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General Certificate of Education (A-level)
June 2013

Mathematics MM04

(Specification 6360)

Mechanics 4

Final

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

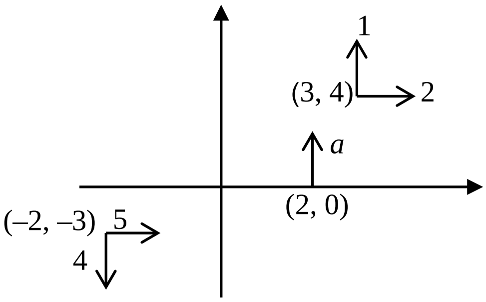
Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

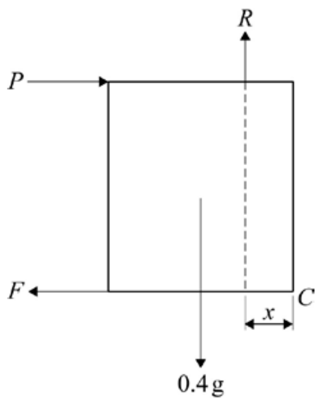
Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

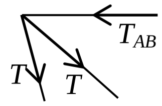
Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
1(a)	$\int xy \, \mathrm{d}x = \int_0^1 kx^{1.5} \, \mathrm{d}x$	M1		Use of $\int xy \, \mathrm{d}x$ or full first principles approach using strips and moments leading to appropriate integral
	$= \left[\dfrac{kx^{2.5}}{2.5}\right]_0^1$			
	$= \dfrac{k}{2.5} = \dfrac{2k}{5}$	A1		Correct integration and substitution of limits
	$\Rightarrow \bar{x} = \dfrac{\int xy \, \mathrm{d}x}{\int y \, \mathrm{d}x} = \dfrac{\frac{2k}{5}}{A} = \dfrac{2k}{5A}$	A1	3	Printed answer – fully correct justification required
(b)	$\int \dfrac{1}{2} y^2 \, \mathrm{d}x = \int_0^1 \dfrac{k^2 x}{2} \, \mathrm{d}x$	M1		Use of $\int \dfrac{1}{2} y^2 \, \mathrm{d}x$ or $\int y^2 \, \mathrm{d}x$ Allow full first principles approach using strips and moments leading to appropriate integral
	$= \left[\dfrac{k^2 x^2}{4}\right]_0^1$	A1		Correct integration using fully correct integral
	$= \dfrac{k^2}{4}$			
	$\Rightarrow \bar{y} = \dfrac{k^2}{4A}$	A1	3	Correct substitution of limits and division by area to obtain correct answer
(c)	On line $y = x \Rightarrow \bar{x} = \bar{y}$			
	$\dfrac{2k}{5A} = \dfrac{k^2}{4A}$	M1		Use of $\bar{x} = \bar{y}$
	$k = \dfrac{8}{5}$	A1	2	CAO
	Total		**8**	

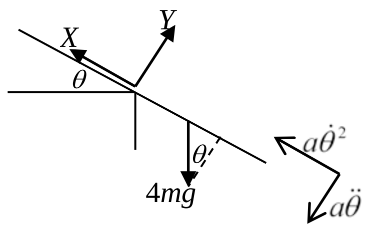
Q	Solution	Marks	Total	Comments
2				
(a)	Taking moments about *O*:			
	$a(2)-2(4)+1(3)+5(3)+4(2) = 2a + 18$	M1 A2,1		M1 for at least two correct F × *d* pairings A1 all pairs correct, A1 all signs correct 2*a* + 18 seen implies M1A2 If **r** × **F** used then award M1 for first correct moment, A1 for each of the others (23**k**, 2*a***k** and –5**k**)
	Couple magnitude $24\,\text{N}m \Rightarrow C = \pm 24$			
	$\Rightarrow 2a+18 = 24$ or -24	M1		Forms equation and finds one solution to 'their total moment' = 24
	$\Rightarrow a = 3$ or -21	A2,1	6	Both correct values for A2 **NB -** A1 only possible if both ± 24 are considered and one solution is correct
	$a = 3$, $\mathbf{F} = \begin{bmatrix} 7 \\ 0 \end{bmatrix}$	B1F		ft part (a) values – **only if** both M1s are scored
	$a = -21$, $\mathbf{F} = \begin{bmatrix} 7 \\ -24 \end{bmatrix}$	B1F	2	
	Total		**8**	

