A-LEVEL
MATHEMATICS
Further Pure 4 – MFP4
Mark scheme

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–x EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Mark	Total	Comment
1				
a)	x axis	**B1**	**1**	
b)	$\cos\theta = -\,0.6$ and $\sin\theta = 0.8$	**M1**		Values of sine **and** cosine correctly identified and use of inverse trig to find angle
	$\theta = 127^0$	**A1**	**2**	**SC1 –B1** for **NMS** or only $\cos\theta = -\,0.6$ seen**.** Accept -233^0 **NB** 53^0 scores **M0 A0** **CAO -** Must be to the nearest degree
	Total		**3**	

Q	Solution	Mark	Total	Comment
2				
a)	Row 2 →row 2 – row 1 Row 3 →row 3 – row 1			
	$= \begin{vmatrix} 1 & x & x^2 \\ 0 & y-x & y^2-x^2 \\ 0 & z-x & z^2-x^2 \end{vmatrix}$			
	$= (y-x)(z-x)\begin{vmatrix} 1 & y+x \\ 1 & z+x \end{vmatrix}$	**M1** **A1**		Finding one factor correctly Two factors correct – correct working
	$= (y-x)(z-x)(z-y)$	**m1** **A1**	**4**	Complete method to find third factor All correct – any equivalent form
b)	$(y-x)(z-x)(z-y) = (z^2-y^2) \times \det \mathbf{B}$	**M1**		Use of det **AB** = det **A** × det **B –** alternatives are det **A** = det **AB** × det **B^{-1}** or det (**AB)$^{-1}$** = det **B^{-1}** × det **A^{-1}**
	Hence det **B** = $\dfrac{(y-x)(z-x)(z-y)}{(z-y)(z+y)}$			
	det **B**$^{-1}$ = $\dfrac{(z-y)(z+y)}{(y-x)(z-x)(z-y)}$	**M1**		Correct use of det **B**$^{-1}$ = $\frac{1}{\det \mathbf{B}}$ to obtain their expression for det **B**$^{-1}$. Numerator does not need to be factorised.
	det **B**$^{-1}$ = $\dfrac{(z+y)}{(y-x)(z-x)}$	**A1**	**3**	**CSO -** Fully correct with factor cancelled
	Total		**7**	

Q	Solution	Mark	Total	Comment
3a)	det M = $\begin{vmatrix} k & 3 \\ k & 1 \end{vmatrix} - 3\begin{vmatrix} 4 & 2 \\ k & 1 \end{vmatrix} + 2\begin{vmatrix} 4 & 2 \\ k & 3 \end{vmatrix}$	**M1**		Correct expansion by row or column
	$= (k-3k) - 3(4-2k) + 2(12-2k)$			
	= 12	**A1**		**CAO**
	(Constant/Independent of k and) therefore can never equal zero – hence non singular	**E1**	**3**	Explanation – must refer to non-zero answer and **M1A1** must have been scored
b)	$\begin{bmatrix} -2k & 3 & k \\ 2k-4 & -3 & 8-k \\ 12-2k & 3 & k-12 \end{bmatrix}$	**M1** **A(2,1)**		**M1** Cofactor matrix - one full row or column correct. **A1**at least six entries correct **A2** all entries correct
	$\mathbf{M}^{-1} = \frac{1}{12}\begin{bmatrix} -2k & 2k-4 & 12-2k \\ 3 & -3 & 3 \\ k & 8-k & k-12 \end{bmatrix}$	**m1** **A1F**	**5**	**m1** Divide by determinant and transpose their matrix Follow through their determinant answer in part a) – must be non-zero
c)	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathbf{M}^{-1}\begin{pmatrix} 25 \\ 3 \\ 2 \end{pmatrix}$			
	$= \frac{1}{12}\begin{pmatrix} -50k+6k-12+24-4k \\ 75-9+6 \\ 25k+24-3k+2k-24 \end{pmatrix}$	**M1A1F**		**M1 -** Attempt at $\mathbf{M}^{-1}$ **v –** One of their components correct – can be unsimplified **A1** Two of their components correct – can be unsimplified. Follow through their $\mathbf{M}^{-1}$
	$= \frac{1}{12}\begin{pmatrix} 12-48k \\ 72 \\ 24k \end{pmatrix}$	**A1**		Three components correct – terms collected
	$= \begin{pmatrix} 1-4k \\ 6 \\ 2k \end{pmatrix}$			
	Hence $x = 1-4k$ $y = 6$ $z = 2k$	**A1**	**4**	Fully correct and simplified – **CSO**
				Any method with does not use $\mathbf{M}^{-1}$ **v** scores zero marks
	Total		**12**	

