Version 1.0

General Certificate of Education
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Mathematics MPC2

Pure Core 2

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Set and published by the Assessment and Qualifications Alliance.

Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MPC2

Q	Solution	Marks	Total	Comments
1(a)	{Area of sector =} $\frac{1}{2}r^2\theta$	M1		$\frac{1}{2}r^2\theta$ seen or used for the area
	$= \frac{1}{2}\times 8^2\times 1.4 = 44.8$ {m^2}	A1	2	Must be exact, not rounded to
(b)(i)	{Arc =} $r\theta$	M1		$r\theta$ seen or used for the arc length
	…. = 11.2	A1		PI Condone AWRT 11.2
	Perimeter of sector = 16+11.2 = 27.2 {m}	A1F	3	Ft on c's evaluation of 8×1.4
(ii)	$27.2 = 2\pi x$	M1		[c's numerical answer for (b)(i)] = $2\pi x$
	$x = \frac{27.2}{2\pi} = 4.329\ldots = 4.33$ to 3sf	A1	2	Condone >3sf
	Total		**7**	
2(a)	$u_2 = 6.8$	B1		OE eg 34/5
	$u_3 = 8.72$	B1F	2	Ft on 6+0.4×c's u_2
(b)	$L = 6 + 0.4L$	M1		Replacing u_{n+1} and u_n by L
	$L = \frac{6}{1-0.4}$	m1		PI provided M scored
	$L = 10$	A1	3	Must form an equation in L otherwise 0/3
	Total		**5**	

MPC2 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$\dfrac{6}{\sin\theta} = \dfrac{15}{\sin 150}$	M1		Sine rule OE PI
	$\sin\theta = \dfrac{6\times\sin 150}{15}$ {= 0.2}	m1		Rearrangement
	θ = 11.53(6..) = 11.5º {to nearest 0.1º}	A1	3	AG Must see at least 4sf value or an exact value for $\sin\theta$ (0.2, 3/15, OE) before seeing the printed value 11.5
(b)	Angle B = 180 − (150+θ) = 18.5 {to 3sf}	B1		Award for B = any value between 18 and 19 inclusive [18.463041….]
	Area = $\dfrac{1}{2}\times 6\times 15\sin B$	M1		
	= 14.3 {cm^2} to 3sf	A1	3	Accept a value 14.2 to 14.3 inclusive Note: For methods involving AC, for the M1 need both a correct method to find AC and a correct area formula
	Total		**6**	

MPC2 (cont)

Q	Solution	Marks	Total	Comments
4(a)	$p = -3$; $q = 3$	B1;B1	2	Accept even if just embedded in the expansion
(b)(i)	$\int \left(1 - \frac{1}{x^2}\right)^3 \mathrm{d}x =$			
	$\int \left(1 - 3x^{-2} + 3x^{-4} - x^{-6}\right) \mathrm{d}x$	M1		Uses (a) with indication of integration and indication of $\frac{1}{x^n} = x^{-n}$ PI
	$= x + 3x^{-1} - x^{-3} + \frac{1}{5}x^{-5}$ $\{+ c\}$	m1		At least three powers of x correctly obtained
		A2F,1F	4	Ft on c's non-zero integers p and q. A1F if 3 of the 4 terms are correct (ft) or if all correct (ft) but left unsimplified Condone missing $+c$.
(ii)	$\int_{\frac{1}{2}}^{1} \left(1 - \frac{1}{x^2}\right)^3 \mathrm{d}x =$			
	$\left(1 + 3 - 1 + \frac{1}{5}\right) - \left(\frac{1}{2} + 6 - 8 + \frac{32}{5}\right)$	M1		Attempting to calculate F(1)−F(1/2) where F is c's answer to part (b)(i) provided F is not the integrand or the c's equivalent of the integrand $(1 - \frac{1}{x^2})^3$.
	$= -\frac{17}{10}$	A1	2	OE exact answer eg −1.7
	Total		**8**	

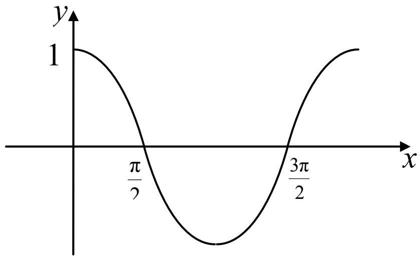
MPC2 (cont)