Q	Solution	Marks	Total	Comments
3	On point of sliding: 			
	$\left.\begin{array}{l} P = F \\ R = 0.4g \\ F = \mu R \end{array}\right\} P = 0.4g\mu$	B1		Obtaining P in terms of μ
	Moments about C:			
	$P(0.28) + Rx = 0.4g(0.10)$	M1		M1 Forming a moment equation at least one term correct
	$(0.4g\mu)(0.28) + (0.4g)x = 0.4g(0.10)$	A2,1		A1 – two terms correct A2 - all terms correct x, μ only
	$x > 0 \Rightarrow x = 0.1 - 0.28\mu > 0$	M1		Using $x > 0$
	$\mu < \dfrac{0.1}{0.28} = \dfrac{10}{28} = \dfrac{5}{14}$			
	$k = \dfrac{5}{14}$ or 0.357	A1	6	CAO
	Alternative			
	To slide: $\left.\begin{array}{l} F = \mu R \\ R = 0.4g \\ F = P \end{array}\right\} P = 3.92\mu$	(B1)		Obtaining P in terms of μ
	Consider toppling: P needed			
	Take moments about C to find P: $0.4g(0.10) = P(0.28)$	(M1A1)		M1 - One side correct, A1 all correct
	$1.4 = P$	(A1)		A1 - Correct P value
	$P_{\text{sliding}} < P_{\text{toppling}}$ Slides before topples $\Rightarrow 3.92\mu < 1.4$	(M1)		Inequality for μ using $P_{\text{sliding}} < P_{\text{toppling}}$
	$\mu < \dfrac{1.4}{3.92} = \dfrac{5}{14} = k$	(A1)	(6)	Accept 0.357 (NB M0 if inequality reversed in equation)
	Total		**6**	

Q	Solution	Marks	Total	Comments
4(a)	Using $T_{AC} = T_{BC} = T_1$ and resolving vertically at C to give $2T_1 \cos 60° = x$	B1		Resolves at C to obtain x
	Using $T_{AD} = T_{BD} = T_2$ and resolving vertically at D to give $2T_2 \cos 30° = y$	B1		Resolves at D to obtain y
	Using $T_1 = T_2 = T$ to get	M1		Uses full symmetry to directly compare
	$x : y = T : \sqrt{3}T = 1 : \sqrt{3}$	A1	4	expressions and establish ratio stated
(b)	Resolve vertically at A (or B)			
	$T_{AD} \sin 60° + T_{AC} \sin 30° = 100$	M1		M1 -Resolving – all terms present
	$T \sin 60° + T \sin 30° = 100$	A1		A1 - Use of equal tensions
	$T = \dfrac{100}{\sin 60° + \sin 30°}$ = 73.2 N	A1	3	Or surd form equivalent, eg $\dfrac{200}{1+\sqrt{3}}$, $100\sqrt{3} - 100$ etc (CAO)
	Alternative			
	Resolve vertically for system $x + y = 200$ and combines with $y = \sqrt{3}x$	(M1)		Must write down/establish both equations
	to get $x + \sqrt{3}x = 200$			
	leading to $x = \dfrac{200}{1+\sqrt{3}}$ or $y = \dfrac{200\sqrt{3}}{1+\sqrt{3}}$	(A1)		Correctly combined to find x or y
	$T_{\text{AC}} = \dfrac{200}{1+\sqrt{3}}$ from part (a)	(A1)	(3)	T_{AC} obtained - CAO
(c)				
	$T_{AB} = T_{AC} \cos 30° + T_{AD} \cos 60°$	M1		Resolves horizontally at A – with or without equal tensions
	$T_{AB} = T \cos 30° + T \cos 60°$			
	= 100 N	A1		
	AB is in compression	E1	3	CAO – must be positive
	Total		**10**	

Q	Solution	Marks	Total	Comments
5(a)(i)	$20m = \pi a^2 \rho \Rightarrow \rho = \dfrac{20m}{\pi a^2}$	B1		ρ and m linked
	Mass of elemental piece = $2\pi x\ \delta x \rho$	M1		Attempt at mass of elemental piece
	MI of elemental piece = $(2\pi x\ \delta x \rho)x^2$	A1		Use of mr^2
	MI of disc $= \int_o^a 2\pi x^3 \rho\ \mathrm{d}x = \int_0^a \dfrac{40mx^3}{a^2}\ \mathrm{d}x$			
	$= \left[\dfrac{40mx^4}{4a^2}\right]_0^a$	m1		Attempt to integrate, dependent on first M1
	$= 10ma^2$	A1	5	AG
(ii)	Using the perpendicular axis theorem	E1		Clearly stated
	$MI_{DISC\ DIA} + MI_{DISC\ DIA} = 10ma^2$	M1		Forms equation with $10ma^2$
	$MI_{DISC\ DIA} = 5ma^2$	A1	3	
(b)	$MI_{ROD\ EF} = 2ma^2$	B1		MI of rod EF correct
	$MI_{ROD\ AB} = MI_{ROD\ CD}$ $= \dfrac{4(2m)a^2}{3} = \dfrac{8ma^2}{3}$	B1		Rod AB and CD correct
	$MI_{DISC} = MI_{DISC\ DA} + 20m(2a)^2$	M1		Use of parallel axis, correct form
	$= 5ma^2 + 80ma^2$ $= 85ma^2$	A1		
	$MI_{SIGN} = 2ma^2 + \dfrac{8ma^2}{3} + \dfrac{8ma^2}{3} + 85ma^2$	M1		Sum of three rods and disc – axes consistent - must have attempted parallel axis theorem
	$= \dfrac{277ma^2}{3}$	A1	6	CAO
	Total		**14**	

