A-LEVEL
MATHEMATICS

Statistics 4 – MS04
Mark scheme

6360
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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Mark	Total	Comment
1 (a)	$\mathrm{F}(t) = \int_0^t 5\mathrm{e}^{-5t}\mathrm{d}t = [-\mathrm{e}^{-5t}]_0^t$ $= 1 - \mathrm{e}^{-5t} \qquad t \geq 0$	M1A1 A1		If result quoted without proof award B1. Incorrect notation A0, unless recovery is clear. Need not see $t \geq 0$ for A1 .
	F(t) = 0 otherwise, or t < 0.	B1	4	
(b)	$1 - (1 - \mathrm{e}^{-1}) = \mathrm{e}^{-1}$ (0.368)	B1	1	
(c)	$\mathrm{e}^{-5c} = 0.05 \quad \Rightarrow \mathrm{e}^{5c} = 20$ $\Rightarrow c = \frac{1}{5}\ln 20$ (0.599)	M1 A1	2	Accept 0.6. Some attempt to simplify a logarithmic answer is required.
	Total		**7**	

Q	Solution	Mark	Total	Comment
2(a)	Journey times are normally distributed.	E1	1	
(b)	s = 1.793 v = 9	B1 B1		$s^2 = 3.215 \quad \sum(x-\bar{x})^2 = 28.936$ (Accept 3.22.)
	χ_9^2 (0.01) = 2.088 χ_9^2 (0.99) = 21.666	B1		Both
	98% CL for σ^2 are			
	$\dfrac{28.936}{21.666}$ and $\dfrac{28.936}{2.088}$	M1A1√		ft on χ^2 values.
	98% CI for σ^2 is (1.34,13.9)	A1	6	CAO
	Total		**7**	

Q	Solution	Mark	Total	Comment
3(a)	$S_X^2 = 0.41636$ $S_Y^2 = 0.04778$	M1 A1		Either Both correct. SC B1 for one only.
	$v_1 = 11$, $v_2 = 8$ $F_{11,8} = 7.104$, $F_{8,11} = 5.682$	B1 B1		Both Both Dfs can be implied by correct CVs.
	$F_{\text{calc}} = \frac{0.41636}{0.04778} = 8.7146$	M1		
	$\frac{1}{7.104} \le \frac{VR}{8.7146} \le 5.682$	A1ft		ft on v_1 and v_2. Accept 5.7
	$\Rightarrow$ $1.23 \le VR \le 49.5$	A1	**7**	CAO
(b)	$1 \notin$ CI $\Rightarrow$ broadband speed is more variable in villages.	E1√ e1√	**2**	Accept 1 is below the CI.
	Total		**9**	

Q	Solution	Mark	Total	Comment
4(a)	Independent (and/or) random samples. Normal distributions with common variance.	B1 B1	2	If 'independent' and 'random' only, award B1.Second B1 req. 'Normal' & 'Common Var'.
(b)	H_0: $\mu_A = \mu_B$ H_1: $\mu_A \ne \mu_B$	B1		Both
	$\bar{x}_A = 9.7$ $s_A = 0.56315$ $(s_A^{\ 2} = 0.3171$ $\bar{x}_B = 8.7$ $s_B = 0.61319$ $(s_B^{\ 2} = 0.376)$	B1 B1		Both. (or 0.2887) Both (or 0.2737)
	$s^2 = \frac{7\times0.5632^2 + 5\times0.6132^2}{8+6-2} = 0.3417$	M1A1		OE
	$t_{calc} = \frac{1-0}{0.5845\sqrt{\frac{1}{8}+\frac{1}{6}}} = 3.17$	M1A1		awrt
	$v = 12$ $t_{crit} = \pm 2.681$	B1B1		Both signs not required.Df can be implied by correct CV.
	$3.17 > 2.681 \Rightarrow$ reject H_0. Sufficient evidence to indicate that means are different at 2% level of significance.	A1√	10	Compares and states conclusion context. √ on t.
	Total		**12**	

Q	Solution	Mark	Total	Comment
5(a)	$\bar{x} = \dfrac{360}{100} = 3.6$	B1		CAO
	$12p = 3.6 \Rightarrow p = 0.3$	B1	2	CSO (AG)
(b)	H_0: B(12,*p*) is an appropriate model.	B1		
	Distribution B(12,0.3): 0.0138 0.0712 0.1678 0.2397 <u>0.2311</u> 0.1585 0.1179	M1A1		Attempt at probabilities; ≥ 4 correct for M1; A1 if all correct. (Note: Tables give 0.2312)
	Expected frequencies are: 1.38 7.12 16.78 23.97 23.11 15.85 11.79	M1		Probabilities × 100.
	O: 6 14 28 27 16 9 **E**: 8.5 16.78 23.97 23.11 15.85 11.79	M1		Combines first two classes.
	$\chi^2_{\text{calc}} = \sum\left\{\dfrac{(\text{O}-\text{E})^2}{\text{E}}\right\} = 3.190$	M1A1		Attempt at formula ; awfw 3.15 to 3.25.
	$\nu = 6 - 2 = 4$ $\chi^2_{\text{crit}} = 7.779$	B1B1√		Ft on $\nu = 7 - 2 = 5$ and 9.236 (When classes not combined.)
	$3.190 < 7.779 \Rightarrow$ Accept H_0 B(12,*p*) is a suitable model.	E1√	10	Compare and state conclusion in context. √ on χ^2
	Total		**12**	