Q	Solution	Marks	Total	Comments
5(a)(i)	$\{S_\infty =\}\ \dfrac{a}{1-r} = \dfrac{10}{1-r}$	M1		$\dfrac{a}{1-r}$ used
	$\dfrac{10}{1-r} = 50$ so $1 - r = \dfrac{10}{50} \Rightarrow r = \dfrac{4}{5}$	A1	2	AG Condone verification with the correct final statement but be convinced.
(ii)	2nd term = ar	M1		ar stated or used for the 2nd term. PI by ans'8'
	$= 8$	A1	2	
(b)(i)	4th term = $a + 3d$; 8th term = $a + 7d$	M1		Uses $a + (n - 1)d$ correctly at least once
	$a + 3d = 10$, $a + 7d = 8$	A1F		Both eqns. correct ft on c's (a)(ii) OE eg $8 = 10 + 4d$
	$\Rightarrow 4d = -2 \quad \Rightarrow d = -0.5$	A1	3	OE fraction.
(ii)	$a + 3(-0.5) = 10$	M1		An appreciation that a is required in **(b)(ii)** and a valid method to find a anywhere or PI if $a = 11.5$ seen/used
	$\Rightarrow a = 11.5$	A1F		Ft on c's non-zero value for d ie using $a = 10-3d$ or $a=$ c's $8 -7d$. [c's 8 is candidate's answer to (a)(ii)]
	$\displaystyle\sum_{n=1}^{40} u_n = S_{40} = \frac{40}{2}\left[2a + (40-1)d\right]$	M1		$\dfrac{40}{2}\left[2a + (40-1)d\right]$ OE
	$= 70$	A1	4	
	Total		**11**	

MPC2 (cont)

Q	Solution	Marks	Total	Comments
6(a)	$\sqrt{x} = x^{\frac{1}{2}}$	B1		PI
	$\frac{x^3+\sqrt{x}}{x} = \frac{x^3}{x} + \frac{\sqrt{x}}{x} = x^2 + x^{-\frac{1}{2}}$	B1;B1	3	Accept $p = 2$; $q = -\frac{1}{2}$
(b)(i)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 2x \quad -\frac{1}{2}x^{-\frac{3}{2}}$	M1 A1	2	Reduces both powers by 1 ACF
(ii)	When $x = 1$, $y = 2$	B1		PI if not stated explicitly eg the '2' may appear in the correct posn. in later eqn.
	When $x = 1$, $\frac{\mathrm{d}y}{\mathrm{d}x} = 2 - \frac{1}{2} = \frac{3}{2}$	M1		Attempt to find $\frac{\mathrm{d}y}{\mathrm{d}x}$ when $x = 1$ PI
	Gradient of normal = $-\frac{2}{3}$	m1		− 1/(c's value of dy/dx when $x = 1$) either stated as the gradient of the normal or used as the gradient in the equation of the normal
	Equation of normal: $y - 2 = -\frac{2}{3}(x-1)$	A1F	4	Only ft on c's $\frac{\mathrm{d}y}{\mathrm{d}x}$ in part (b)(i). ACftF
(c)(i)	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 2 + \frac{3}{4}x^{-\frac{5}{2}}$	M1 A1F	2	Reduces both powers by 1. Ft on (b)(i) provided at least one power to be differentiated is both negative and fractional
(ii)	(Since x>0,) $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$>0 For a maximum point $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ is **not** positive so C has no maximum points	E2,1,0	2	E1 for attempt to find the sign of $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$; either in general terms or at the pt(s) where c's dy/dx = 0 for the remaining E mark a correct justification for why $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ >0 and also a full correct concluding statement must be made.
	Total		**13**	

MPC2 (cont)

