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General Certificate of Education (A-level)
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Mathematics MPC4

(Specification 6360)

Pure Core 4

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MPC4

Q	Solution	Marks	Total	Comments
1(a)	$R=\sqrt{29}$	B1		Accept 5.4 or 5.38, 5.39, 5.385….
	$R\sin\alpha=5$ or $R\cos\alpha=2$ or $\tan\alpha=\frac{5}{2}$	M1		
	$\alpha=68.2^{\circ}$	A1	3	Condone $\alpha=68.20^{\circ}$
(b)(i)	(maximum value =) $\sqrt{29}$	B1ft	1	ft on R
(ii)	$\sin(x+\alpha)=1$	M1		Or $x+\alpha=90$, $x+\alpha=\frac{\pi}{2}$
	$x=21.8^{\circ}$ only	A1	2	No ISW
	Total		**6**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
2 (a)(i)	$\mathrm{f}\left(-\frac{1}{3}\right)=9\left(-\frac{1}{3}\right)^3+18\left(-\frac{1}{3}\right)^2-\left(-\frac{1}{3}\right)-2$ $=9\left(-\frac{1}{27}\right)+18\left(\frac{1}{9}\right)-\left(-\frac{1}{3}\right)-2$ $=-\frac{1}{3}+2+\frac{1}{3}-2=0$	M1		$\mathrm{f}\left(-\frac{1}{3}\right)$ attempted **NOT** long division
	$\Rightarrow(3x+1)$ is a factor	A1	2	Shown = 0 plus statement
(ii)	$(\mathbf{f}(x)=)\ (3x+1)\left(3x^2+kx-2\right)$	M1		3 and -2
	$k=5$	A1		
	$(\mathbf{f}(x)=)\ (3x+1)(3x-1)(x+2)$	A1	3	
(iii)	$9x^3+21x^2+6x=x\left(9x^2+21x+6\right)$	M1		x and attempt to factorise quadratic equation.
	$=3x(3x+1)(x+2)$	A1		Correct factors
	$\dfrac{9x^3+21x^2+6x}{\mathrm{f}(x)}=\dfrac{3x}{3x-1}$	A1	3	cso no ISW
(b)	$9\left(\frac{2}{3}\right)^3+p\left(\frac{2}{3}\right)^2-\frac{2}{3}-2=-4$	M1		Condone missing brackets, but must have $=-4$
	$p=-9$	A1	2	
			10	
2(a)(ii)	**Alternative** Using long division			
	$3x^2+5x-2$ over $3x+1\overline{)9x^3+18x^2-x-2}$	(M1)		$3x^2+ax+b$
	$9x^3+3x^2$ $\overline{15x^2-x}$ $15x^2+5x$ $\overline{-6x-2}$	(A1)		$3x^2+5x-2$
	$-6x-2$			
	$(\mathrm{f}(x)=)\ (3x+1)(3x-1)(x+2)$	(A1)	(3)	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
2(a)(iii)	**Alternative** $\dfrac{\mathrm{f}(x)+\mathrm{q}(x)}{\mathrm{f}(x)}$, where q is a quadratic expression $= 1+\dfrac{(3x+1)(x+2)}{(3x+1)(3x-1)(x+2)}$ $=1+\dfrac{1}{3x-1}$	(M1) (A1) (A1)	 (3)	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$3+9x = A(3+5x)+B(1+x)$	M1		PI by correct A and B
	$x=-1 \qquad x=-\frac{3}{5}$	m1		Substitute two values of x and solve for A and B.
	$A=3 \qquad B=-6$	A1	3	
	Alternative Equating coefficients $3+9x = A(3+5x)+B(1+x)$	(M1)		
	$3=3A+B$ $9=5A+B$	(m1)		Set up simultaneous equations and solve. Condone 1 error.
	$A=3 \qquad B=-6$	(A1)	(3)	
	Alternative Cover up rule $x=-1 \qquad A=\dfrac{3-9}{3-5}$ $x=-\dfrac{3}{5} \qquad B=\dfrac{3-\frac{27}{5}}{1-\frac{3}{5}}$	(M1)		$x=-1$ and $x=-\frac{3}{5}$ and attempt to find A and B.
	$A=3 \qquad B=-6$	(A1 A1)	(3)	SC NMS A and B both correct; 3/3 One of A and B correct 1/3
(b)	$(1+x)^{-1} = 1-x+kx^2$ $= 1-x+x^2$	M1 A1		
	$(3+5x)^{-1} = 3^{-1}\left(1+\frac{5}{3}x\right)^{-1}$	B1		
	$\left(1+\frac{5}{3}x\right)^{-1} = 1-\frac{5}{3}x+\left(\frac{5}{3}x\right)^2$ $= 1-\frac{5}{3}x+\frac{25}{9}x^2$	M1 A1		Condone missing brackets; allow one sign error
	$\dfrac{3+9x}{(1+x)(3+5x)}$ $= 3(1-x+x^2) - 6\times 3^{-1}\left(1-\dfrac{5}{3}x+\dfrac{25}{9}x^2\right)$	M1		Use PFs and simplify to $a+bx+cx^2$ **or** expand product of $(3+9x)$ and binomial expansions and simplify to $a+bx+cx^2$
	$= 1+\dfrac{1}{3}x-\dfrac{23}{9}x^2$	A1	7	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
(c)	$\frac{5x}{3} < 1$ oe or $\frac{5x}{3} > -1$ oe	M1		Condone $\le$ instead of $<$
	$\lvert x \rvert < \frac{3}{5}$ or $-\frac{3}{5} < x < \frac{3}{5}$	A1	**2**	CAO
			12	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
4(a)(i)	$\dfrac{dx}{dt} = 3e^t$ $\quad\dfrac{dy}{dt} = 2e^{2t} + 2e^{-2t}$	M1 A1		Both derivatives attempted and one correct Both correct
	$t = 0$ $\quad$ gradient $= \dfrac{4}{3}$	A1	3	cso Condone $\dfrac{dy}{dx} = \dfrac{4}{3}$
(ii)	$y = \dfrac{4}{3}(x-3)$ oe	B1ft	1	ft on non-zero gradient
(b)	$e^{2t} = \dfrac{x^2}{9}$ or $9e^{2t} = x^2$ or $e^t = \dfrac{x}{3}$ or $e^{2t} = \left(\dfrac{x}{3}\right)^2$ or $t = \ln\left(\dfrac{x}{3}\right)$ or $2t = \ln\left(\dfrac{x^2}{9}\right)$	M1		
	$y = \dfrac{x^2}{9} - \dfrac{9}{x^2}$	A1	2	Equation required
			6	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$m = 10 \times 2^{-\frac{14}{8}}$	M1		
	≈ 3 (gm)	A1	2	Condone 2.97 or better NOT 2.9 as final answer
(b)	$2^{-\frac{d}{8}} = \frac{1}{16}$	M1		
	$\frac{d}{8} = 4 \Rightarrow d = 32$	A1	2	cso
(c)	$0.01m_0 = m_0 \times 2^{-\frac{t}{8}}$	M1		m_0 can be numerical
	$\ln(0.01) = -\frac{t}{8}\ln(2)$	M1		Take logs correctly from their equation leading to a linear equation in t.
	$t = 53.15$			
	$n = 54$	A1	3	cso
			7	

