Centre Number						Candidate Number				
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For Examiner's Use	
Examiner's Initials	
Question	Mark
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TOTAL	

AQA

General Certificate of Education
Advanced Level Examination
January 2013

Mathematics MFP4

Unit Further Pure 4

Wednesday 30 January 2013 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 Two planes have equations

$$x + 2y + 2z = 5 \quad \text{and} \quad px + 3y = 10$$

where p is a non-zero constant.

Given that the acute angle, θ, between the planes is such that $\cos\theta = \frac{2}{3}$, find the value of p. *(5 marks)*

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2 It is given that $\mathbf{A}$ and $\mathbf{B}$ are 3×3 matrices such that

$$\det(\mathbf{AB}) = 24 \quad \text{and} \quad \det(\mathbf{A}^{-1}) = -3$$

(a) State the value of $\det \mathbf{A}$. *(1 mark)*

(b) A three-dimensional shape S, with volume $20\,\text{cm}^3$, is transformed using matrix $\mathbf{B}$.

Find the volume of the image of S. *(3 marks)*

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3 (a) Expand and simplify, as far as possible,

$$(\mathbf{a} - 4\mathbf{b}) \times (\mathbf{a} + 3\mathbf{b})$$

where **a** and **b** are vectors. *(3 marks)*

(b) Given that **a** and **b** are perpendicular, deduce that

$$|(\mathbf{a} - 4\mathbf{b}) \times (\mathbf{a} + 3\mathbf{b})| = \lambda|\mathbf{a}|\,|\mathbf{b}|$$

where λ is an integer. *(2 marks)*

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4 The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$$

(a) Given that $\mathbf{A}^2 = \begin{bmatrix} p & -2 & -4 \\ 5 & 6 & 4 \\ 10 & q & 9 \end{bmatrix}$, find the value of p and the value of q. *(2 marks)*

(b) Given that $\mathbf{A}^3 - 6\mathbf{A}^2 + 11\mathbf{A} - 6\mathbf{I} = \mathbf{0}$, prove that

$$\mathbf{A}^{-1} = \frac{1}{6}(\mathbf{A}^2 - 6\mathbf{A} + 11\mathbf{I})$$ *(2 marks)*

(c) Given that $\mathbf{A}^{-1} = \frac{1}{6}\begin{bmatrix} r & -2 & 2 \\ -1 & 5 & -2 \\ -2 & s & 2 \end{bmatrix}$, find the value of r and the value of s.

(2 marks)

(d) Hence, or otherwise, find the solution of the system of equations

$$\begin{aligned} x \qquad - z &= k \\ x + 2y + z &= 5 \\ 2x + 2y + 3z &= 7 \end{aligned}$$

giving your answers in terms of k. *(3 marks)*

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5 (a) By direct expansion, or otherwise, show that the value of $\begin{vmatrix} -2 & 1 & 2k \\ -1 & 1 & k+1 \\ 2 & k-1 & 1 \end{vmatrix}$ is independent of k. *(4 marks)*

(b) State, with a reason, whether the vectors

$$\begin{bmatrix} -2 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \text{ and } \begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix}$$

are linearly dependent or linearly independent. *(2 marks)*

(c) (i) State, with a reason, whether the equations

$$\begin{aligned} -2x + y + 6z &= 1 \\ -x + y + 4z &= 0 \\ 2x + 2y + z &= -1 \end{aligned}$$

are consistent or inconsistent. *(2 marks)*

(ii) The three equations given in part **(c)(i)** are the Cartesian equations of three planes.

State the geometrical configuration of these three planes. *(1 mark)*

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6 The linear transformations T_1 and T_2 are represented by the matrices

$$\mathbf{M}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \text{ and } \mathbf{M}_2 = \begin{bmatrix} \frac{1}{2} & 0 & \frac{\sqrt{3}}{2} \\ 0 & 1 & 0 \\ -\frac{\sqrt{3}}{2} & 0 & \frac{1}{2} \end{bmatrix}$$

respectively.

(a) Give a full geometrical description of the transformations:

(i) T_1 ; *(2 marks)*

(ii) T_2 . *(3 marks)*

(b) Find the matrix which represents the transformation T_1 followed by T_2 . *(2 marks)*

(c) The linear transformation T_3 is represented by the matrix

$$\mathbf{M}_3 = \begin{bmatrix} k & 2 & -1 \\ 1 & 1 & 1 \\ 3 & 4 & 1 \end{bmatrix}$$

where k is a constant.

For one particular value of k, T_3 has a line L of invariant points.

(i) Find k.

(ii) Find the Cartesian equations of L in the form $\frac{x}{p} = \frac{y}{q} = \frac{z}{r}$. *(7 marks)*

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7 The matrix $\mathbf{M}$ is defined by

$$\mathbf{M} = \begin{bmatrix} -a & 0 & a \\ 0 & 6 & 0 \\ a & 0 & 2 \end{bmatrix}$$

where a is a real number. The distinct eigenvalues of $\mathbf{M}$ are λ_1, λ_2 and λ_3 with corresponding eigenvectors $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$.

(a) Given that $\mathbf{v}_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, find λ_1. *(2 marks)*

(b) Given that $\mathbf{v}_2 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$, find the value of a. *(3 marks)*

(c) Given that $\lambda_3 = -6$, find a possible eigenvector $\mathbf{v}_3$. *(3 marks)*

(d) The matrix $\mathbf{M}$ can be expressed as $\mathbf{UDU}^{-1}$, where $\mathbf{D}$ is a diagonal matrix.

Write down possible matrices $\mathbf{D}$ and $\mathbf{U}$. *(3 marks)*

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8 The four vertices of a parallelogram $ABCD$ have coordinates

$$A(1,\ 0,\ 2),\ B(3,\ -1,\ 5),\ C(7,\ 2,\ 4) \text{ and } D(5,\ 3,\ 1)$$

(a) (i) Find $\overrightarrow{AB} \times \overrightarrow{AD}$. *(3 marks)*

(ii) Show that the area of the parallelogram is $p\sqrt{10}$, where p is an integer to be found. *(2 marks)*

(b) The diagonals AC and BD of the parallelogram meet at the point M. The line L passes through M and is perpendicular to the plane $ABCD$.

Find an equation for the line L, giving your answer in the form $(\mathbf{r} - \mathbf{u}) \times \mathbf{v} = \mathbf{0}$. *(4 marks)*

(c) The plane Π is parallel to the plane $ABCD$ and passes through the point $Q(6,\ 5,\ 17)$.

(i) Find the coordinates of the point of intersection of the line L with the plane Π. *(6 marks)*

(ii) One face of a parallelepiped is $ABCD$ and the opposite face lies in the plane Π.

Find the volume of the parallelepiped. *(3 marks)*

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