



**General Certificate of Education (A-level)  
June 2012**

**Mathematics**

**MFP2**

**(Specification 6360)**

**Further Pure 2**

***Mark Scheme***

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## Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
✓ or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
-x EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

## No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

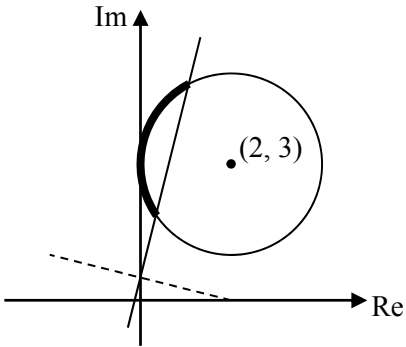
Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

## MFP2

Q	Solution	Marks	Total	Comments
1(a)	Sketch of $y = \cosh x$	B1	1	approximately correct with minimum point above the $x$ -axis, symmetrical about $y$ -axis
(b)	Attempt to factorise $(3 \cosh x - 5)(2 \cosh x + 1) = 0$ $\cosh x \neq -\frac{1}{2}$ $x = \ln\left(\frac{5}{3} + \sqrt{\frac{25}{9} - 1}\right)$ $= \pm \ln 3$  <b>Alternative:</b> $3\left(\frac{e^x + e^{-x}}{2}\right) = 5$ $3e^{2x} - 10e^x + 3 = 0$ $(3e^x - 1)(e^x - 3) = 0$ $x = \ln\frac{1}{3}$ or $\ln 3$  NB if $\cosh x = \frac{e^x + e^{-x}}{2}$ used initially, M0 until quartic in $e^x$ is factorised	M1 A1 E1 M1 A1F A1F  (M1) (A1F) (A1F)	6	or complete square or use (correct unsimplified) formula  indicated or stated (not merely neglected)  evidence of use of formula. Must see $-1$ or equivalent ft incorrect factorisation A1 for $\pm$  Correct factors  for both  M1 for $e^x - 3$ is a factor A1 if correct M1 for $3e^x - 1$ is a factor A1 if correct A1 for $x = \pm \ln 3$ E1 for showing remaining quadratic has no real roots
	<b>Total</b>		<b>7</b>	

## MFP2

Q	Solution	Marks	Total	Comments
2(a)				
(i)	Circle Correct centre Touching Im axis	B1 B1 B1	3	Convex loop Some indication of position of centre
(ii)	Straight line well to left of centre through $(0, \frac{1}{2})$ $\perp$ to line joining $(-2, 1)$ and $(2, 0)$ NB 0/3 for line parallel to $x$ -axis  0/3 for line joining the two points $(-2, 1)$ and $(2, 0)$  0/3 for line joining $(0, 0)$ to centre of circle	B1 B1 B1	3	$\frac{1}{2}$ line through $(0, \frac{1}{2})$ B0 Point approximately between 0 and 1
(b)	Minor arc indicated	B1F	1	ft incorrect position of line or circle
	<b>Total</b>		<b>7</b>	