Q	Solution	Marks	Total	Comments
6(a)(i)	$\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$	B1		Condone numerator as tan x + tan x
	$2\tan x + \tan x\left(1-\tan^2 x\right) = 0$	M1		Multiplying throughout by their denominator
	$\tan x = 0$			
	or $\left(2+1-\tan^2 x\right) = 0 \Rightarrow \tan^2 x = 3$	A1	3	**AG** Must show tan x = 0 **and** $\tan^2 x = 3$
	Alternative			
	$\tan 2x = \dfrac{\sin 2x}{\cos 2x} = \dfrac{2\sin x\cos x}{\cos^2 x - \sin^2 x}$			
	$\dfrac{2\sin x\cos x}{\cos^2 x-\sin^2 x} + \dfrac{\sin x}{\cos x} = 0$	(B1)		
	$2\sin x\cos^2 x + \sin x\left(\cos^2 x - \sin^2 x\right) = 0$			
	$\sin x(2\cos^2 x + \cos^2 x - \sin^2 x) = 0$	(M1)		
	$\left.\begin{array}{l}\Rightarrow \sin x = 0\\ \Rightarrow \tan x = 0\end{array}\right\}$ $\begin{array}{l}\text{and}\\ \text{and}\end{array}$ $\left.\begin{array}{r}3\cos^2 x = \sin^2 x\\ \tan^2 x = 3\end{array}\right\}$	(A1)	(3)	
(ii)	$x = 60$ **AND** $x = 120$	B1	1	Condone extra answers outside interval eg 0 and 180
(b)(i)	$2\sin x\cos x = \cos x.\mathrm{f}(x)$	M1		Where f(x) = $\cos^2 x - \sin^2 x$ or $2\cos^2 x - 1$ or $1 - 2\sin^2 x$
	$2\sin x\cos x = \cos x\left(1-2\sin^2 x\right)$	A1		
	$\left(\cos x \neq 0\right)$ $2\sin x = 1 - 2\sin^2 x$			
	$2\sin^2 x + 2\sin x - 1 = 0$	A1	3	**AG**

