

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
TOTAL	



General Certificate of Education
Advanced Level Examination
June 2010

Mathematics

MFP3

Unit Further Pure 3

Friday 11 June 2010 9.00 am to 10.30 am

For this paper you must have:

- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed

- 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.



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2 (a) Find the value of the constant k for which $k \sin 2x$ is a particular integral of the differential equation

$$\frac{d^2y}{dx^2} + y = \sin 2x \quad (3 \text{ marks})$$

(b) Hence find the general solution of this differential equation. (4 marks)

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3 (a) Explain why $\int_1^{\infty} 4xe^{-4x} dx$ is an improper integral. (1 mark)

(b) Find $\int 4xe^{-4x} dx$. (3 marks)

(c) Hence evaluate $\int_1^{\infty} 4xe^{-4x} dx$, showing the limiting process used. (3 marks)

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4 By using an integrating factor, find the solution of the differential equation

$$\frac{dy}{dx} + \frac{3}{x}y = (x^4 + 3)^{\frac{3}{2}}$$

given that $y = \frac{1}{5}$ when $x = 1$.

(9 marks)

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5 (a) Write down the expansion of $\cos 4x$ in ascending powers of x up to and including the term in x^4 . Give your answer in its simplest form. (2 marks)

(b) (i) Given that $y = \ln(2 - e^x)$, find $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ and $\frac{d^3y}{dx^3}$.

(You may leave your expression for $\frac{d^3y}{dx^3}$ unsimplified.) (6 marks)

(ii) Hence, by using Maclaurin's theorem, show that the first three non-zero terms in the expansion, in ascending powers of x , of $\ln(2 - e^x)$ are

$$-x - x^2 - x^3 \quad (2 \text{ marks})$$

(c) Find

$$\lim_{x \rightarrow 0} \left[\frac{x \ln(2 - e^x)}{1 - \cos 4x} \right] \quad (3 \text{ marks})$$

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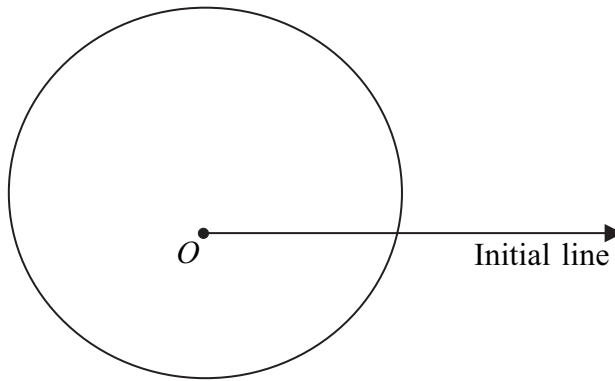
6 The polar equation of a curve C_1 is

$$r = 2(\cos \theta - \sin \theta), \quad 0 \leq \theta \leq 2\pi$$

- (a) (i)** Find the cartesian equation of C_1 . *(4 marks)*
- (ii)** Deduce that C_1 is a circle and find its radius and the cartesian coordinates of its centre. *(3 marks)*

(b) The diagram shows the curve C_2 with polar equation

$$r = 4 + \sin \theta, \quad 0 \leq \theta \leq 2\pi$$



- (i)** Find the area of the region that is bounded by C_2 . *(6 marks)*
- (ii)** Prove that the curves C_1 and C_2 do not intersect. *(4 marks)*
- (iii)** Find the area of the region that is outside C_1 but inside C_2 . *(2 marks)*

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7 (a) Given that $x = t^{\frac{1}{2}}$, $x > 0$, $t > 0$ and y is a function of x , show that:

(i) $\frac{dy}{dx} = 2t^{\frac{1}{2}} \frac{dy}{dt}$; (2 marks)

(ii) $\frac{d^2y}{dx^2} = 4t \frac{d^2y}{dt^2} + 2 \frac{dy}{dt}$. (3 marks)

(b) Hence show that the substitution $x = t^{\frac{1}{2}}$ transforms the differential equation

$$x \frac{d^2y}{dx^2} - (8x^2 + 1) \frac{dy}{dx} + 12x^3y = 12x^5$$

into

$$\frac{d^2y}{dt^2} - 4 \frac{dy}{dt} + 3y = 3t$$
 (2 marks)

(c) Hence find the general solution of the differential equation

$$x \frac{d^2y}{dx^2} - (8x^2 + 1) \frac{dy}{dx} + 12x^3y = 12x^5$$

giving your answer in the form $y = f(x)$. (7 marks)

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END OF QUESTIONS

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