General Certificate of Education

Mathematics 6360

MM04 Mechanics 4

Mark Scheme

2009 examination - June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
–*x* EE	deduct *x* marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

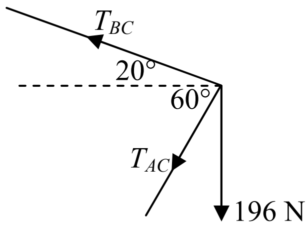
Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

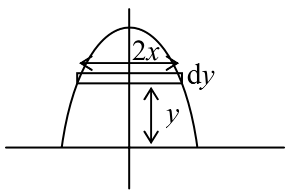
MM04

Q	Solution	Marks	Total	Comments
1(a)	1200 rev per min = $\dfrac{1200\times 2\pi}{60}$ rad s^{-1}	M1		Attempt to convert to rad s^{-1}
	$= 40\pi$	A1		
	Using $\omega = \omega_0 + \ddot{\theta}t$	M1		Use of constant acceleration formula
	$\ddot{\theta} = \dfrac{40\pi - 0}{10}$			
	$= 4\pi$	A1	4	AG
(b)	Using $C = I\ddot{\theta}$	M1		Attempt to use $C = I\ddot{\theta}$
	$100\pi = 4\pi I$			
	$I = 25$ (kg m^2)	A1F	2	ft $\ddot{\theta}$ from (a)
	Total		**6**	
2				
	Resolve horizontally at *C*	M1		Resolve in one direction – one correct component
	$T_{BC}\cos 20° + T_{AC}\cos 60° = 0$	A1		Fully correct equation
	Resolving vertically at *C*	M1		Resolve in second direction – one correct component
	$T_{BC}\sin 20° = T_{AC}\sin 60° + 196$	A1		Fully correct equation
	Solving gives:	M1		Attempt to solve their pair of equations – eliminate a variable
	$\lvert T_{AC}\rvert = 187\,\text{N}$			
	$\lvert T_{BC}\rvert = 99.5\,\text{N}$	A1		Both correct; accept ±
	AC in compression **and** *BC* in tension	B1	7	Both correct
	Total		**7**	

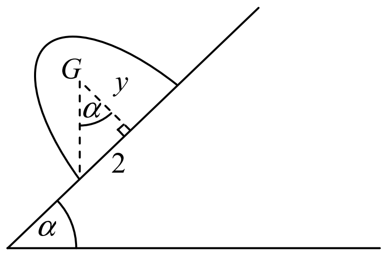
MM04 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$\mathbf{F} = \begin{pmatrix}1\\5\\-3\end{pmatrix} + \begin{pmatrix}2\\-7\\-1\end{pmatrix} + \begin{pmatrix}0\\4\\-2\end{pmatrix} = \begin{pmatrix}3\\2\\-6\end{pmatrix}$	B1		Correct total
	$\lvert\mathbf{F}\rvert = \sqrt{3^2+2^2+6^2}$	M1		Attempt to find $\lvert\mathbf{F}\rvert$
	$= 7$	A1	3	AG
(b)(i)	$\mathbf{r} = \begin{pmatrix}6\\4\\-7\end{pmatrix} - \begin{pmatrix}3\\6\\-3\end{pmatrix} = \begin{pmatrix}3\\-2\\-4\end{pmatrix}$	B1		Correct **r**
	Moment $= \mathbf{r}\times\mathbf{F}$ $= \begin{vmatrix}\mathbf{i} & 3 & 0\\ \mathbf{j} & -2 & 4\\ \mathbf{k} & -4 & -2\end{vmatrix}$	M1		Attempt at $\mathbf{r}\times\mathbf{F}$ or $\mathbf{F}\times\mathbf{r}$
	$= \begin{pmatrix}20\\6\\12\end{pmatrix}$	A2,1,0	4	One component correct $\Rightarrow$ A1 All components correct $\Rightarrow$ A2 SC1: $\mathbf{F}\times\mathbf{r} \Rightarrow$ M1A1A0 SC2: Use of $\begin{pmatrix}-3\\2\\4\end{pmatrix}$ to get $\begin{pmatrix}-20\\-6\\-12\end{pmatrix}$ scores B0 M1 A1 A1F SC3: Use of $\mathbf{r} = \begin{pmatrix}3\\6\\-3\end{pmatrix}$ to get $\begin{pmatrix}0\\6\\12\end{pmatrix}$ scores B0 M1 A1F A0
(ii)	Moment $= \begin{pmatrix}20\\6\\12\end{pmatrix}$	B1F		ft (b)(i)
	since resultant of other two forces acts through given point (therefore 0 moment)	E1	2	
	Total		**9**	

