{3}$	m1		Dependent on first **M1**
	Coordinates of centre of mass **=** $(\frac{a}{3}, \frac{a}{3})$	A1	5	Coordinates must be clearly stated
b)i)	Moments about B			
	$P(a\sin 45^0) = W(\frac{a}{3})$	M1A1		**M1** one side correct - **A1** all correct
	$P = \frac{W\sqrt{2}}{3}$	A1	3	Printed answer
ii)	Resolve horizontally $F = P\sin45^0$ Resolve vertically $P\cos45^0 + R = W$ Law of friction $F = \mu R$	M1A1		**M1** Three equations seen **–** **A1** all correct
	Combining gives $P = \frac{W\sqrt{2}\mu}{1+\mu}$	M1A1		**M1** Combining to obtain *P* - **A1** if correct
	Slides before toppling hence			
	$\frac{W\sqrt{2}\mu}{1+\mu} < \frac{W\sqrt{2}}{3}$	m1		Inequality using *P* expressions formed dependent on both previous **M1**s
	$\mu < \frac{1}{2}$	A1	6	Correctly solving for μ - **CSO**
	Total		**15**	

Q4	Solution	Mark	Total	Comment
a)	Use of **r** x **F**	M1		Use of **r x F** or **F x r** three times
	$\begin{vmatrix} \mathbf{i} & -1 & 3 \\ \mathbf{j} & 1 & -2 \\ \mathbf{k} & 0 & 0 \end{vmatrix} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$	A1		**CAO**
	$\begin{vmatrix} \mathbf{i} & 2 & 4 \\ \mathbf{j} & 0 & -1 \\ \mathbf{k} & 5 & 2 \end{vmatrix} = \begin{bmatrix} 5 \\ 16 \\ -2 \end{bmatrix}$	A1		**CAO**
	$\begin{vmatrix} \mathbf{i} & -6 & 0 \\ \mathbf{j} & 2 & 3 \\ \mathbf{k} & 1 & -4 \end{vmatrix} = \begin{bmatrix} -11 \\ -24 \\ -18 \end{bmatrix}$	A1		**CAO**
	Total $= \begin{bmatrix} -6 \\ -8 \\ -21 \end{bmatrix}$	A1F	5	Sum of their three **r x F**
b)i)	$\begin{bmatrix} 7 \\ 0 \\ -2 \end{bmatrix}$	B1	1	Sum of three given forces
(b)(ii)				
	$\begin{vmatrix} \mathbf{i} & x & 7 \\ \mathbf{j} & y & 0 \\ \mathbf{k} & z & -2 \end{vmatrix} = \begin{pmatrix} -6 \\ -8 \\ -21 \end{pmatrix}$	M1		Setting up equation to find point
	$\begin{pmatrix} -2y \\ 2x+7z \\ -7y \end{pmatrix} = \begin{pmatrix} -6 \\ -8 \\ -21 \end{pmatrix}$	A1		Evaluation of determinant – LHS
	so $y = 3$ and	M1		Equating components and finding correct *y* value
	$x = -4$ $z = 0$	A1		Any correct valid combination of *x* and *z* seen
	$\mathbf{r} = \begin{pmatrix} -4 \\ 3 \\ 0 \end{pmatrix} + t \begin{pmatrix} 7 \\ 0 \\ -2 \end{pmatrix}$	M1 A1		**M1** Correct structure of a straight line with their **a** and **b** used **A1** Fully correct - **CSO**
			6	
	Total		**12**	

