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General Certificate of Education (A-level)
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Mathematics MFP4

(Specification 6360)

Further Pure 4

Final

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MFP4

Q	Solution	Marks	Total	Comments
1	Use of $ab\cos\theta$ = **a . b** = 21 $\Rightarrow \cos\theta = \dfrac{7}{5\sqrt{2}}$	M1 A1		
	$\Rightarrow \sin\theta = \dfrac{1}{5\sqrt{2}}$	B1 ft		FT exact only
	Use of $\lvert \mathbf{a} \times \mathbf{b} \rvert = ab\sin\theta = 3$	M1 A1	5	CSO
	Total		**5**	
2(a)	Reflection in $x = z$	M1 A1	2	
(b)	Rotation about the y-axis	M1 A1		
	Through $\cos^{-1}0.6$ ($\approx 53.13^{\circ}$)	A1	3	Ignore direction
	Total		**5**	
3(a)	Char. Eqn. is $\lambda^2 - 8\lambda - 9 = 0$	M1		Attempted
	Quadratic solved to get two roots	dM1		
	$\Rightarrow \lambda = 9, -1$	A1		
	Subst^{g}. back λ (at least once) :	M1		
	$\lambda = 9 \Rightarrow -x + y = 0$ $\Rightarrow \lambda = 9$ has evecs. $\alpha\begin{bmatrix}1\\1\end{bmatrix}$	A1		any $\alpha \neq 0$
	$\lambda = -1 \Rightarrow x + y = 0$ $\Rightarrow \lambda = -1$ has evecs. $\beta\begin{bmatrix}1\\-1\end{bmatrix}$	A1	6	any $\beta \neq 0$
(b)	(0, 0)	B1	1	
	Total		**7**	

MFP4 (cont)

Q	Solution	Marks	Total	Comments
4(a)	$\mathbf{X}\,\mathbf{X}^{\mathrm{T}} = \begin{bmatrix} 3 & x \\ -1 & 7 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ x & 7 \end{bmatrix}$	M1		Attempted multn. with $\mathbf{X}^{\mathrm{T}}$ correct
	$= \begin{bmatrix} x^2+9 & 7x-3 \\ 7x-3 & 50 \end{bmatrix}$	A1	2	
(b)	$\mathbf{X}^{\mathrm{T}}\mathbf{X} = \begin{bmatrix} 3 & -1 \\ x & 7 \end{bmatrix}\begin{bmatrix} 3 & x \\ -1 & 7 \end{bmatrix}$			
	$= \begin{bmatrix} 10 & 3x-7 \\ 3x-7 & x^2+49 \end{bmatrix}$	M1		Good attempt
	$\mathbf{X}\,\mathbf{X}^{\mathrm{T}} - \mathbf{X}^{\mathrm{T}}\mathbf{X} = \begin{bmatrix} x^2-1 & 4x+4 \\ 4x+4 & 1-x^2 \end{bmatrix}$	M1		Good attempt
	$\mathrm{Det}(\mathbf{X}\,\mathbf{X}^{\mathrm{T}} - \mathbf{X}^{\mathrm{T}}\mathbf{X}) = \begin{vmatrix} x^2-1 & 4x+4 \\ 4x+4 & 1-x^2 \end{vmatrix}$			
	$= (x+1)^2 \begin{vmatrix} x-1 & 4 \\ 4 & 1-x \end{vmatrix}$	M1		Good attempt to factorise/expand the determinant
	$= -(x+1)^2\left\{(x-1)^2+16\right\} \le 0$ for all real x	E1	4	Explained/demonstrated fully
(c)	$x = -1$	B1	1	CSO
	Total		**7**	

MFP4 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$\begin{vmatrix} 2 & n & 1 \\ 3 & -1 & n \\ -1 & 7 & 1 \end{vmatrix}$			
	Expanding the det. of the coefft. mtx. Setting it = 0 Obtaining & solving a quadratic eqn. in n	M1 M1 M1		
	$0 = n^2 + 17n - 18 = (n + 18)(n - 1)$ $\Rightarrow\ n = 1, -18$	A1	4	CSO
(b)	$n = 1$ gives $2x + y + z = 5$ $3x - y + z = 1$ $-x + 7y + z = 1$	B1		ft their chosen integer n
	Eliminating one variable from a pair of equations, twice	M1		
	e.g. ② – ① $\Rightarrow$ $x - 2y = -4$ and ② – ③ $\Rightarrow$ $4x - 8y = 0$	A1 ft A1 ft		
	Inconsistency clearly demonstrated from fully correct working	E1		
	3 planes have no common intersection (or form a Δ^r prism)	B1 ft	**6**	Also ft "3 planes meet in a common line" or "3 planes form a sheaf" if consistency conclusion made
	Total		**10**	

MFP4 (cont)