Q	Solution	Marks	Total	Comments
7(a)		B1 B1	 2	Correct shape meeting positive y-axis and only one oscillation within interval 0 to 2π The three correct intercepts stated; Accept 1.57 for π/2 and 4.71 for 3π/2 but must be evidence of radian vals. not just degrees Ignore any parts of the graph clearly indicated as outside the given interval
(b)(i)	$1-\cos^2\theta = \cos\theta(2-\cos\theta)$ $1 = 2\cos\theta \Rightarrow \cos\theta = \frac{1}{2}$	M1 A1	 2	$\cos^2\theta + \sin^2\theta = 1$ used CSO AG Completion
(ii)	$\sin^2 2x = \cos 2x(2-\cos 2x)$ $\Rightarrow \cos 2x = \frac{1}{2}$ $\{2x =\}$ $\cos^{-1}\left(\frac{1}{2}\right) = 1.04(7..)$ $x = 0.524,\ 2.62$ $x = 0.523(59..),\ 2.61(7..)$	 M1 m1 A2,1,0	 4	 Uses (b)(i) PI Accept 1.05, $\frac{\pi}{3}$; Condone 60º Condone >3sf; Condone x = 0.525, 2.62 Accept truncated '3sf' vals x =0.523, 2.61 Deduct 1 mark for each extra (>2 solns) in the given interval from A marks to a min of A0. Ignore any solns outside the given interval 0 to π. Accept, as equivalent, the exact answers $x = \frac{\pi}{6}$ and $x = \frac{5\pi}{6}$ when seen and apply ISW if 'errors' converting these to decimals. If not A2 then A1 if • one soln correct. • 30º , 150º ie solns left in degrees • AWRT 0.52, 2.6 ie correct vals to only 2sf. Must see an indication that (b)(i) has been used otherwise 0/4 so just stating the two correct answers with nothing else scores 0/4.
	Total		**8**	

MPC2 (cont)

Q	Solution	Marks	Total	Comments
8(a)	(y =) 1	B1	1	
(b)	$h = 0.2$	B1		PI
	f(x) = 2^{4x} I $\approx$ h/2{…} {.}=f(0)+f(1)+2[f(0.2)+f(0.4)+f(0.6)+f(0.8)]	M1		OE summing of areas of the 'trapezia'..
	{.} = 1 + 16+2($2^{0.8}$ + $2^{1.6}$ +$2^{2.4}$ +$2^{3.2}$) =1+16+2(1.741..+3.031..+5.278..+9.1895..) = [17+2×19.24...]	A1		OE Accept 2dp rounded or truncated evidence
	I = 5.55 (to2dp)	A1	4	Must be 5.55
(c)	Stretch(**I**) in y-direction(**II**) scale	M1		Need (**I**) **and** either (**II**) or (**III**)
	factor $\frac{1}{8}$ (**III**)	A1	2	Need (**I**) and (**II**) and (**III**)
	ALTn: Translation with an indication that the translation is in the x-direction (B1) $\begin{bmatrix} \frac{3}{4} \\ 0 \end{bmatrix}$ (B1)			Combination of **different** transformations scores 0/2
(d)	g(x) = $2^{4(x-1)} - \frac{1}{2}$	B2,1,0		B1 for either $2^{4(x+1)} - \frac{1}{2}$ or for $2^{4(x-1)} + \frac{1}{2}$ or for $2^{4x-1} - \frac{1}{2}$
	At Q, $y = 0 \Rightarrow 2^{4(x-1)} = 2^{-1}$	M1		Reaches a stage from which linear eqn can be stated directly eg an alternative stage is $4(x-1)\log 2 = -\log 2$
	$\Rightarrow 4x - 4 = -1 \Rightarrow x = 0.75$	A1	4	NMS mark as 4 or 0
(e)(i)	$\log_a k = \log_a 2^3 + \log_a 5 - \log_a 4$	M1		One law of logs used
	$\log_a k = \log_a (2^3 \times 5) - \log_a 4$	M1		A second law of logs used; could be $\log_a k = \log_a 2^3 + \log_a (\frac{5}{4})$
	$\log_a k = \log_a (\frac{2^3 \times 5}{4}) = \log_a 10 \Rightarrow k = 1$	A1	3	CSO AG
(ii)	$2^{4x-3} = \frac{5}{4}$ so $(4x-3)\log_{10} 2 = \log_{10} \frac{5}{4}$	M1		Equate y's, take logs (to any base) of both sides **and** apply 3rd law of logs. Altn $4x \log 2 = \log\left(\frac{5}{4} \times 2^3\right)$
	$x = \dfrac{3\log_{10} 2 + \log_{10}\left(\frac{5}{4}\right)}{4\log_{10} 2}$	m1		Rearrange correctly to x = …. Altn $4x \log 2 = \log 10$ In both cases, log term(s) must have same base and expressions must be in an exact form, ie not approx. dec. vals
	$x = \dfrac{\log_{10} 10}{4\log_{10} 2}$ so $x = \dfrac{1}{4\log_{10} 2}$	A1	3	CSO AG Must be clear evidence that base 10 is used, also be convinced
	Total		**17**	
	TOTAL		**75**	