

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
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6	
7	
8	
TOTAL	



General Certificate of Education
Advanced Level Examination
January 2013

Mathematics

MFP3

Unit Further Pure 3

Friday 25 January 2013 1.30 pm to 3.00 pm

For this paper you must have:

- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed

- 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.



J A N 1 3 M F P 3 0 1

Answer **all** questions.

Answer each question in the space provided for that question.

1 It is given that $y(x)$ satisfies the differential equation

$$\frac{dy}{dx} = f(x, y)$$

where $f(x, y) = \sqrt{2x + y}$

and $y(3) = 5$

(a) Use the Euler formula

$$y_{r+1} = y_r + hf(x_r, y_r)$$

with $h = 0.2$, to obtain an approximation to $y(3.2)$, giving your answer to four decimal places. *(3 marks)*

(b) Use the formula

$$y_{r+1} = y_{r-1} + 2hf(x_r, y_r)$$

with your answer to part **(a)**, to obtain an approximation to $y(3.4)$, giving your answer to three decimal places. *(3 marks)*

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2 (a) Write down the expansion of e^{3x} in ascending powers of x up to and including the term in x^2 . *(1 mark)*

(b) Hence, or otherwise, find the term in x^2 in the expansion, in ascending powers of x , of $e^{3x}(1 + 2x)^{-\frac{3}{2}}$. *(4 marks)*

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3 It is given that the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$$

is $y = e^x(Ax + B)$. Hence find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 6e^x \quad (5 \text{ marks})$$

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4 (a) Explain why $\int_0^1 x^4 \ln x \, dx$ is an improper integral. (1 mark)

(b) Evaluate $\int_0^1 x^4 \ln x \, dx$, showing the limiting process used. (6 marks)

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6 (a) It is given that $y = \ln(e^{3x} \cos x)$.

(i) Show that $\frac{dy}{dx} = 3 - \tan x$. (3 marks)

(ii) Find $\frac{d^4y}{dx^4}$. (3 marks)

(b) Hence use Maclaurin's theorem to show that the first three non-zero terms in the expansion, in ascending powers of x , of $\ln(e^{3x} \cos x)$ are $3x - \frac{1}{2}x^2 - \frac{1}{12}x^4$. (3 marks)

(c) Write down the expansion of $\ln(1 + px)$, where p is a constant, in ascending powers of x up to and including the term in x^2 . (1 mark)

(d) (i) Find the value of p for which $\lim_{x \rightarrow 0} \left[\frac{1}{x^2} \ln \left(\frac{e^{3x} \cos x}{1 + px} \right) \right]$ exists.

(ii) Hence find the value of $\lim_{x \rightarrow 0} \left[\frac{1}{x^2} \ln \left(\frac{e^{3x} \cos x}{1 + px} \right) \right]$ when p takes the value found in part **(d)(i)**. (4 marks)

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7 (a) Find the general solution of the differential equation

$$\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 10y = e^{2t}$$

giving your answer in the form $y = f(t)$. (6 marks)

(b) Given that $x = t^{\frac{1}{2}}$, $x > 0$, $t > 0$ and y is a function of x , show that

$$\frac{d^2y}{dx^2} = 4t \frac{d^2y}{dt^2} + 2 \frac{dy}{dt} \quad (5 \text{ marks})$$

(c) Hence show that the substitution $x = t^{\frac{1}{2}}$ transforms the differential equation

$$x \frac{d^2y}{dx^2} - (12x^2 + 1) \frac{dy}{dx} + 40x^3y = 4x^3e^{2x^2}$$

into

$$\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 10y = e^{2t} \quad (2 \text{ marks})$$

(d) Hence **write down** the general solution of the differential equation

$$x \frac{d^2y}{dx^2} - (12x^2 + 1) \frac{dy}{dx} + 40x^3y = 4x^3e^{2x^2} \quad (1 \text{ mark})$$

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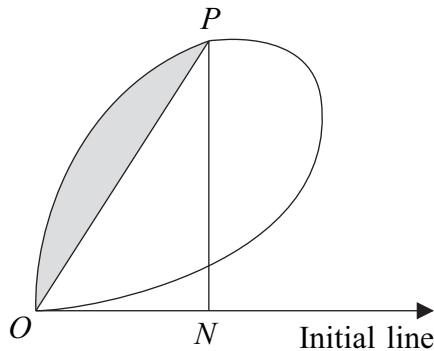
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- 8 The diagram shows a sketch of a curve.



The polar equation of the curve is

$$r = \sin 2\theta \sqrt{\left(2 + \frac{1}{2} \cos \theta\right)}, \quad 0 \leq \theta \leq \frac{\pi}{2}$$

The point P is the point of the curve at which $\theta = \frac{\pi}{3}$.

The perpendicular from P to the initial line meets the initial line at the point N .

- (a) (i) Find the exact value of r when $\theta = \frac{\pi}{3}$. (2 marks)
- (ii) Show that the polar equation of the line PN is $r = \frac{3\sqrt{3}}{8} \sec \theta$. (2 marks)
- (iii) Find the area of triangle ONP in the form $\frac{k\sqrt{3}}{128}$, where k is an integer. (2 marks)
- (b) (i) Using the substitution $u = \sin \theta$, or otherwise, find $\int \sin^n \theta \cos \theta \, d\theta$, where $n \geq 2$. (2 marks)
- (ii) Find the area of the shaded region bounded by the line OP and the arc OP of the curve. Give your answer in the form $a\pi + b\sqrt{3} + c$, where a , b and c are constants. (8 marks)



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END OF QUESTIONS



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