## MFP2

Q	Solution	Marks	Total	Comments
3(a)	Attempt to put LHS over common denominator $\frac{2^{r+1}(r+1) - 2^r(r+2)}{(r+1)(r+2)}$ $= \frac{r(2^{r+1} - 2^r)}{(r+1)(r+2)}$ $= \frac{r2^r}{(r+1)(r+2)}$ must see $r2^{r+1} = 2r2^r$	M1  A1  A1	3	any form  clearly shown as AG
(b)	$\frac{2^2}{3} - \frac{2}{2}$ $\frac{2^3}{4} - \frac{2^2}{3}$ ..... $\frac{2^{31}}{32} - \frac{2^{30}}{31}$ $S_{30} = \frac{2^{31}}{32} - 1$ or $S_n = \frac{2^{n+1}}{n+2} - 1$ $= 2^{26} - 1$	M1  A1 A1	3	3 rows indicated (PI)  CAO
<b>Total</b>			<b>6</b>	
4(a)(i)	$\alpha + \beta + \gamma = 0$	B1	1	
(ii)	$\alpha\beta\gamma = -q$	B1	1	
(b)	$\alpha^3 + p\alpha + q = 0$ $\sum \alpha^3 + p\sum \alpha + 3q = 0$ $\alpha^3 + \beta^3 + \gamma^3 = 3\alpha\beta\gamma$	M1 m1 A1	3	AG
	<b>Alternative to (b)</b> Use of $(\sum \alpha)^3 = (\sum \alpha^3) + 6\alpha\beta\gamma + 3(\sum \alpha \sum \alpha\beta - 3\alpha\beta\gamma)$ Substitution of $\sum \alpha = 0$ Result	(M1) (m1) (A1)		
(c)(i)	$\beta = 4 - 7i, \gamma = -8$	B1, B1	2	
(ii)	Attempt at either $p$ or $q$ $p = 1$ $q = 520$	M1 A1F A1F	3	ft incorrect roots provided $p$ and $q$ are real
(d)	Replace $z$ by $\frac{1}{z}$ in cubic equation  $520z^3 + z^2 + 1 = 0$ coefficients must be integers	M1 A1F  A1	3	or $\sum \frac{1}{\alpha} = -\frac{p}{q}, \sum \frac{1}{\alpha\beta} = 0, \frac{1}{\alpha\beta\gamma} = -\frac{1}{q}$ ft on incorrect $p$ and/or $q$ CAO
<b>Total</b>			<b>13</b>	

## MFP2

Q	Solution	Marks	Total	Comments
5(a)	$\frac{1}{x} = \cos y$ or $\frac{1}{y} = \cos x$	M1	2	CSO
	$y = \cos^{-1} \frac{1}{x}$ ie result	A1		
(b)	$\frac{d}{dx}(\sec^{-1} x) = \frac{d}{dx}\left(\cos^{-1} \frac{1}{x}\right)$	M1	4	clearly shown (AG)  Use of $\sec y = x$ M0
	$= -\frac{1}{\sqrt{1-\frac{1}{x^2}}}$ if in terms of $u$ A0	A1		
	$\times \left(-\frac{1}{x^2}\right)$	A1		
	$= \frac{1}{\sqrt{x^4 - x^2}}$	A1		
	<b>Alternative</b> $\cos y = \frac{1}{x}$	(M1)		
	$-\sin y \frac{dy}{dx} = \frac{-1}{x^2}$	(A1)		
Substitute for $\sin y$	(A1)			
Result	(A1)			
	<b>Total</b>		<b>6</b>	

## MFP2

Q	Solution	Marks	Total	Comments
6(a)	Use of $\cosh 2x = 2 \cosh^2 x - 1$ $\text{RHS} = \frac{1}{2} \cosh 2x + \frac{1}{2} \cosh^2 2x$ $= \frac{1}{4} (1 + 2 \cosh 2x + \cosh 4x)$ <p>If substituted for both <math>\cosh 4x</math> and <math>\cosh 2x</math> in LHS M1 only, until corrected            If RHS is put in terms of <math>e^x</math>            M1 for correct substitution            A1 for correct expansion            A1 for correct result</p>	M1 A1 A1	3	or $\cosh 4x = 2 \cosh^2 2x - 1$  allow A1 for $1 + \left(\frac{dy}{dx}\right)^2 = 1 - 4 \cosh^2 x + 4 \cosh^4 x$ Incorrect form for $\cosh^2 x$ in terms of $\cosh 2x$ M1 only
(b)	$\frac{dy}{dx} = 2 \cosh x \sinh x = \sinh 2x$  <b>Or</b> $y = \left(\frac{e^x + e^{-x}}{2}\right)^2 = \frac{e^{2x} + 2 + e^{-2x}}{4}$ $\frac{dy}{dx} = \frac{2e^{2x} - 2e^{-2x}}{4}$ $= \sinh 2x$  $1 + \left(\frac{dy}{dx}\right)^2 = 1 + \sinh^2 2x = \cosh^2 2x$	M1A1  (M1) (A1) A1	3	AG
(c)	$S = 2\pi \int_{(0)}^{(\ln 2)} \cosh^2 x \cosh 2x dx$ $= 2\pi \int_0^{\ln 2} \frac{1}{4} (1 + 2 \cosh 2x + \cosh 4x) dx$ $= \frac{2\pi}{4} \left[ x + \frac{2 \sinh 2x}{2} + \frac{\sinh 4x}{4} \right]$ <p>Correct use of limits  <math>a = 128, b = 495</math></p>	M1A1 m1 A1 m1 A1,A1	7	allow even if limits missing  Integrated correctly  accept correct answers written down with no working. Only one A1 if $2\pi$ not used
<b>Total</b>			<b>13</b>	