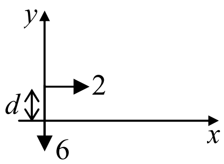
MM04 (cont)

Q	Solution	Marks	Total	Comments
4(a)	$\frac{1}{2}\int_{-2}^{2} y^2\,\mathrm{d}x$			
	$=\frac{1}{2}\int_{-2}^{2}\left(16-8x^2+x^4\right)\,\mathrm{d}x$	M1		Attempt to integrate y^2 as a function of x
	$=\frac{1}{2}\left[16x-\frac{8x^3}{3}+\frac{x^5}{5}\right]_{-2}^{2}$	A1		Correct integration
	$=\frac{256}{15}$	A1F		Correct limits applied to their integral
	$\bar{y}=\frac{256/15}{32/3}$	m1		$\bar{y}=\frac{\frac{1}{2}\int_a^b y^2\,\mathrm{d}x}{\text{Area}}$
	$=\frac{8}{5}$	A1	5	
	Alternative 1: 			
	$I=\int_{x=2}^{x=0} 2xy\,\mathrm{d}y=\int_{2}^{0} 2x\left(4-x^2\right)(-2x)\,\mathrm{d}x$	(M1)		Attempt to integrate $2xy$ as a function of x
	$=\int_{2}^{0} 4x^4-16x^2\,\mathrm{d}x$			
	$I=\left[\frac{4x^5}{5}-\frac{16x^3}{3}\right]_{2}^{0}$	(A1)		Correct integration
	$I=\frac{256}{15}$	(A1F)		Correct limit applied to their integral
	$\bar{y}=\frac{I}{32/3}=\frac{8}{5}$	(m1) (A1)		Their evaluated $I \div$ area
	Alternative 2:			
	$I=\int_{y=0}^{y=4} 2xy\,\mathrm{d}y=\int_{0}^{4} 2\sqrt{(4-y)}\,y\,\mathrm{d}y$	(M1)		Attempt to integrate $2xy$ as a function of y by parts
	$I=\left[-\frac{4y}{3}(4-y)^{\frac{3}{2}}\right]_0^4+\int_0^4 \frac{4}{3}(4-y)^{\frac{3}{2}}\,\mathrm{d}y$			
	$I=\left[-\frac{8}{15}(4-y)^{\frac{5}{2}}\right]_0^4$	(A1)		Fully integrated
	$I=\frac{256}{15}$	(A1F)		Correct limit applied to their integral
	$\bar{y}=\frac{I}{32/3}=\frac{8}{5}$	(m1) (A1)		Their evaluated $I \div$ area

MM04 (cont)

Q	Solution	Marks	Total	Comments
4(a) cont	**General for all three versions:** M1 Attempt to integrate an appropriate function of x or y (must apply a full method) A1 Correct integration A1F Correct limits applied to their integral m1 Their evaluated integral $\div$ area A1 Correct answer $\frac{8}{5}$			
(b)				
	$\tan\alpha = \frac{2}{\bar{y}}$	M1 A1F		$\tan\alpha$ seen Correct structure – ft error in (a)
	$\tan\alpha = \frac{2}{8/5} = 1.25$	m1		Substitute and use of $\tan^{-1}$ – dependent on first M1
	$\alpha = 51°$	A1F	4	ft error in (a)
	Total		**9**	

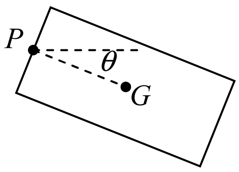
MM04 (contd)

