

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	



General Certificate of Education  
Advanced Level Examination  
June 2012

# Mathematics

# MFP3

## Unit Further Pure 3

Thursday 14 June 2012 9.00 am to 10.30 am

**For this paper you must have:**

- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

### Time allowed

- 1 hour 30 minutes

### Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

### Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

### Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.



J U N 1 2 M F P 3 0 1

Answer **all** questions.

Answer each question in the space provided for that question.

**1** The function  $y(x)$  satisfies the differential equation

$$\frac{dy}{dx} = f(x, y)$$

where  $f(x, y) = \sqrt{(2x)} + \sqrt{y}$

and  $y(2) = 9$

Use the improved Euler formula

$$y_{r+1} = y_r + \frac{1}{2}(k_1 + k_2)$$

where  $k_1 = hf(x_r, y_r)$  and  $k_2 = hf(x_r + h, y_r + k_1)$  and  $h = 0.25$ , to obtain an approximation to  $y(2.25)$ , giving your answer to two decimal places. (5 marks)

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**2 (a)** Write down the expansion of  $\sin 2x$  in ascending powers of  $x$  up to and including the term in  $x^5$ . (1 mark)

**(b)** Show that, for some value of  $k$ ,

$$\lim_{x \rightarrow 0} \left[ \frac{2x - \sin 2x}{x^2 \ln(1 + kx)} \right] = 16$$

and state this value of  $k$ . (4 marks)

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QUESTION  
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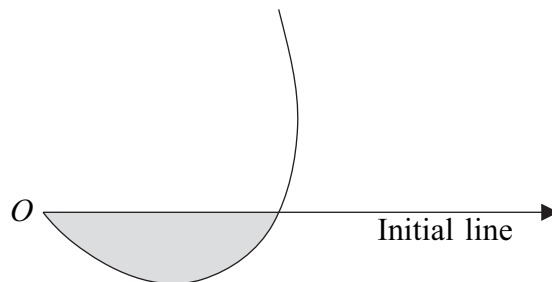
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- 3 The diagram shows a sketch of a curve  $C$ , the pole  $O$  and the initial line.



The polar equation of  $C$  is

$$r = 2\sqrt{1 + \tan \theta}, \quad -\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4}$$

Show that the area of the shaded region, bounded by the curve  $C$  and the initial line, is  $\frac{\pi}{2} - \ln 2$ . (4 marks)

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5 (a) Find  $\int x^2 e^{-x} dx$ . (4 marks)

(b) Hence evaluate  $\int_0^{\infty} x^2 e^{-x} dx$ , showing the limiting process used. (3 marks)

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- 6 It is given that  $y = \ln(1 + \sin x)$ .
- (a) Find  $\frac{dy}{dx}$ . (2 marks)
- (b) Show that  $\frac{d^2y}{dx^2} = -e^{-y}$ . (3 marks)
- (c) Express  $\frac{d^4y}{dx^4}$  in terms of  $\frac{dy}{dx}$  and  $e^{-y}$ . (3 marks)
- (d) Hence, by using Maclaurin's theorem, find the first four non-zero terms in the expansion, in ascending powers of  $x$ , of  $\ln(1 + \sin x)$ . (3 marks)

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**7 (a)** Show that the substitution  $x = e^t$  transforms the differential equation

$$x^2 \frac{d^2y}{dx^2} - 4x \frac{dy}{dx} + 6y = 3 + 20 \sin(\ln x)$$

into  $\frac{d^2y}{dt^2} - 5 \frac{dy}{dt} + 6y = 3 + 20 \sin t$  (7 marks)

**(b)** Find the general solution of the differential equation

$$\frac{d^2y}{dt^2} - 5 \frac{dy}{dt} + 6y = 3 + 20 \sin t$$
 (11 marks)

**(c)** Write down the general solution of the differential equation

$$x^2 \frac{d^2y}{dx^2} - 4x \frac{dy}{dx} + 6y = 3 + 20 \sin(\ln x)$$
 (1 mark)

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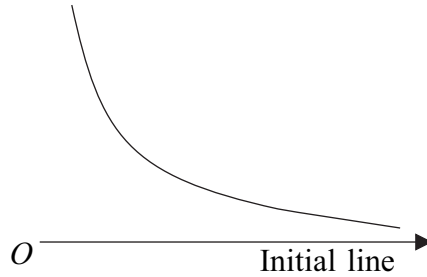
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**8 (a)** A curve has cartesian equation  $xy = 8$ . Show that the polar equation of the curve is  $r^2 = 16 \operatorname{cosec} 2\theta$ . (3 marks)

**(b)** The diagram shows a sketch of the curve,  $C$ , whose polar equation is

$$r^2 = 16 \operatorname{cosec} 2\theta, \quad 0 < \theta < \frac{\pi}{2}$$



- (i)** Find the polar coordinates of the point  $N$  which lies on the curve  $C$  and is closest to the pole  $O$ . (2 marks)
- (ii)** The circle whose polar equation is  $r = 4\sqrt{2}$  intersects the curve  $C$  at the points  $P$  and  $Q$ . Find, in an exact form, the polar coordinates of  $P$  and  $Q$ . (4 marks)
- (iii)** The obtuse angle  $PNQ$  is  $\alpha$  radians. Find the value of  $\alpha$ , giving your answer to three significant figures. (5 marks)

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**END OF QUESTIONS**



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