

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
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8	
TOTAL	



General Certificate of Education  
Advanced Level Examination  
June 2013

# Mathematics

# MPC4

## Unit Pure Core 4

Monday 10 June 2013 9.00 am to 10.30 am

**For this paper you must have:**

- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

### Time allowed

- 1 hour 30 minutes

### Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

### Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

### Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.



J U N 1 3 M P C 4 0 1

Answer **all** questions.

Answer each question in the space provided for that question.

**1 (a) (i)** Express  $\frac{5 - 8x}{(2 + x)(1 - 3x)}$  in the form  $\frac{A}{2 + x} + \frac{B}{1 - 3x}$ , where  $A$  and  $B$  are integers. (3 marks)

**(ii)** Hence show that  $\int_{-1}^0 \frac{5 - 8x}{(2 + x)(1 - 3x)} dx = p \ln 2$ , where  $p$  is rational. (4 marks)

**(b) (i)** Given that  $\frac{9 - 18x - 6x^2}{2 - 5x - 3x^2}$  can be written as  $C + \frac{5 - 8x}{2 - 5x - 3x^2}$ , find the value of  $C$ . (1 mark)

**(ii)** Hence find the exact value of the area of the region bounded by the curve  $y = \frac{9 - 18x - 6x^2}{2 - 5x - 3x^2}$ , the  $x$ -axis and the lines  $x = -1$  and  $x = 0$ .

You may assume that  $y > 0$  when  $-1 \leq x \leq 0$ . (2 marks)

QUESTION  
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**2** The acute angles  $\alpha$  and  $\beta$  are given by  $\tan \alpha = \frac{2}{\sqrt{5}}$  and  $\tan \beta = \frac{1}{2}$ .

**(a) (i)** Show that  $\sin \alpha = \frac{2}{3}$ , and find the exact value of  $\cos \alpha$ . (2 marks)

**(ii)** Hence find the exact value of  $\sin 2\alpha$ . (2 marks)

**(b)** Show that the exact value of  $\cos(\alpha - \beta)$  can be expressed as  $\frac{2}{15}(k + \sqrt{5})$ , where  $k$  is an integer. (4 marks)

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**3 (a)** Find the binomial expansion of  $(1 + 6x)^{-\frac{1}{3}}$  up to and including the term in  $x^2$ .  
(2 marks)

**(b) (i)** Find the binomial expansion of  $(27 + 6x)^{-\frac{1}{3}}$  up to and including the term in  $x^2$ ,  
simplifying the coefficients. (3 marks)

**(ii)** Given that  $\sqrt[3]{\frac{2}{7}} = \frac{2}{\sqrt[3]{28}}$ , use your binomial expansion from part **(b)(i)** to obtain an  
approximation to  $\sqrt[3]{\frac{2}{7}}$ , giving your answer to six decimal places. (2 marks)

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- 4** A curve is defined by the parametric equations  $x = 8e^{-2t} - 4$ ,  $y = 2e^{2t} + 4$ .
- (a) Find  $\frac{dy}{dx}$  in terms of  $t$ . (3 marks)
- (b) The point  $P$ , where  $t = \ln 2$ , lies on the curve.
- (i) Find the gradient of the curve at  $P$ . (1 mark)
- (ii) Find the coordinates of  $P$ . (2 marks)
- (iii) The normal at  $P$  crosses the  $x$ -axis at the point  $Q$ . Find the coordinates of  $Q$ . (3 marks)
- (c) Find the Cartesian equation of the curve in the form  $xy + 4y - 4x = k$ , where  $k$  is an integer. (3 marks)

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- 5 The polynomial  $f(x)$  is defined by  $f(x) = 4x^3 - 11x - 3$ .
- (a) Use the Factor Theorem to show that  $(2x + 3)$  is a factor of  $f(x)$ . (2 marks)
- (b) Write  $f(x)$  in the form  $(2x + 3)(ax^2 + bx + c)$ , where  $a$ ,  $b$  and  $c$  are integers. (2 marks)
- (c) (i) Show that the equation  $2 \cos 2\theta \sin \theta + 9 \sin \theta + 3 = 0$  can be written as  $4x^3 - 11x - 3 = 0$ , where  $x = \sin \theta$ . (3 marks)
- (ii) Hence find all solutions of the equation  $2 \cos 2\theta \sin \theta + 9 \sin \theta + 3 = 0$  in the interval  $0^\circ < \theta < 360^\circ$ , giving your solutions to the nearest degree. (4 marks)

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- 6** The points  $A$ ,  $B$  and  $C$  have coordinates  $(3, -2, 4)$ ,  $(1, -5, 6)$  and  $(-4, 5, -1)$  respectively.

The line  $l$  passes through  $A$  and has equation  $\mathbf{r} = \begin{bmatrix} 3 \\ -2 \\ 4 \end{bmatrix} + \lambda \begin{bmatrix} 7 \\ -7 \\ 5 \end{bmatrix}$ .

- (a) Show that the point  $C$  lies on the line  $l$ . (2 marks)
- (b) Find a vector equation of the line that passes through points  $A$  and  $B$ . (3 marks)
- (c) The point  $D$  lies on the line through  $A$  and  $B$  such that the angle  $CDA$  is a right angle.  
Find the coordinates of  $D$ . (5 marks)
- (d) The point  $E$  lies on the line through  $A$  and  $B$  such that the area of triangle  $ACE$  is three times the area of triangle  $ACD$ .  
Find the coordinates of the two possible positions of  $E$ . (4 marks)

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**7** The height of the tide in a certain harbour is  $h$  metres at time  $t$  hours. Successive high tides occur every 12 hours.

The **rate of change** of the height of the tide can be modelled by a function of the form  $a \cos(kt)$ , where  $a$  and  $k$  are constants. The largest value of this rate of change is 1.3 metres per hour.

Write down a differential equation in the variables  $h$  and  $t$ . State the values of the constants  $a$  and  $k$ . (3 marks)

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**8 (a)** Find  $\int t \cos\left(\frac{\pi}{4}t\right) dt$ . (4 marks)

**(b)** The platform of a theme park ride oscillates vertically. For the first 75 seconds of the ride,

$$\frac{dx}{dt} = \frac{t \cos\left(\frac{\pi}{4}t\right)}{32x}$$

where  $x$  metres is the height of the platform above the ground after time  $t$  seconds.

At  $t = 0$ , the height of the platform above the ground is 4 metres.

Find the height of the platform after 45 seconds, giving your answer to the nearest centimetre. (6 marks)

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**END OF QUESTIONS**