Q	Solution	Marks	Total	Comments
5(a)	Let resultant be $\begin{pmatrix} X \\ Y \end{pmatrix} = R$			
	$X = 8 + 6 - 15\cos\theta$ $Y = 1 + 2 - 15\sin\theta$	M1		Attempt at X and Y; must involve use of $15\sin\theta$ or $15\cos\theta$
	with $\cos\theta = \frac{8}{10}$ and $\sin\theta = \frac{6}{10}$ or $\theta = 36.9°$	A1		Either 12 or 9 seen as components of the 15N force
	$\Rightarrow X = 2,\ Y = -6$	A1		Both X and Y correctly evaluated including direction
	$\lvert R \rvert = \sqrt{2^2 + 6^2} = \sqrt{40}$	m1		Attempt at $\lvert R \rvert$
	$= 2\sqrt{10}$	A1	5	AG; must see $\sqrt{40}$ or $\sqrt{4 \times 10}$
	Alternative – using diagrams:			
		(M1) (A1) (A1) (m1) (A1)		4 components shown 12 or 9 seen Resultant components - correct direction shown As above As above
(b)(i)	Moments about O for system $20 + 2(8) - 8(6) = -12$	M1 A2,1,0		Attempt at moments for system –1 each error or omission
	(ie 12 Nm clockwise)			
	 Moment of resultant			
	$2d = 12$	M1		Form equation – must be of form x-component$\times d$ = moment for system
	$d = 6$	A1	5	
(ii)	$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 6 \end{pmatrix} + t\begin{pmatrix} 1 \\ -3 \end{pmatrix}$	M1		Correct structure on RHS (**a** + t**b**)
		A1F		$\begin{pmatrix} 0 \\ 6 \end{pmatrix}$; ft d value from (b)(i)
		A1F	3	$\begin{pmatrix} 1 \\ -3 \end{pmatrix}$ OE; ft components from (a) Condone omission of $\begin{pmatrix} x \\ y \end{pmatrix}$ or **r** on LHS
	Total		**13**	

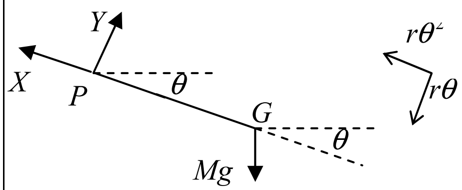
MM04 (cont)

Q	Solution	Marks	Total	Comments
6(a)	$m = \pi r^2 \rho \Rightarrow \rho = \dfrac{m}{\pi r^2}$	B1		ρ and m linked – used anywhere
	Mass of elemental ring $= 2\pi x\,\delta x\,\rho$	M1		Attempt at mass
	MI of elemental ring = $(2\pi x\,\delta x\,\rho)x^2$	A1		Correct use of mr^2
	MI of disc $= \int_0^r 2\pi x^3 \rho\,\mathrm{d}x = \int_0^r \dfrac{2mx^3}{r^2}\mathrm{d}x$	m1		Attempt to integrate – dependent on first M1and **must** be of form $\int kx^3\,\mathrm{d}x$
	$= \left[\dfrac{mx^4}{2r^2}\right]_0^r = \dfrac{mr^2}{2}$	A1	5	AG
(b)	MI required = $\mathrm{MI}_{\text{large disc}} - \mathrm{MI}_{\text{small disc}}$			
	$= \dfrac{(4a)^2\pi\rho(4a)^2}{2} - \dfrac{(2a)^2\pi\rho(2a)^2}{2}$	M1		Attempt at difference of MIs – $4a$, $2a$ substituted for r_1, r_2; $\dfrac{M(4a)^2}{2} - \dfrac{m(2a)^2}{2}$ ok for M1
		A1		Correct MI for either disc - must involve correct masses or ratios
	$= 120\pi a^4 \rho$	A1		Correct difference
	$M = 12a^2\pi\rho$	B1		Mass of ring
	$\Rightarrow$ MI $= 10Ma^2$	A1	5	AG
	Alternative 1:			
	$M = 12a^2\pi\rho \Rightarrow \rho = \dfrac{M}{12a^2\pi}$	(B1)		ρ and M linked – used anywhere
	MI of hoop =			
	$\int_{2a}^{4a} 2\pi x^3\rho\,\mathrm{d}x = \int_{2a}^{4a} \dfrac{2\pi x^3 M}{12a^2\pi}\mathrm{d}x = \int_{2a}^{4a} \dfrac{Mx^3}{6a^2}\mathrm{d}x$	(M1)		Integral with correct limits - any form given here
	$= \left[\dfrac{Mx^4}{24a^2}\right]_{2a}^{4a} = \dfrac{M(4a)^4}{24a^2} - \dfrac{M(2a)^4}{24a^2}$	(A1)		Correct integration
	$= 10Ma^2$	(M1) (A1)		Use of correct limits AG
	Alternative 2: Mass removed = ¼ of mass of whole disc (as mass is proportional to radius ^{2})			
	Let masses be $4m$ and m; remaining mass = $3m$	(B1)		Ratio of masses
	$\mathrm{MI}_{\text{large disc}} = \dfrac{4m(4a)^2}{2} = 32ma^2$	(M1)		MI of either
	$\mathrm{MI}_{\text{small disc}} = \dfrac{m(2a)^2}{2} = 2ma^2$	(A1)		Both correct
	Difference = $30ma^2$	(A1)		Difference
	$= 10(3m)a^2 = 10Ma^2$	(A1)		Converting answer