Q5	Solution	Mark	Total	Comment
a)	MI = $\frac{4}{3}(2m)(l\sqrt{2})^2 = \frac{16}{3}ml^2$	M1A1	2	**M1** Correct structure for MI of rod **A1** correct length – can be unsimplified
b)	MI = $\frac{1}{3}\mathrm{m}l^2 + m\left(\sqrt{5}l\right)^2 = \frac{16}{3}ml^2$	M1 A1	2	Must use I_G + md^2 Correct MI obtained – fully simplified **CSO**
c)	MI = $2(\frac{4}{3}ml^2) + 3(\frac{16}{3}ml^2)$	M1A1F		Five MI combined for **M1** **A1** fully correct – follow through part b) above
	= $\frac{56ml^2}{3}$	A1	3	Printed answer – **CSO –** must have part b) fully correct
d)	Gain in KE = $\frac{1}{2}I\omega^2 = \frac{1}{2}(\frac{56}{3}ml^2)\omega^2 = \frac{28}{3}ml^2\omega^2$	B1		Correct gain in KE – can be unsimplified
	Loss in PE: For rods *AB* and *AD* = *mgl* For rods *BC* and *CD* = 3*mgl* For rod *AC* = 4*mgl*	M1 A1		Considering change in PE for five rods – at least three correct All correct or correct total (12*mgl*) seen
	Using conservation of energy $\frac{28}{3}ml^2\omega^2$ = 12*mgl*	m1		Use of KE gained = PE lost – dependent on first **M1**
	Hence $\omega = \sqrt{\frac{9g}{7l}}$	A1	5	Any equivalent form
e)	Angular momentum immediately before = $\left(\frac{56ml^2}{3}\right)\left(\sqrt{\frac{9g}{7l}}\right)$	B1F		FT their ω
	MI of framework **<u>and</u>** particle = $\frac{56}{3}ml^2 + (3m)\left(\frac{l}{3}\right)^2 = 19ml^2$	M1A1		**M1** Finding new MI or use of moment of momentum – **A1** fully correct
	Conservation of angular momentum $\left(\frac{56ml^2}{3}\right)\left(\sqrt{\frac{9g}{7l}}\right) = 19ml^2\omega'$	m1		Forming an equation using momentum – dependent on **M1** above
	Hence $\omega' = \left(\frac{56}{57}\right)\left(\sqrt{\frac{9g}{7l}}\right) = \frac{8}{19}\sqrt{\frac{7g}{l}}$	A1	5	Any equivalent form - **CSO**
	Total		**17**	

Q6	Solution	Mark	Total	Comment
a)	$\rho = \frac{m}{8a}$	B1		Connecting ρ and m
	MI of elemental piece = $(\rho dx)x^2$			
	MI of rod = $\int_{-2a}^{6a} x^2 pdx = \int_{-2a}^{6a} x^2(\frac{m}{8a})dx$	M1		Use of correct elemental piece and evidence of integration
	$= \left[\frac{mx^3}{24a}\right]_{-2a}^{6a}$	A1		Correct integration
	$= \frac{m(6a)^3}{24a} - \frac{m(-2a)^3}{24a}$			
	$= \frac{28ma^2}{3}$	A1	4	Correct limits to obtain printed answer
b)i)	$mg(2a\sin\theta) = \frac{28}{3}ma^2\ddot{\theta}$	M1A1		Use of C = **I**$\ddot{\theta}$ **M1** one side correct - **A1** fully correct
	$\ddot{\theta} = \frac{3g\sin\theta}{14a}$	A1	3	**CAO**
	ALTERNATIVE			
	$\frac{1}{2}(\frac{28}{3}ma^2)\dot{\theta}^2 = 2mga(\cos 60^0 - \cos\theta)$			
	$2mga\sin\theta\dot{\theta} = \frac{28}{3}ma^2\dot{\theta}\ddot{\theta}$	(M1A1)		Use of conservation of energy and differentiation **M1** one side correct - **A1** fully correct
	$\ddot{\theta} = \frac{3g\sin\theta}{14a}$	(A1)	(3)	**CAO**
ii)	Gain in KE = $\frac{1}{2}I\dot{\theta}^2 = \frac{1}{2}(\frac{28}{3}ma^2)\dot{\theta}^2$	B1		Correct KE seen
	Change in PE = $2mga(\cos 60^0 - \cos\theta)$	M1A1		**M1** either potential energy term seen – **A1** fully correct
	$\frac{1}{2}(\frac{28}{3}ma^2)\dot{\theta}^2 = 2mga(\cos 60^0 - \cos\theta)$	M1		Use of conservation of energy with their PE and KE terms
	$\frac{14}{3}a\dot{\theta}^2 = 2g(\frac{1}{2} - \cos\theta)$	A1		Evidence of correct substitution, simplification and cancelling
	Hence $\dot{\theta} = \sqrt{\frac{3g(1-2\cos\theta)}{14a}}$	A1	6	Fully correct rearrangement - **CSO**

iii)	Using $F = ma$ along the rod $mg\cos\theta - X = 2am\dot{\theta}^2$ $mg\cos\theta - X = (2am)\dfrac{3g}{14a}(1-2\cos\theta)$ Hence $X = \dfrac{mg}{7}(13\cos\theta - 3)$	**M1** **A1F** **A1**	**3**	Correct structure of $F = ma$ Substitution of their expression **CAO** – can be unsimplified
	Total		**16**	
	TOTAL		**75**	