Q	Solution	Mark	Total	Comment
4	$\mathbf{u} \times \mathbf{v} = \mathbf{u} \times \mathbf{w}$ $\mathbf{u} \times \mathbf{v} - \mathbf{u} \times \mathbf{w} = \mathbf{0}$ $\mathbf{u} \times (\mathbf{v} - \mathbf{w}) = \mathbf{0}$	M1		Collect together on one side and factorise – must include x sign and ether **v – w** or **w – v**
	(Either $\mathbf{u} = \mathbf{0}$ **or** $\mathbf{v} - \mathbf{w} = \mathbf{0}$ or) the angle between **u** and **v – w** is 0^0 or 180^0	E1		Correct deduction about the angle between the vectors **u** and **v – w.** Condone reference to parallel vectors.
	Given $\mathbf{u} \neq \mathbf{0}$ and $\mathbf{v} \neq \mathbf{w}$ hence $\mathbf{v} - \mathbf{w} = \lambda\, \mathbf{u}$	A1	3	Fully correct proof
	Total		**3**	

Q	Solution	Mark	Total	Comment
5a)	$\overrightarrow{AB} = \begin{pmatrix} 3 \\ 2 \\ 2-p \end{pmatrix}$	M1		Any one vector correct
	$\overrightarrow{AC} = \begin{pmatrix} 1 \\ -2 \\ -1-p \end{pmatrix}$			
	$\overrightarrow{AD} = \begin{pmatrix} 5 \\ 2 \\ -p \end{pmatrix}$	A1	2	All three vectors fully correct and consistent with labels or clearly listed in the order stated.
b)	$(\overrightarrow{AB} \times \overrightarrow{AC}).\overrightarrow{AD}$ $= \begin{vmatrix} 5 & 3 & 1 \\ 2 & 2 & -2 \\ -p & 2-p & -1-p \end{vmatrix}$			
	$5\begin{vmatrix} 2 & -2 \\ 2-p & -1-p \end{vmatrix} -2\begin{vmatrix} 3 & 1 \\ 2-p & -1-p \end{vmatrix}$ $-p\begin{vmatrix} 3 & 1 \\ 2 & -2 \end{vmatrix}$	M1		**M1 -** Correct expansion of appropriate determinant by row or column
	$= 5(2-4p) - 2(-5-2p) - p(-6-2)$	m1A1		**m1** Expansion of 2 by 2 determinants with two correct. **A1** all three correct.
	$= 20 - 8p$	A1		Correct linear expression in p
	$= 4(5-2p)$, hence $m = 4$	A1	5	Correct value of m stated or implied by factorisation
	ALTERNATIVE for b) $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} i & 3 & 1 \\ j & 2 & -2 \\ k & 2-p & -1-p \end{vmatrix}$			
	$= \begin{pmatrix} 2-4p \\ 5+2p \\ -8 \end{pmatrix}$	(M1A1)		**M1** Use of vector product – two components correct. **A1** – all components correct
	$(\overrightarrow{AB} \times \overrightarrow{AC}).\overrightarrow{AD} =$ $\begin{pmatrix} 2-4p \\ 5+2p \\ -8 \end{pmatrix}.\begin{pmatrix} 5 \\ 2 \\ -p \end{pmatrix}$			
	$= 20 - 8p$	(m1A1)		**m1** Scalar product – must obtain linear expression in p. **A1** Correct expression
	$= 4(5-2p)$, hence $m = 4$	(A1)	(5)	Correct value of m stated or implied by factorisation
c)	when $p = 2.5$ $(\overrightarrow{AB} \times \overrightarrow{AC}).\overrightarrow{AD} = 0$ and as this represents the volume of a parallelepiped	E1		Reference to volume being zero or triple scalar product being zero
	the points lie in a single plane (coplanar)	E1	2	Deduction about coplanar points
d)	$20 - 8p = \pm 60$			
	$p = -5$ or $p = 10$	M1 A1, A1	3	**M1** Attempt to solve **both** equations **A1** each answer **SC1–B1** for one solution $p = -5$ or $p = 10$

