Centre Number			Candidate Number		
Surname					
Other Names					
Candidate Signature					



General Certificate of Education Advanced Level Examination June 2011

Mathematics

MFP3

Unit Further Pure 3

Thursday 16 June 2011 1.30 pm to 3.00 pm

For this paper you must have:

• the blue AQA booklet of formulae and statistical tables. You may use a graphics calculator.

Time allowed

• 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer all questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice

 Unless stated otherwise, you may quote formulae, without proof, from the booklet.

For Examiner's Use						
Examiner's Initials						
Question	Mark					
1						
2						
3						
4						
5						
6						
7						
TOTAL						



Answer all questions in the spaces provided.

1 The function y(x) satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x, y)$$

where

$$f(x, y) = x + \ln(1 + y)$$

and

$$y(2) = 1$$

Use the improved Euler formula

$$y_{r+1} = y_r + \frac{1}{2}(k_1 + k_2)$$

where $k_1 = hf(x_r, y_r)$ and $k_2 = hf(x_r + h, y_r + k_1)$ and h = 0.2, to obtain an approximation to y(2.2), giving your answer to four decimal places. (5 marks)

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2 (a) Find the values of the constants p and q for which $p + qxe^{-2x}$ is a particular integral of the differential equation

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 4 - 9e^{-2x}$$
 (5 marks)

- (b) Hence find the general solution of this differential equation. (3 marks)
- (c) Hence express y in terms of x, given that y = 4 when x = 0 and that $\frac{dy}{dx} \to 0$ as $x \to \infty$.

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3 (a	$\int x^2 \ln x \mathrm{d}x.$	(3 marks)
(b	Explain why $\int_0^e x^2 \ln x dx$ is an improper integral.	(1 mark)
(с	Evaluate $\int_0^e x^2 \ln x dx$, showing the limiting process used.	(3 marks)
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4	By using an	integrating	factor,	find the	solution	of the	differential	equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + (\cot x)y = \sin 2x, \quad 0 < x < \frac{\pi}{2}$$

given that
$$y = \frac{1}{2}$$
 when $x = \frac{\pi}{6}$. (10 marks)

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(You may leave your expression for
$$\frac{d^2y}{dx^2}$$
 unsimplified.) (4 marks)

- (b) Hence, using Maclaurin's theorem, find the first two non-zero terms in the expansion, in ascending powers of x, of $\ln(1 + 2\tan x)$. (2 marks)
- (c) Find

$$\lim_{x \to 0} \left[\frac{\ln(1 + 2\tan x)}{\ln(1 - x)} \right] \tag{4 marks}$$

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6 A differential equation is given by

$$(x^3 + 1)\frac{d^2y}{dx^2} - 3x^2\frac{dy}{dx} = 2 - 4x^3$$

(a) Show that the substitution

$$u = \frac{\mathrm{d}y}{\mathrm{d}x} - 2x$$

transforms this differential equation into

$$(x^3 + 1)\frac{\mathrm{d}u}{\mathrm{d}x} = 3x^2u\tag{4 marks}$$

(b) Hence find the general solution of the differential equation

$$(x^3 + 1)\frac{d^2y}{dx^2} - 3x^2\frac{dy}{dx} = 2 - 4x^3$$

giving your answer in the form y = f(x). (8 marks)

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7 The curve C_1 is defined by $r = 2 \sin \theta$, $0 \le \theta < \frac{\pi}{2}$.

The curve C_2 is defined by $r = \tan \theta$, $0 \le \theta < \frac{\pi}{2}$.

(a) Find a cartesian equation of C_1 .

(3 marks)

- (b) (i) Prove that the curves C_1 and C_2 meet at the pole O and at one other point, P, in the given domain. State the polar coordinates of P. (4 marks)
 - (ii) The point A is the point on the curve C_1 at which $\theta = \frac{\pi}{4}$.

The point B is the point on the curve C_2 at which $\theta = \frac{\pi}{4}$.

Determine which of the points A or B is further away from the pole O, justifying your answer. (2 marks)

(iii) Show that the area of the region bounded by the arc OP of C_1 and the arc OP of C_2 is $a\pi + b\sqrt{3}$, where a and b are rational numbers. (10 marks)

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