Q	Solution	Marks	Total	Comments
6(a)	Use of scalar product on $\begin{pmatrix}2\\1\\7\end{pmatrix}$ and $\begin{pmatrix}3\\1\\-4\end{pmatrix}$	M1		
	Sc.Prod. = $\pm$ 21	B1		
	Moduli $\sqrt{54}$ and $\sqrt{26}$ correct	B1		Accept 7.348… & 5.099…
	AWRT 56°	A1	4	From correct working
(b)	$2x + y + 7t = 10$ and $3x + y - 4t = 7$ noted or used	M1		
	Eliminating (say) y to get x as a fn. of t	M1		
	$x = 11t - 3$	A1		CAO
	Subst^g. back for y	M1		
	$y = 16 - 29t$	A1		CAO
	$\frac{x+3}{11} = \frac{y-16}{-29} = z \quad (= t)$	B1 ft	6	
(c)	Attempt at either $\begin{pmatrix}\lambda+20\\9\lambda-1\\4\lambda+7\end{pmatrix} \bullet \begin{pmatrix}2\\1\\7\end{pmatrix} = 10$ or $\begin{pmatrix}\lambda+20\\9\lambda-1\\4\lambda+7\end{pmatrix} \bullet \begin{pmatrix}3\\1\\-4\end{pmatrix} = 7$	M1		
	Solving either $40 + 2\lambda - 1 + 9\lambda + 49 + 28\lambda = 10$ or $60 + 3\lambda - 1 + 9\lambda - 28 - 16\lambda = 7$	M1		
	$\lambda_1 = -2$ $\quad\quad$ $\lambda_2 = 6$	A1A1		
	$P = (18, -19, -1)$ and $Q = (26, 53, 31)$			
	$PQ = \sqrt{8^2 + 72^2 + 32^2} = \sqrt{6272} = 56\sqrt{2}$	M1A1	6	**NB** P, Q not required: $d = \lvert \lambda_1 - \lambda_2 \rvert \times \lvert \mathbf{i} + 9\mathbf{j} + 4\mathbf{k} \rvert$ $= 8 \times 7\sqrt{2} = 56\sqrt{2}$ **M1 A1**
	Total		**16**	

MFP4 (cont)

Q	Solution	Marks	Total	Comments
7(a)	$\begin{bmatrix} c & s \\ -s & c \end{bmatrix}$	B1		
	$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c & s \\ -s & c \end{bmatrix}\begin{bmatrix} X \\ Y \end{bmatrix}$	M1		
	$= \begin{bmatrix} cX + sY \\ -sX + cY \end{bmatrix}$	A1	3	
(b)(i)	$(cX+sY)^2 - 6(cX+sY)(-sX+cY) - 7(-sX+cY)^2 = 8$			
	$(c^2X^2 + 2csXY + s^2Y^2) - 6([c^2 - s^2]XY + sc[Y^2 - X^2])$ $-7(s^2X^2 - 2csXY + c^2Y^2) = 8$	M1		Substn. for *x* & *y* in eqn. *and* multiplying out
	$p = c^2 + 6sc - 7s^2$	A1		
	$q = 16cs - 6(c^2 - s^2)$	A1		AG
	$r = s^2 - 6sc - 7c^2$	A1	4	
(ii)	Factorising: $3s^2 + 8sc - 3c^2 = (3s - c)(s + 3c) = 0$	M1A1		Or by double angles
	Deducing a tan value	M1		
	$\tan\theta = \frac{1}{3}$ (θ acute)	A1		
	$\cos\theta = \frac{3}{\sqrt{10}}$, $\sin\theta = \frac{1}{\sqrt{10}}$	A1		Both
	Subst^{g}. sensible values back for *p* and *r*	M1		
	$2X^2 - 8Y^2 = 8$	A1		
	$\frac{X^2}{2^2} - \frac{Y^2}{1^2} = 1$	A1	8	CSO
(iii)	Since C' is a hyperbola, and it is just C rotated, it follows that C is a hyperbola	E1	1	
	Total		**16**	

MFP4 (cont)

Q	Solution	Marks	Total	Comments
8	For considering $\begin{vmatrix} 1 & 2n & n-1 \\ n & 2n^2+n & n^2-1 \\ n^2 & -1 & 1-n^2 \end{vmatrix}$	B1		Or by scalar triple product
	$= (n-1)\begin{vmatrix} 1 & 2n & 1 \\ n & 2n^2+n & n+1 \\ n^2 & -1 & -1-n \end{vmatrix}$	M1A1		For 1st factor
	$= (n-1)\begin{vmatrix} 1 & 2n & 1 \\ n & 2n^2+n & n+1 \\ n(n+1) & (n+1)(2n-1) & 0 \end{vmatrix}$ $R_3' = R_3 + R_2$	M1		Row ops. for 2nd factor
	$= (n-1)(n+1)\begin{vmatrix} 1 & 2n & 1 \\ n & 2n^2+n & n+1 \\ n & 2n-1 & 0 \end{vmatrix}$	M1A1		
	$= (n-1)(n+1)$ $\{2n^3+2n^2+2n^2-n-2n^3-n^2-2n^2-n+1\}$ **OR** $= (n-1)(n+1)\begin{vmatrix} 1 & 2n & 1 \\ 0 & n & 1 \\ n & 2n-1 & 0 \end{vmatrix}$			
	$R_2' = R_2 - nR_1$ = $(n-1)(n+1)\{2n^2-n^2-2n+1\}$	M1		Full method for remaining factors
	$= (n-1)(n+1)(n-1)^2$	A1		
	$n = -1$	B1	9	CSO **Note**: Expanding straightaway scores **B1 M1** and then **A1** for $n^4 - 2n^3 + 2n - 1$ Thereafter, **M1 A1** for 1st factor, **M1** for 2nd factor attempted and **M1** for full method for remaining factors plus **A1** and **B1 cso** at the end, as above.
	Total		**9**	
	TOTAL		**75**	