| Centre Number | | | | | | Candidate Number | | | | |
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| For Examiner's Use | |
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| Examiner's Initials | |
| Question | Mark |
| 1 | |
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| TOTAL | |

General Certificate of Education
Advanced Level Examination
January 2012

# Mathematics MFP3

## Unit Further Pure 3

## Monday 23 January 2012 9.00 am to 10.30 am

**For this paper you must have:**
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

**Time allowed**
- 1 hour 30 minutes

**Instructions**
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

**Information**
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

**Advice**
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions in the spaces provided.

**1** The function $y(x)$ satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x, y)$$

where $$\mathrm{f}(x, y) = \frac{y - x}{y^2 + x}$$

and $$y(1) = 2$$

**(a)** Use the Euler formula

$$y_{r+1} = y_r + h\,\mathrm{f}(x_r, y_r)$$

with $h = 0.1$, to obtain an approximation to $y(1.1)$. *(3 marks)*

**(b)** Use the formula

$$y_{r+1} = y_{r-1} + 2h\,\mathrm{f}(x_r, y_r)$$

with your answer to part **(a)**, to obtain an approximation to $y(1.2)$, giving your answer to three decimal places. *(3 marks)*

QUESTION PART REFERENCE

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**2** Find

$$\lim_{x \to 0} \left[ \frac{\sqrt{4+x} - 2}{x + x^2} \right]$$

*(3 marks)*

QUESTION PART REFERENCE

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**3** Solve the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 10y = 26e^x$$

given that $y = 5$ and $\frac{dy}{dx} = 11$ when $x = 0$. Give your answer in the form $y = f(x)$. *(10 marks)*

QUESTION PART REFERENCE

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**4 (a)** By using an integrating factor, find the general solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{2}{x}y = \ln x \qquad \textit{(7 marks)}$$

**(b)** Hence, given that $y \to 0$ as $x \to 0$, find the value of $y$ when $x = 1$. *(3 marks)*

QUESTION PART REFERENCE

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**5 (a)** Explain why $\int_{\frac{1}{2}}^{\infty} \frac{x(1-2x)}{x^2+3e^{4x}}\,dx$ is an improper integral. *(1 mark)*

**(b)** By using the substitution $u = x^2e^{-4x} + 3$, find

$$\int \frac{x(1-2x)}{x^2+3e^{4x}}\,dx$$ *(3 marks)*

**(c)** Hence evaluate $\int_{\frac{1}{2}}^{\infty} \frac{x(1-2x)}{x^2+3e^{4x}}\,dx$, showing the limiting process used. *(4 marks)*

QUESTION PART REFERENCE

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**6 (a)** Given that $y = \ln \cos 2x$, find $\dfrac{\mathrm{d}^4 y}{\mathrm{d}x^4}$. *(6 marks)*

**(b)** Use Maclaurin's theorem to show that the first two non-zero terms in the expansion, in ascending powers of $x$, of $\ln \cos 2x$ are $-2x^2 - \frac{4}{3}x^4$. *(3 marks)*

**(c)** **Hence** find the first two non-zero terms in the expansion, in ascending powers of $x$, of $\ln \sec^2 2x$. *(2 marks)*

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**7** It is given that, for $x \neq 0$, $y$ satisfies the differential equation

$$x\frac{d^2y}{dx^2} + 2(3x+1)\frac{dy}{dx} + 3y(3x+2) = 18x$$

**(a)** Show that the substitution $u = xy$ transforms this differential equation into

$$\frac{d^2u}{dx^2} + 6\frac{du}{dx} + 9u = 18x \qquad \textit{(4 marks)}$$

**(b)** Hence find the general solution of the differential equation

$$x\frac{d^2y}{dx^2} + 2(3x+1)\frac{dy}{dx} + 3y(3x+2) = 18x$$

giving your answer in the form $y = f(x)$. *(8 marks)*

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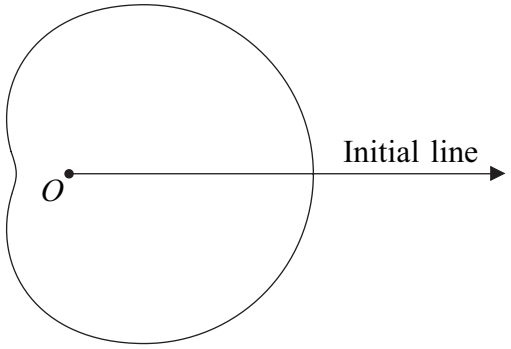
**8** The diagram shows a sketch of the curve $C$ with polar equation

$$r = 3 + 2\cos\theta\,, \quad 0 \leqslant \theta \leqslant 2\pi$$




**(a)** Find the area of the region bounded by the curve $C$. *(6 marks)*

**(b)** A circle, whose cartesian equation is $(x-4)^2 + y^2 = 16$, intersects the curve $C$ at the points $A$ and $B$.

**(i)** Find, in surd form, the length of $AB$. *(6 marks)*

**(ii)** Find the perimeter of the segment $AOB$ of the circle, where $O$ is the pole. *(3 marks)*

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**END OF QUESTIONS**

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