Centre Number						Candidate Number				
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Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
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6	
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TOTAL	

AQA

General Certificate of Education
Advanced Level Examination
June 2014

Mathematics MM04

Unit Mechanics 4

Tuesday 24 June 2014 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\mathrm{m\,s^{-2}}$, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

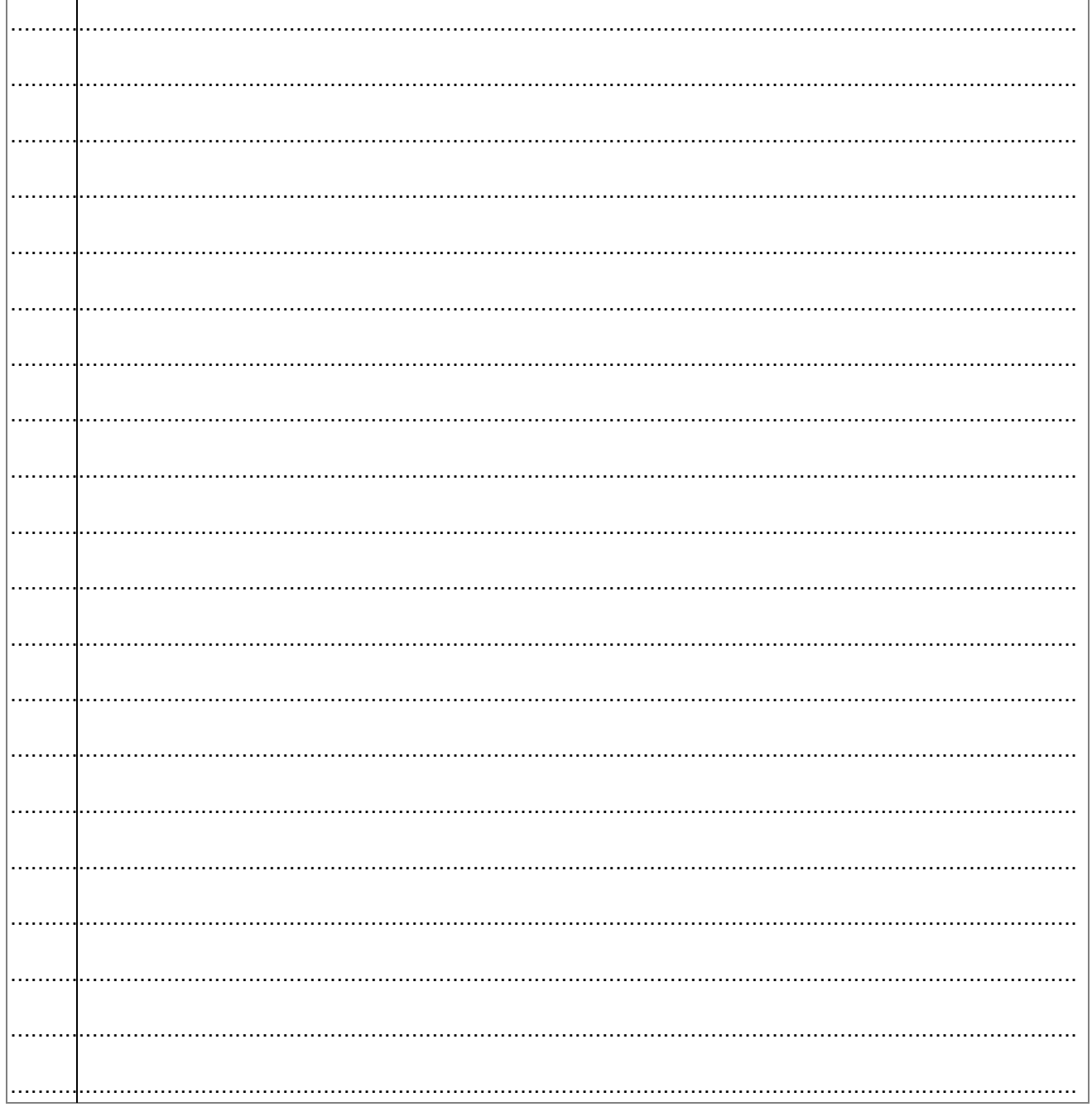
1 The forces $3\mathbf{i} + \mathbf{j} + 4\mathbf{k}$, $\frac{1}{2}\mathbf{i} - \mathbf{j} + \frac{1}{2}a\mathbf{k}$ and $a\mathbf{i} + 2\mathbf{j} - a\mathbf{k}$ act at the points with coordinates $(1, -2, a)$, $(6, 2, 8)$ and $(1, 0, -1)$ respectively, where a is a constant.