(ii)	$\sin x = \dfrac{-2 \pm \sqrt{4 - 4\times 2\times(-1)}}{2\times 2}$	M1		Correct use of quadratic formula or completing the square or correct factors
	$\sin x = \dfrac{-2 \pm 2\sqrt{3}}{4}$	A1		$\sqrt{12}$ must be simplified and must have $\pm$
	$\left.\begin{array}{l}\sin x = \dfrac{-1-\sqrt{3}}{2} \text{ has no solution} \\ \sin x = \dfrac{\sqrt{3}-1}{2}\end{array}\right\}$	E1	3	Reject one solution and state correct solution.
			10	

MPC4

Q	Solution	Marks	Total	Comments
7 (a)(i)	$\int \frac{\mathrm{d}x}{\sqrt{x}} = \int \sin\left(\frac{t}{2}\right)\,\mathrm{d}t$	B1		Correct separation; condone missing integral signs.
	$2\sqrt{x} = -2\cos\left(\frac{t}{2}\right)\ (+k)$	M1		$p\sqrt{x} = q\cos\left(\frac{t}{2}\right)$ Condone missing $+\,k$
	$x = \left(-\cos\left(\frac{t}{2}\right) + C\right)^2$	A1	3	Must have previous line correct
(ii)	$(1,0)\quad 2 = -2 + k$ or $1 = (-1 + C)^2$	M1		Use $(1,0)$ to find a constant
	$k = 4$ or $C = 2$	A1ft		ft on $C = p - q$ from (a)(i)
	$x = \left(2 - \cos\left(\frac{t}{2}\right)\right)^2$	A1	3	cso applies to (a)(ii)
(b)(i)	Greatest height when $\cos(bt) = -1$	M1		
	Greatest height = 9 (m)	A1ft	2	ft is (their a + 1)^2
(ii)	$\cos\left(\frac{t}{2}\right) = 2 - \sqrt{5}$	M1		$\cos bt = a - \sqrt{5}$
	$t = 2\cos^{-1}\left(2 - \sqrt{5}\right) = 3.6$ (seconds 1dp)	A1	2	condone 3.6 or better (3.618…..)
			10	

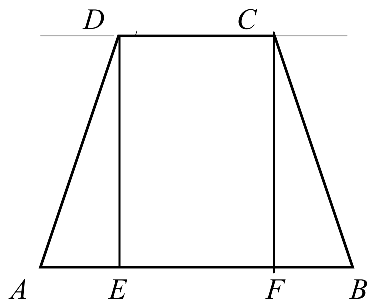
MPC4 (cont)