## MFP2

Q	Solution	Marks	Total	Comments
7(a)	<p>Assume true for <math>n = k</math></p> <p>Then <math>\sum_{r=1}^{k+1} \frac{2r+1}{r^2(r+1)^2}</math></p> $= 1 - \frac{1}{(k+1)^2} + \frac{2k+3}{(k+1)^2(k+2)^2}$ $= 1 - \frac{1}{(k+1)^2} \left( 1 - \frac{2k+3}{(k+2)^2} \right)$ $= 1 - \frac{1}{(k+1)^2} \left( \frac{k^2+2k+1}{(k+2)^2} \right)$ $= 1 - \frac{1}{(k+2)^2}$ <p>True for <math>n = 1</math> LHS = RHS = <math>\frac{3}{4}</math></p> <p>Method of induction set out properly</p>	<p>M1A1</p> <p>m1</p> <p>A1</p> <p>A1</p> <p>B1</p> <p>E1</p>	<p>7</p>	<p>M1A0 if no LHS</p> <p>attempt to factorise or put over a common denominator</p> <p>any correct combination starting 1–</p> <p>must score all 6 previous marks for this mark</p>
(b)	$(n+1)^2 > 10^5 \text{ or } \frac{1}{(n+1)^2} > 10^{-5}$ <p><math>n+1 &gt; 316.2</math></p> <p><math>n &gt; 315.2</math></p> <p><math>n = 316</math></p>	<p>M1</p> <p>A1</p>	<p>2</p>	<p>Condone equals</p>
	<b>Total</b>		<b>9</b>	

## MFP2

Q	Solution	Marks	Total	Comments
8(a)	Use of $(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta$	M1	3	Stated or used
	$\cos(-n\theta) + i \sin(-n\theta) = \cos n\theta - i \sin n\theta$	A1		
	$z^n + \frac{1}{z^n} = 2 \cos n\theta$	A1		allow $\frac{2}{3}$ if this line is assumed allow if complex conjugate used AG
(b)(i)	$z^8 + 4z^4 + 6 + 4z^{-4} + z^{-8}$	B1	1	allow in retrospect
(ii)	$z^2 + \frac{1}{z^2} = 2 \cos 2\theta$ used	B1	4	Can be implied from (b)(i)
	$(2 \cos 2\theta)^4 = 2 \cos 8\theta + 8 \cos 4\theta + 6$	M1A1		
	$\cos^4 2\theta = \frac{1}{8} \cos 8\theta + \frac{1}{2} \cos 4\theta + \frac{3}{8}$	A1F		M1 for RHS A1 for whole line ft coefficients on previous line
	<b>Alternative to (b)(ii)</b>			
	$\cos^4 2\theta = \left(\frac{1 + \cos 4\theta}{2}\right)^2$	(M1) (A1)		
	$\cos^2 4\theta = \frac{1}{2}(1 + \cos 8\theta)$	(B1)		
	Final result	(A1)		
(c)	$8 \cos^4 2\theta = \cos 8\theta + 5 \rightarrow \cos 4\theta = \frac{1}{2}$	M1 A1F	3	ft provided simplifies to $\cos 4\theta = p$ CAO
	$k = \frac{1}{12}, \frac{5}{12}, \frac{7}{12}, \frac{11}{12}$	A1		
(d)	$\int_0^{\frac{\pi}{2}} \cos^4 2\theta d\theta =$			
	$\left[ \frac{\sin 8\theta}{64} + \frac{\sin 4\theta}{8} + \frac{3}{8}\theta \right]_0^{\frac{\pi}{2}}$	M1 A1F		ie their $\cos^4 2\theta$
	$= \frac{3\pi}{16}$	A1	3	AG
	<b>Total</b>		<b>14</b>	
	<b>TOTAL</b>		<b>75</b>	