General Certificate of Education
Advanced Level Examination
January 2010

Mathematics MPC4

Unit Pure Core 4

Tuesday 19 January 2010 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The **Examining Body** for this paper is AQA. The **Paper Reference** is MPC4.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The polynomial $f(x)$ is defined by $f(x) = 15x^3 + 19x^2 - 4$.

(a) (i) Find $f(-1)$. *(1 mark)*

(ii) Show that $(5x - 2)$ is a factor of $f(x)$. *(2 marks)*

(b) Simplify

$$\frac{15x^2 - 6x}{f(x)}$$

giving your answer in a fully factorised form. *(5 marks)*

2 (a) Express $\cos x + 3\sin x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give your value of α, in radians, to three decimal places. *(3 marks)*

(b) (i) Hence write down the minimum value of $\cos x + 3\sin x$. *(1 mark)*

(ii) Find the value of x in the interval $0 \leqslant x \leqslant 2\pi$ at which this minimum occurs, giving your answer, in radians, to three decimal places. *(2 marks)*

(c) Solve the equation $\cos x + 3\sin x = 2$ in the interval $0 \leqslant x \leqslant 2\pi$, giving all solutions, in radians, to three decimal places. *(4 marks)*

3 (a) (i) Find the binomial expansion of $(1 + x)^{-\frac{1}{3}}$ up to and including the term in x^2. *(2 marks)*

(ii) Hence find the binomial expansion of $\left(1 + \frac{3}{4}x\right)^{-\frac{1}{3}}$ up to and including the term in x^2. *(2 marks)*

(b) Hence show that $\sqrt[3]{\frac{256}{4 + 3x}} \approx a + bx + cx^2$ for small values of x, stating the values of the constants a, b and c. *(3 marks)*

4 The expression $\dfrac{10x^2 + 8}{(x+1)(5x-1)}$ can be written in the form $2 + \dfrac{A}{x+1} + \dfrac{B}{5x-1}$, where A and B are constants.

(a) Find the values of A and B. *(4 marks)*

(b) Hence find $\displaystyle\int \frac{10x^2 + 8}{(x+1)(5x-1)}\,\mathrm{d}x$. *(4 marks)*

5 A curve is defined by the equation

$$x^2 + xy = \mathrm{e}^y$$

Find the gradient at the point $(-1, 0)$ on this curve. *(5 marks)*

6 (a) (i) Express $\sin 2\theta$ and $\cos 2\theta$ in terms of $\sin\theta$ and $\cos\theta$. *(2 marks)*

(ii) Given that $0 < \theta < \dfrac{\pi}{2}$ and $\cos\theta = \dfrac{3}{5}$, show that $\sin 2\theta = \dfrac{24}{25}$ and find the value of $\cos 2\theta$. *(2 marks)*

(b) A curve has parametric equations

$$x = 3\sin 2\theta, \quad y = 4\cos 2\theta$$

(i) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of θ. *(3 marks)*

(ii) At the point P on the curve, $\cos\theta = \dfrac{3}{5}$ and $0 < \theta < \dfrac{\pi}{2}$. Find an equation of the tangent to the curve at the point P. *(3 marks)*

7 Solve the differential equation $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{y}\cos\left(\dfrac{x}{3}\right)$, given that $y = 1$ when $x = \dfrac{\pi}{2}$.

Write your answer in the form $y^2 = \mathrm{f}(x)$. *(6 marks)*

Turn over for the next question

8 The points A, B and C have coordinates $(2,\ -1,\ -5)$, $(0,\ 5,\ -9)$ and $(9,\ 2,\ 3)$ respectively.

The line l has equation $\mathbf{r} = \begin{bmatrix} 2 \\ -1 \\ -5 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}$.

(a) Verify that the point B lies on the line l. *(2 marks)*

(b) Find the vector $\overrightarrow{BC}$. *(2 marks)*

(c) The point D is such that $\overrightarrow{AD} = 2\overrightarrow{BC}$.

(i) Show that D has coordinates $(20,\ -7,\ 19)$. *(2 marks)*

(ii) The point P lies on l where $\lambda = p$. The line PD is perpendicular to l. Find the value of p. *(5 marks)*

9 A botanist is investigating the rate of growth of a certain species of toadstool. She observes that a particular toadstool of this type has a height of 57 millimetres at a time 12 hours after it begins to grow.

She proposes the model $h = A\left(1 - \mathrm{e}^{-\frac{1}{4}t}\right)$, where A is a constant, for the height h millimetres of the toadstool, t hours after it begins to grow.

(a) Use this model to:

(i) find the height of the toadstool when $t = 0$; *(1 mark)*

(ii) show that $A = 60$, correct to two significant figures. *(2 marks)*

(b) Use the model $h = 60\left(1 - \mathrm{e}^{-\frac{1}{4}t}\right)$ to:

(i) show that the time T hours for the toadstool to grow to a height of 48 millimetres is given by

$$T = a \ln b$$

where a and b are integers; *(3 marks)*

(ii) show that $\dfrac{\mathrm{d}h}{\mathrm{d}t} = 15 - \dfrac{h}{4}$; *(3 marks)*

(iii) find the height of the toadstool when it is growing at a rate of 13 millimetres per hour. *(1 mark)*

END OF QUESTIONS