Q	Solution	Marks	Total	Comments
8(a)(i)	$\overrightarrow{AB}=\begin{bmatrix}6\\0\\3\end{bmatrix}-\begin{bmatrix}3\\-2\\4\end{bmatrix}=\begin{bmatrix}3\\2\\-1\end{bmatrix}$	M1 A1	2	$\pm\left(\overrightarrow{OB}-\overrightarrow{OA}\right)$ implied by 2 correct components
(ii)	$\begin{bmatrix}3\\2\\-1\end{bmatrix}\bullet\begin{bmatrix}2\\-1\\3\end{bmatrix}=6-2-3=1$	M1 A1ft		Scalar product with correct vectors; allow one component error. ft on $\overrightarrow{AB}$
	$\cos\theta=\dfrac{sp}{\sqrt{14}\sqrt{14}}$	m1		Correct form for $\cos\theta$ with one correct modulus
	$\cos\theta=\dfrac{1}{14}\qquad\theta=85.9^{\circ}$	A1	4	cso 85.9 or better
(b)(i)	$\overrightarrow{OD}=\begin{bmatrix}3\\-2\\4\end{bmatrix}+2\begin{bmatrix}2\\-1\\3\end{bmatrix}=\begin{bmatrix}7\\-4\\10\end{bmatrix}$	M1		Implied by 2 correct components
	line $l_2\qquad\mathbf{r}=\begin{bmatrix}7\\-4\\10\end{bmatrix}+\mu\begin{bmatrix}3\\2\\-1\end{bmatrix}$	A1ft	2	$\mathbf{r}=$ or $\begin{bmatrix}x\\y\\z\end{bmatrix}$ required ft on $\overrightarrow{AB}$
(ii)	$\overrightarrow{BC}=\overrightarrow{OC}-\overrightarrow{OB}=\begin{bmatrix}1+3p\\-4+2p\\7-p\end{bmatrix}$	M1		$\mu=p$ at C Find $\overrightarrow{BC}$ in terms of p
	$\overrightarrow{AD}=\begin{bmatrix}4\\-2\\6\end{bmatrix}\qquad\left\lvert\overrightarrow{BC}\right\rvert=\sqrt{56}$	B1ft		PI B1 is for $\left\lvert\overrightarrow{BC}\right\rvert=\sqrt{56}$
	$(1+3p)^2+(-4+2p)^2+(7-p)^2=56$	m1		
	$14p^2-24p+66=56$ $7p^2-12p+5=0$ $(7p-5)(p-1)=0$	m1		ft on $\overrightarrow{BC}$ Simplification to quadratic equation with all terms on one side
	$p=\dfrac{5}{7}$ and $p=1$	A1		Exact fraction required
	C is at $\left(9\frac{1}{7},-2\frac{4}{7},9\frac{2}{7}\right)$	A1	6	cso Accept as column vector
			14	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
8(b)(ii)	**Alternative** : Using equal angles $\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = \begin{bmatrix} 1+3p \\ -4+2p \\ 7-p \end{bmatrix}$	(M1)		$\mu = p$ at C Find $\overrightarrow{BC}$ in terms of p
	$\overrightarrow{AD} = \begin{bmatrix} 4 \\ -2 \\ 6 \end{bmatrix}$ $\quad \left\lvert\overrightarrow{BC}\right\rvert = \sqrt{56}$	(B1ft)		
	$(\cos\theta) = \dfrac{\overrightarrow{BA} \bullet \overrightarrow{BC}}{\sqrt{14}\sqrt{56}} = \dfrac{\begin{bmatrix} -3 \\ -2 \\ 1 \end{bmatrix} \bullet \begin{bmatrix} 1+3p \\ -4+2p \\ 7-p \end{bmatrix}}{\sqrt{14}\sqrt{56}} = \dfrac{1}{14}$	(m1)		Condone $\overrightarrow{AB}$ used. Allow $\left\lvert\overrightarrow{BC}\right\rvert$ in terms of p, in which case previous B1 is implied
	$-3-9p+8-4p+7-p=2$	(m1)		Reduce to linear or quadratic equation in p.
	$p = \dfrac{5}{7}$	(A1)		
	C is at $\left(9\dfrac{1}{7}, -2\dfrac{4}{7}, 9\dfrac{2}{7}\right)$	(A1)	(6)	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
8(b)(ii)	**Alternative** : using symmetry (i)			
	$\left\|\overrightarrow{AD}\right\| = \left\|\overrightarrow{BC}\right\| = \sqrt{56}$	(B1ft)		$\overrightarrow{AD} = \begin{bmatrix} 4 \\ -2 \\ 6 \end{bmatrix}$
	$\left\|\overrightarrow{DC}\right\| = \left\|\overrightarrow{AB}\right\| - \left\|\overrightarrow{AD}\right\|\cos\theta - \left\|\overrightarrow{BC}\right\|\cos\theta$	(M1)		Substitute values and evaluate $\left\|\overrightarrow{AB}\right\| - \left\|\overrightarrow{AD}\right\|\cos\theta - \left\|\overrightarrow{BC}\right\|\cos\theta$
	$\left\|\overrightarrow{DC}\right\| = \dfrac{10}{\sqrt{14}}$	(A1ft)		F on $\overrightarrow{AB}$ and $\cos\theta$
	$\left\|\overrightarrow{DC}\right\| = p\left\|\overrightarrow{AB}\right\| \Rightarrow \dfrac{10}{\sqrt{14}} = p\sqrt{14}$	(m1)		Set up equation in p
	$p = \dfrac{5}{7}$	(A1)		
	C is at $\left(9\frac{1}{7}, -2\frac{4}{7}, 9\frac{2}{7}\right)$	(A1)	(6)	
	Alternative using symmetry (ii)			
	$\left\|\overrightarrow{AD}\right\| = \sqrt{56}$	(B1ft)		
	$\left\|\overrightarrow{AE}\right\| = \left\|\overrightarrow{AD}\right\|\cos\theta = \sqrt{56} \times \dfrac{1}{14} = \dfrac{2}{\sqrt{14}}$	(M1) (A1ft)		Substitute values and evaluate for $\left\|\overrightarrow{AD}\right\|\cos\theta$. F on $\cos\theta$
	$\left\|\overrightarrow{AE}\right\| = q\left\|\overrightarrow{AB}\right\| \Rightarrow \dfrac{2}{\sqrt{14}} = q\sqrt{14}$	(m1)		Set up equation to find p
	and $\left\|\overrightarrow{AE}\right\| = \left\|\overrightarrow{FB}\right\| \Rightarrow p = 1 - 2q$			
	$q = \dfrac{2}{14}$ $\quad p = \dfrac{5}{7}$	(A1)		
	C is at $\left(9\frac{1}{7}, -2\frac{4}{7}, 9\frac{2}{7}\right)$	(A1)	(6)	
	TOTAL		**75**	