Q	Solution	Marks	Total	Comments
6(a)	$4\mathbf{i} + 4\mathbf{k}$	B1	1	
(b)	$\begin{vmatrix} \mathbf{i} & 1 & 2 \\ \mathbf{j} & 0 & 3 \\ \mathbf{k} & 0 & 0 \end{vmatrix} = 3\mathbf{k}$	M1		At least one $\mathbf{r} \times \mathbf{F}$ (or $\mathbf{F} \times \mathbf{r}$) correct
	$\begin{vmatrix} \mathbf{i} & 1 & 2 \\ \mathbf{j} & 1 & -3 \\ \mathbf{k} & 0 & 0 \end{vmatrix} = -5\mathbf{k}$			
	$\begin{vmatrix} \mathbf{i} & 0 & 0 \\ \mathbf{j} & 0 & -1 \\ \mathbf{k} & 1 & 2 \end{vmatrix} = \mathbf{i}$			
	$\begin{vmatrix} \mathbf{i} & 0 & 0 \\ \mathbf{j} & 1 & 1 \\ \mathbf{k} & 1 & 2 \end{vmatrix} = \mathbf{i}$	A2,1		A1 three correct – all four correct for A2
	Total $= 2\mathbf{i} - 2\mathbf{k}$	m1 A1	 5	Totalling their four determinants – dependent on first M1(max 3/5 for $\mathbf{F} \times \mathbf{r}$)
(c)	$\mathbf{P} = -4\mathbf{i} - 4\mathbf{k}$	B1F		ft part (a): $-1 \times$ their answer from (a)
	Moment $= -2\mathbf{i} + 2\mathbf{k}$	B1F		ft part (b): $-1 \times$ their answer from (b) Moment can be implied by appropriate sum equal to **0**
	Coords $= (0, y, 0)$			
	Hence $\begin{vmatrix} \mathbf{i} & 0 & -4 \\ \mathbf{j} & y & 0 \\ \mathbf{k} & 0 & -4 \end{vmatrix} = -2\mathbf{i} + 2\mathbf{k}$	M1 A1		Forming an $\mathbf{r} \times \mathbf{F}$ equation using their **P**, point on y-axis and their moment Fully correct equation
	Determinant $= -4y\mathbf{i} + 4y\mathbf{k}$	A1F		Their $\mathbf{r} \times \mathbf{F}$ evaluated but $\mathbf{j}$ component must be 0
	$\therefore\ y = \frac{1}{2}$			
	Coords at $\left(0, \frac{1}{2}, 0\right)$	A1	6	CSO
	Total		**12**	

Q	Solution	Marks	Total	Comments
7(a)	Ratio of masses = 3 : 1 so G divides CB in ratio 1 : 3	M1		Or $\left(\sum m\right)\bar{X} = \left(\sum mX\right)$
	$\Rightarrow CG = \frac{1}{4}(4a) = a$	A1	2	Printed answer
(b)	$MI_{ROD} = \frac{3m(4a)^2}{3} = 16ma^2$	M1		Attempt to total MI of rod and particle
	$MI = m(4a)^2 = 16ma^2$	A1		Correct MI of particle
	Total = $32ma^2$	A1	3	Total correct – printed answer
(c)(i)	KE gained = $\frac{1}{2}I\dot{\theta}^2$			
	$= \frac{1}{2}(32ma^2)\dot{\theta}^2 = 16ma^2\dot{\theta}^2$	B1		Correct KE obtained
	PE lost = $mgh = 4mg\,a\sin\theta$	B1		Correct PE obtained
	Conservation of energy $\Rightarrow 16ma^2\dot{\theta}^2 = 4mg\,a\sin\theta$	M1		Equation formed – conservation of energy
	$\Rightarrow \dot{\theta}^2 = \frac{4mg\,a\sin\theta}{16ma^2}$			
	$\Rightarrow \dot{\theta}^2 = \frac{g\sin\theta}{4a}$			
	$\Rightarrow \dot{\theta} = \sqrt{\frac{g\sin\theta}{4a}}$	A1	4	Printed answer – must show convincing steps of cancelling/simplification
(ii)	Differentiating $2\dot{\theta}\ddot{\theta} = \frac{g\cos\theta\,\dot{\theta}}{4a}$	M1		Differentiating or equivalent
	$\ddot{\theta} = \frac{g\cos\theta}{8a}$	A1	2	**Alternative:** use of $C = I\ddot{\theta}$ $I\ddot{\theta} = 4mga\cos\theta$ M1 $\ddot{\theta} = \frac{4mga\cos\theta}{32ma^2} = \frac{g\cos\theta}{8a}$ A1
(iii)				
	Perp to rod: $4mg\cos\theta - Y = 4ma\ddot{\theta}$	M1A1		M1 structurally and dimensionally correct A1 – fully correct
	$Y = 4mg\cos\theta - \frac{mg}{2}\cos\theta = \frac{7mg\cos\theta}{2}$	A1		CSO
	Parallel to rod $X - 4mg\sin\theta = 4ma\dot{\theta}^2$	M1A1		M1 structurally and dimensionally correct A1 – fully correct
	$X = 4mg\sin\theta + mg\sin\theta = 5mg\sin\theta$	A1	6	CSO
	Total		**17**	
	TOTAL		**75**	