MM04 (cont)

Q	Solution	Marks	Total	Comments
6 cont (c)	Using the perpendicular axis theorem	E1		
	$10Ma^2 = I_D + I_D$	M1		
	$\therefore\ I_D = 5Ma^2$	A1	3	
	Total		**13**	
7(a)(i)	Use $I = \frac{1}{3}m\left(a^2+b^2\right)$			
	With $'a' = 2a\quad 'b' = 3a$	M1		Use of formulae booklet
	$I = \frac{1}{3}M\left(4a^2+9a^2\right) = \frac{13Ma^2}{3}$	A1	2	AG
(ii)	$I_M = I_G + Md^2$			
	$= \frac{13Ma^2}{3} + M\left(2a\right)^2$	M1		Use of Parallel Axis Theorem
	$= \frac{25Ma^2}{3}$	A1	2	
(b)(i)	 KE gained $= \frac{1}{2}I\dot{\theta}^2$			
	$= \frac{25Ma^2}{6}\dot{\theta}^2$	B1F		ft from (a)(ii)
	PE lost $= mgh = Mg\,2a\sin\theta$	B1		
	C of E $\Rightarrow\ \frac{25Ma^2}{6}\dot{\theta}^2 = 2Mga\sin\theta$	M1 A1F		Forms equation: KE gained = PE lost ft their expressions - 2 terms
	$\dot{\theta}^2 = \frac{12g\sin\theta}{25a}$	A1	5	AG
(ii)	Differentiating $\quad 2\dot{\theta}\ddot{\theta} = \frac{12g}{25a}\cos\theta\dot{\theta}$	M1A1		M1 RHS, A1 LHS
	Cancelling $\quad \ddot{\theta} = \frac{6g}{25a}\cos\theta$	A1	3	
	Alternative:			
	$C = I\ddot{\theta}$ gives $Mg\cos\theta.2a = \frac{25Ma^2}{3}\ddot{\theta}$	(M1) (A1)		M1 one side correct A1 fully correct
	$\ddot{\theta} = \frac{6g}{25a}\cos\theta$	(A1)		

MM04 (cont)

Q	Solution	Marks	Total	Comments
7 cont (b)(iii)				
	Along *GP*:			
	$X - Mg\sin\theta = M(2a)\dfrac{12g}{25a}\sin\theta$	M1A1		$X \pm$ component $= \pm Mr\dot{\theta}^2$ M1 one side, A1 both sides correct (structure)
	$X = Mg\sin\theta + \dfrac{24Mg}{25}\sin\theta = \dfrac{49Mg}{25}\sin\theta$	A1	3	AG
(iv)	Along *PR*:			
	$Y - Mg\cos\theta = -M(2a)\dfrac{6g}{25a}\cos\theta$	M1 A1		$Y \pm$ component $= \pm Mr\ddot{\theta}$ M1 one side, A1 both sides correct (structure)
	$Y = -\dfrac{12Mg}{25}\cos\theta + Mg\cos\theta$			
	$= \dfrac{13Mg}{25}\cos\theta$	A1	3	Must be simplified
	Total		**18**	
	TOTAL		**75**	