	Total		12	
Q	**Solution**	**Mark**	**Total**	**Comments**
6a)	Determinant = $-2a - bc$	**M1**		Correct evaluation of the determinant
	Determinant of **shear** = 1/**shear** leaves area unchanged hence $-2a - bc = 1$ Giving $2a + bc = -1$	**A1**	**2**	Set determinant equal to 1 with **justification** and manipulate correctly to obtain result
b)	Fixed point $\begin{pmatrix} a & b \\ c & -2 \end{pmatrix}\begin{pmatrix} 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$			
	Gives $2a + 2b = 2$ and $2c - 4 = 2$ hence	**M1**		Correct use of fixed point to set up two equations
	c = 3 $b = -3$ $a = 4$	**A1** **A1** **A1**	**4**	**A1** each correct value
c)	$\begin{pmatrix} 4 & -3 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} x \\ x+k \end{pmatrix} = \begin{pmatrix} 4x - 3x - 3k \\ 3x - 2x - 2k \end{pmatrix}$	**M1**		Correct substitution and multiplication of their values form part b) – can be unsimplified
	$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x - 3k \\ x - 2k \end{pmatrix}$			
	$x' + k = x - 3k + k$ $= x - 2k$ $= y'$	**m1**		Both components correctly simplified and attempt to show that $y' = x' + k$ works
	Hence $y' = x' + k$	**A1**	**3**	Fully shown – **CSO**
	ALTERNATIVE for c)			
	$\begin{pmatrix} 4 & -3 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} x \\ mx+k \end{pmatrix} = \begin{pmatrix} 4x - 3mx - 3k \\ 3x - 2mx - 2k \end{pmatrix}$	**(M1)**		Correct substitution and multiplication of their values form part b) – can be unsimplified
	Using $y' = mx' + k$ $3x - 2mx - 2k = m(4x - 3mx - 3k) + k$	**(m1)**		Substitution into $y' = mx' + k$ to obtain a quadratic in m – can be unsimplified
	Giving $3(m-1)^2x + 3(m-1)k = 0$			
	Hence $m = 1$ and k can take any value So $y' = x' + k$ is invariant	**(A1)**	**3**	Fully shown - **CSO**
	Total		**9**	

Q	Solution	Mark	Total	Comment
7a)	$\begin{pmatrix}4\\-2\\3\end{pmatrix} \times \begin{pmatrix}5\\-1\\0\end{pmatrix} = \begin{pmatrix}3\\15\\6\end{pmatrix}$	**M1**		Use of vector product – two components correct
	Hence $\mathbf{n} = \begin{pmatrix}1\\5\\2\end{pmatrix}$	**A1**		Correct **n –** accept equivalents
	Hence plane is $x + 5y + 2z = d$ (1, 2, -1) on plane gives $1 + 10 - 2 = d$ Hence $d = 9$	**m1** **A1**	**4**	Substitution of valid point on the plane to find d. Dependant on finding vector product first. Correct value of d - **CAO**
b)	Substitute $x = 4$, $y = 3$ and $z = c$ to get $4 + 15 + 2c = 9$, hence $c = -5$	**B1**	**1**	Substitute point to find correct value of c
c)i)	$\mathbf{a} = \begin{pmatrix}1\\5\\2\end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix}2\\-1\\2\end{pmatrix}$			
	$\therefore$ **a.b** = 1	**B1**		Correct value of **a.b**
	$3\sqrt{30}\cos\theta = 1$	**M1**		Form appropriate scalar product equation using two normals to find cos θ
	$\cos\theta = \dfrac{1}{3\sqrt{30}}$	**A1**		cos θ correct
	$\theta = 86.5^0$	**A1**	**4**	Using $\cos^{-1}$ to find angle - **CAO**
	ALTERNATIVE to c)i)			
	$\therefore\ \|\mathbf{a}\times\mathbf{b}\| = \sqrt{269}$	**(B1)**		Correct value of |**a**x**b** |
	$3\sqrt{30}\sin\theta = \sqrt{269}$	**(M1)**		Form appropriate vector product equation using two normals to find sin θ
	$\sin\theta = \dfrac{\sqrt{269}}{3\sqrt{30}}$	**(A1)**		sin θ correct
	$\theta = 86.5^0$	**(A1)**	**(4)**	Using $\sin^{-1}$ to find angle - **CAO**