Show that the total moment of these forces about the origin is independent of a.

[6 marks]

QUESTION PART REFERENCE

Answer space for question 1



QUESTION PART REFERENCE

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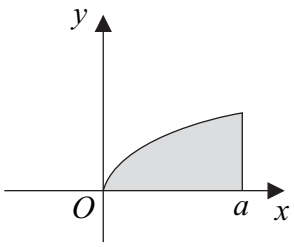
2 The region bounded by the positive x-axis, the line $x = a$ and the curve with equation $y = ax^{\frac{1}{2}}$ is shown in the diagram.



The region is rotated through 2π radians about the x-axis to form a uniform solid.

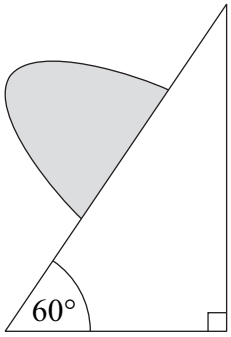
(a) Show that the volume of the solid is $\frac{1}{2}\pi a^4$.

[3 marks]

(b) Find the distance, in terms of a, of the centre of mass of the solid from the origin.

[4 marks]

(c) The solid is placed on a rough plane inclined at an angle of 60° to the horizontal, with the plane face of the solid in contact with the plane, as shown in the diagram.



Given that the solid is at rest and on the point of toppling, find the value of a.

[4 marks]

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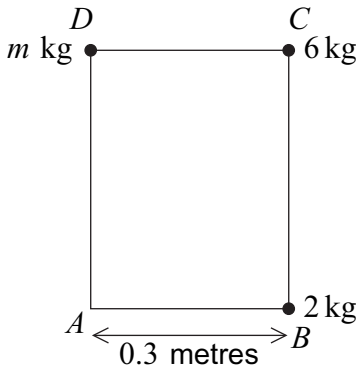
3 A thin wire has length 1.4 metres and is of negligible mass. It is bent to form a rigid rectangular frame $ABCD$ where AB has length 0.3 metres. Particles of mass $2\,\text{kg}$, $6\,\text{kg}$ and $m\,\text{kg}$ are attached to the points B, C and D respectively, as shown in the diagram.



The moment of inertia of the system about an axis through A and perpendicular to the plane $ABCD$ is $2.24\,\text{kg}\,\text{m}^2$.

Find m.

[5 marks]

QUESTION PART REFERENCE

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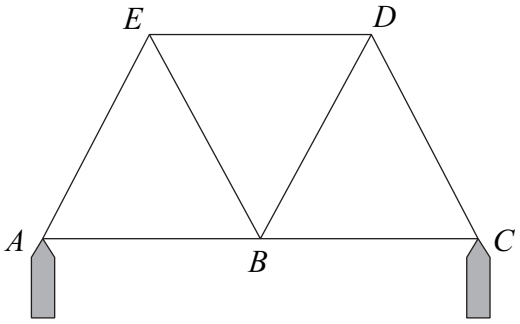
4 A framework consists of seven light inextensible smoothly jointed rods AB, BC, CD, BD, BE, DE and AE. They form three equilateral triangles ABE, BCD and BDE. The framework rests in equilibrium in a vertical plane resting on vertical supports at A and C. The rods AB, BC and ED are horizontal, as shown in the diagram.



When a load of weight $x\,\mathrm{N}$ is attached at B, rod AE is in compression with an internal force of $50\,\mathrm{N}$.

(a) Find the magnitude of the force in rod AB, stating whether it is in tension or compression.

[2 marks]

(b) Find the magnitude of the reaction force on the framework at A.

[1 mark]

(c) Determine the value of x.

[2 marks]

(d) Find the magnitude of the force in rod ED, stating whether it is in tension or compression.