Q	Solution	Mark	Total	Comment
6(a)	$\text{E}(\bar{X}_1) = \frac{n_1\mu}{n_1} = \mu$ and $\text{E}(\bar{X}_2) = \frac{n_2\mu}{n_2} = \mu$ $\text{E}(k\bar{X}_1 + (1-k)\bar{X}_2) = k\text{E}(\bar{X}_1) + (1-k)\text{E}(\bar{X}_2)$ $= k\mu + (1-k)\mu = \mu$	M1 A1	2	Stated or implied.
(b)	$\text{Var}(k\bar{X}_1 + (1-k)\bar{X}_2)$ $= k^2\text{Var}(\bar{X}_1) + (1-k)^2\text{Var}(\bar{X}_2)$	M1		Stated or implied.
	$\text{Var}(\bar{X}_1) = \frac{\sigma^2}{n_1}$ and $\text{Var}(\bar{X}_2) = \frac{\sigma^2}{n_2}$ (OE) $\Rightarrow V = k^2\frac{\sigma^2}{n_1} + (1-k)^2\frac{\sigma^2}{n_2}$ (AG)	A1	2	
(c)	$\frac{\text{d}V}{\text{d}k} = \sigma^2\left\{\frac{2k}{n_1} - \frac{2(1-k)}{n_2}\right\}$ $\frac{k}{n_1} - \frac{(1-k)}{n_2} = 0 \Rightarrow k = \frac{n_1}{n_1+n_2}$	M1A1 A1	3	Using $n_1 = n_2 = n$ from the start ⇒M0.
(d)(i)	$k\bar{X}_1 + (1-k)\bar{X}_2 = \frac{n_1\bar{X}_1 + n_2\bar{X}_2}{n_1+n_2}$ (OE)	M1A1√	2	F.t. on algebraic form. $\frac{1}{2}$ gets A0.
(ii)	This is the weighted average of means.	E1	1	Explanation in terms of proportionality, or 'pooled estimate' OK.
(iii)	$\frac{\text{d}^2V}{\text{d}k^2} = 2\sigma^2\left\{\frac{1}{n_1} + \frac{1}{n_2}\right\} > 0 \Rightarrow$ minimum *V*.	M1A1	2	No omissions.
	Total		**12**	

Q	Solution	Mark	Total	Comment
7(a)(i)	$\mathrm{E}(X^2) = p(1 + 2q + 3q^2 + 4q^3 + \cdots)$ $+2pq(1 + 3q^2 + 6q^3 + 10q^4 + \cdots)$ where $p + q = 1$.	M1A1		Accept proof by generating functions, or any other valid method.
	$= \frac{p}{(1-q)^2} + \frac{2pq}{(1-q)^3} = \frac{1}{p} + \frac{2(1-p)}{p^2}$	M1A1	4	CSO (AG)
(ii)	$\Rightarrow \mathrm{Var}(X) = \frac{1}{p} + \frac{2(1-p)}{p^2} - \frac{1}{p^2}$ $= \frac{p+2-p-1}{p^2} = \frac{1-p}{p^2}$ (AG)	B1	1	
(iii)	$p = \frac{1}{2} \Rightarrow \mathrm{Var}(X) = 2$	B1		
	$\mathrm{P}(X > 2) = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$	M1A1	3	
(b)(i)	$\frac{1}{30} + \frac{2}{3} \times \frac{1}{30} + \left(\frac{2}{3}\right)^2 \times \frac{1}{30}$ $= \frac{19}{270}$	M1A1 A1	3	In a round: P(both miss) = $\frac{4}{5} \times \frac{5}{6} = \frac{2}{3}$. P(both hit) = $\frac{1}{5} \times \frac{1}{6} = \frac{1}{30}$.
(ii)	$\frac{1}{30} \div \left(1 - \frac{2}{3}\right) = \frac{1}{10}$	M1A1	2	Sum to infinity of series started in part (i).
(iii)	$\frac{1}{6} + \frac{2}{3} \times \frac{1}{6} + \left(\frac{2}{3}\right)^2 \times \frac{1}{6} + \cdots$	M1A1		**Alternatively:** P(R hits and W misses) $= \frac{1}{5} \times \frac{5}{6} = \frac{1}{6}$. (B1)
	$= \frac{1}{6} \div \frac{1}{3} = \frac{1}{2}$	A1	3	Then $\mathrm{P}_r = \frac{1}{6} + \frac{2}{3}\mathrm{P}_r \Rightarrow \frac{1}{3}\mathrm{P}_r = \frac{1}{6}$ $\Rightarrow \mathrm{P}_r = \frac{1}{6} \div \frac{1}{3} = \frac{1}{2}$ (M1A1)
	Total		**16**	
	TOTAL		**75**	