Q	Solution	Mark	Total	Comment
7c)ii)	$\begin{pmatrix}1\\5\\2\end{pmatrix} \times \begin{pmatrix}2\\-1\\2\end{pmatrix} = \begin{pmatrix}12\\2\\-11\end{pmatrix}$	M1 A1		**M1** Use of vector product – two components correct. **A1** all correct
	Common point $y = 0$ so $x + 2z = 9$ $2x + 2z = 4$ Gives (-5 , 0 , 7)	M1 A1		Attempt to find common point set one variable = 0 (or other value) Other common possibilities are $\left(0, \frac{5}{6}, \frac{29}{12}\right)$, $\left(\frac{29}{11}, \frac{14}{11}, 0\right)$ and $\left(1, 1, \frac{3}{2}\right)$
	$\left(\boldsymbol{r} - \begin{pmatrix}-5\\0\\7\end{pmatrix}\right) \times \begin{pmatrix}12\\2\\-11\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$	A1F	5	Correct format –their **a** and **b** placed in correct positions – must have scored **both** **M1**s above
	ALTERNATIVE for 7c)ii) Eqn 1 $x + 5y + 2z = 9$ Eqn 2 $2x - y + 2z = 4$ 2 x Eqn 1 – Eqn 2 gives $11y + 2z = 14$			
	Let $z = t$	(M1)		Set one variable to a parameter and attempt to find other variables
	Hence $y = \frac{14-2t}{11}$	(A1)		Correct expression for x
	and $x = \frac{29-12t}{11}$	(A1)		Correct expression for y
	$\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}29/11\\14/11\\0\end{pmatrix} + t\begin{pmatrix}-12/11\\-2/11\\1\end{pmatrix}$	(M1)		Rewriting to identify point and direction – can be implied
	Hence $\left(\boldsymbol{r} - \begin{pmatrix}29/11\\14/11\\0\end{pmatrix}\right) \times \begin{pmatrix}-12\\-2\\11\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$	(A1F)	(5)	Correct format –their **a** and **b** placed in correct positions – must have scored **both** **M1**s above
	Total		**14**	