[3 marks]

QUESTION PART REFERENCE

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5 The forces $\begin{bmatrix}1\\2\end{bmatrix}$, $\begin{bmatrix}-2p\\-5\end{bmatrix}$ and $\begin{bmatrix}5\\p\end{bmatrix}$ act in the x–y plane at the points with coordinates $(0, 3)$, $(2, 0)$ and $(3, 4)$ respectively, where p is a constant.

(a) Write down the resultant of these forces in terms of p.

[1 mark]

(b) Show that the resultant moment of these forces about the origin is $c(p - 11)$, where c is an integer.

[4 marks]

(c) Show that there is one value of p for which this system of forces is equivalent to a couple.

State the magnitude and sense of this couple.

[5 marks]

(d) In the case where $p = 11$, this system of forces is equivalent to a single force $\mathbf{F}$ acting at any point on the line L.

(i) Find $\mathbf{F}$.

[2 marks]

(ii) Determine a Cartesian equation for the line L.

[3 marks]

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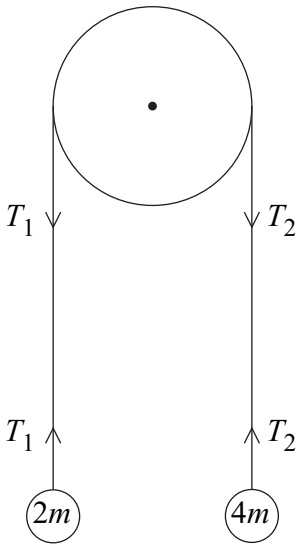
6 A pulley of radius r is free to rotate about an axis through its centre and perpendicular to the circular face of the pulley. The moment of inertia of the pulley about this axis is $6mr^2$. A light inextensible string lies over the pulley and connects particles of masses $2m$ and $4m$ to form a system. The system is released from rest at time $t = 0$. During the subsequent motion, a constant frictional couple of magnitude $\frac{1}{2}mgr$ acts on the pulley. At time t, the pulley has turned through an angle θ. During the motion, the string does not slip on the pulley. The string between the pulley and the $2m$ mass is vertical and has tension T_1, and the string between the pulley and the $4m$ mass is vertical and has tension T_2, as shown in the diagram.



(a) By considering the equation of motion for each of the two particles, show that

$$2T_1 + T_2 = 8mg$$

[3 marks]

(b) Show that $T_2 - T_1 - \frac{1}{2}mg = 6mr\ddot{\theta}$.

[3 marks]

(c) Find T_1 and T_2 in terms of m and g.

[4 marks]

(d) Find $\ddot{\theta}$ in terms of r and g.

[2 marks]

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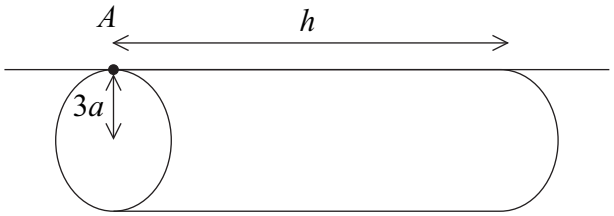
7 (a) A uniform solid right circular cylinder has mass m, height h and radius a. By using integration, show that the moment of inertia of the cylinder along its axis of symmetry is $\frac{1}{2}ma^2$.

[5 marks]

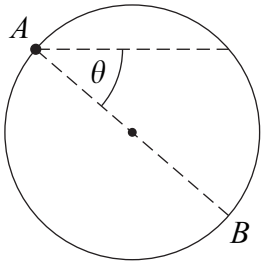
(b) A uniform solid right circular cylinder of mass $2m$, height h and radius $3a$ is hinged so that it can rotate freely about a horizontal axis along its curved surface through the point A, parallel to its axis of symmetry, as shown in the diagram.



Stating any theorems used, find the moment of inertia along this axis.

[4 marks]

(c) The cylinder is initially held at rest, with the diameter of one end of the cylinder, AB, horizontal. It is then released, and at time t the angle between AB and the horizontal is θ, as shown in the diagram.



(i) Find expressions for $\dot{\theta}^2$ and $\ddot{\theta}$ in terms of m, a, g and θ.

[6 marks]

(ii) Find an expression, in terms of m, g and θ, for the component of the reaction in the direction perpendicular to AB exerted on the cylinder at time t.

[3 marks]

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END OF QUESTIONS

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