Q	Solution	Mark	Total	Comment
8a)	Det ($\mathbf{M} - \lambda\,\mathbf{I}$) = 0 $\therefore (p-\lambda)^2 - q^2 = 0$ $\therefore p - \lambda = \pm q$ Hence eigenvalues are $p+q$, $p-q$	 M1 m1 A1,A1	 4	 Correct characteristic equation obtained Correct method used to solve equation to obtain **two distinct** solutions **A1** each correct eigenvalue expression
b)	$\lambda = p + q$ $\therefore \begin{pmatrix} -q & q \\ q & -q \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ Hence $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ is an eigenvector $\lambda = p - q$ $\therefore \begin{pmatrix} q & q \\ q & q \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ Hence $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ is an eigenvector	 M1 A1 A1	 3	 **Correct** equation seen or implied by correct matrix equation. Either $-qx + qy = 0$ or $qx + qy = 0$ One eigenvector correct Second eigenvector correct
c)	$\mathbf{D} = \begin{pmatrix} p+q & 0 \\ 0 & p-q \end{pmatrix}$ $\mathbf{U} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$	B1F B1F	 2	Use of their eigenvalues– must have first **M1** in part a) Use of their numerical eigenvectors – must have first **M1**from part a) **and** part b**)**. Columns must correspond to **D**
d)	$\mathbf{M} = \mathbf{UDU}^{-1}$ Combines $\mathbf{M}^n = \mathbf{UDU}^{-1}\,\mathbf{UDU}^{-1}\ldots.\mathbf{UDU}^{-1}$ and $\mathbf{U}^{-1}\mathbf{U} = \mathbf{I}$ to simplify Extends the process fully to give $\mathbf{M}^n = \mathbf{U}\ \mathbf{DDD}\ldots.\mathbf{U}^{-1} = \mathbf{UD}^n\mathbf{U}^{-1}$	 M1 A1	 2	 **M1 -** Idea of repeated triples and attempt at simplification using $\mathbf{U}^{-1}\mathbf{U} = \mathbf{I}$. Allow **M1** if shown for a particular value of n. **A1** Repeated use of $\mathbf{U}^{-1}\mathbf{U} = \mathbf{I}$ and hence result
e)	$\mathbf{U}^{-1} = \begin{pmatrix} 0.5 & 0.5 \\ 0.5 & -0.5 \end{pmatrix}$ Use of $p+q = 1$ and $p - q = 0.2$ $\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0.2^n \end{pmatrix}\begin{pmatrix} 0.5 & 0.5 \\ 0.5 & -0.5 \end{pmatrix}$ $= \frac{1}{2}\begin{pmatrix} 1+0.2^n & 1-0.2^n \\ 1-0.2^n & 1+0.2^n \end{pmatrix}$ $\mathbf{L} = \begin{pmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix}$	B1 M1 A1 A1	 4	Correct $\mathbf{U}^{-1}$ seen and used in part d) or e) **M1** Multiplying three appropriate matrices ($\mathbf{UD}^n\mathbf{U}^{-1}$) together Obtains the correct single 2 by 2 matrix Correct **L** obtained by letting n approach infinity **- CSO**

	ALTERNATIVE for e) $\mathbf{U}^{-1} = \begin{pmatrix} 0.5 & 0.5 \\ 0.5 & -0.5 \end{pmatrix}$ Use of $p + q = 1$ and $p - q = 0.2$ $\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0.5 & 0.5 \\ 0.5 & -0.5 \end{pmatrix}$ $\mathbf{L} = \begin{pmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix}$	**(B1)** **(M1)** **(A1)** **(A1)**	**(4)**	Correct $\mathbf{U}^{-1}$ seen to be used in part d) or e) **M1** Multiplying three appropriate numerical matrices ($\mathbf{U}\mathbf{D}^n\mathbf{U}^{-1}$) to get a single 2 by 2 matrix **A1** $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ identified as $\mathbf{D}^n$ Correct **L - CSO**
	NB – ALTERNATIVES for c) $\mathbf{D} = \begin{pmatrix} p-q & 0 \\ 0 & p+q \end{pmatrix}$ $\mathbf{U} = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$	**(B1F)** **(B1F)**	**(2)**	Use of their eigenvalues– must have first **M1** in part a) Use of their numerical eigenvectors – must have first **M1** from part a) **and** part b**)**. Columns must correspond to **D**
	Correspondingly in e) $\mathbf{U}^{-1} = \begin{pmatrix} 0.5 & -0.5 \\ 0.5 & 0.5 \end{pmatrix}$ Limiting $\mathbf{D}^n = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ Method as before leading to answer $\mathbf{L} = \begin{pmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix}$	**(B1)** **(M1)** **(A1)** **(A1)**	**(4)**	Correct $\mathbf{U}^{-1}$ seen to be used in part d) or e) Marks allocated as above dependant on method chosen
	Total		**15**	
	TOTAL		**75**	