

General Certificate of Education
June 2009
Advanced Level Examination



MATHEMATICS
Unit Further Pure 4

MFP4

Wednesday 17 June 2009 9.00 am to 10.30 am

For this paper you must have:

- a 12-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP4.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 Let $\mathbf{P} = \begin{bmatrix} 1 & 4 & 2 \\ -1 & 2 & 6 \end{bmatrix}$ and $\mathbf{Q} = \begin{bmatrix} k & 1 \\ 2 & -1 \\ 3 & 1 \end{bmatrix}$, where k is a constant.

(a) Determine the product matrix \mathbf{PQ} , giving its elements in terms of k where appropriate. (3 marks)

(b) Find the value of k for which \mathbf{PQ} is singular. (2 marks)

2 (a) Write down the 3×3 matrices which represent the transformations A and B, where:

(i) A is a reflection in the plane $y = x$; (2 marks)

(ii) B is a rotation about the z -axis through the angle θ , where $\theta = \frac{\pi}{2}$. (1 mark)

(b) (i) Find the matrix \mathbf{R} which represents the composite transformation

‘A followed by B’ (3 marks)

(ii) Describe the single transformation represented by \mathbf{R} . (2 marks)

3 The plane Π has equation $\mathbf{r} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} + \lambda \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} + \mu \begin{bmatrix} 4 \\ -1 \\ 1 \end{bmatrix}$.

(a) Find an equation for Π in the form $\mathbf{r} \cdot \mathbf{n} = d$. (4 marks)

(b) Show that the line with equation $\mathbf{r} = \begin{bmatrix} 7 \\ 1 \\ 4 \end{bmatrix} + t \begin{bmatrix} 10 \\ 1 \\ 5 \end{bmatrix}$ does not intersect Π , and explain the geometrical significance of this result. (4 marks)

- 4 (a) Show that the system of equations

$$\begin{aligned} 3x - y + 3z &= 11 \\ 4x + y - 5z &= 17 \\ 5x - 4y + 14z &= 16 \end{aligned}$$

does not have a unique solution and is consistent.

(You are not required to find any solutions to this system of equations.) (4 marks)

- (b) A transformation T of three-dimensional space maps points (x, y, z) onto image points (x', y', z') such that

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} x - y + 3z - 2 \\ 2x + 6y - 4z + 12 \\ 4x + 11y + 4z - 30 \end{bmatrix}$$

Find the coordinates of the invariant point of T . (8 marks)

- 5 The points A, B, C and D have position vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$ and \mathbf{d} respectively, relative to the origin O , where

$$\mathbf{a} = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix}, \mathbf{c} = \begin{bmatrix} 1 \\ -1 \\ 5 \end{bmatrix} \text{ and } \mathbf{d} = \begin{bmatrix} 5 \\ 5 \\ 11 \end{bmatrix}$$

- (a) Using scalar triple products:

(i) show that \overrightarrow{OA} , \overrightarrow{OB} and \overrightarrow{OC} are coplanar; (2 marks)

(ii) find the volume of the parallelepiped defined by AB , AC and AD . (4 marks)

- (b) (i) Find the direction ratios of the line BD . (2 marks)

(ii) Deduce the direction cosines of the line BD . (2 marks)

Turn over ►

6 The plane transformation T is defined by

$$T : \begin{bmatrix} x' \\ y' \end{bmatrix} = \mathbf{M} \begin{bmatrix} x \\ y \end{bmatrix}$$

where $\mathbf{M} = \begin{bmatrix} -1 & 4 \\ -1 & 3 \end{bmatrix}$.

- (a) Evaluate $\det \mathbf{M}$ and state the significance of this answer in relation to T . *(2 marks)*
- (b) Find the single eigenvalue of \mathbf{M} and a corresponding eigenvector. Describe the geometrical significance of these answers in relation to T . *(5 marks)*
- (c) Show that the image of the line $y = \frac{1}{2}x + k$ under T is $y' = \frac{1}{2}x' + k$. *(3 marks)*
- (d) Given that T is a shear, give a full geometrical description of this transformation. *(2 marks)*

7 The 2×2 matrix \mathbf{M} has an eigenvalue 3, with corresponding eigenvector $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$, and a second eigenvalue -3 , with corresponding eigenvector $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$.

The diagonalised form of \mathbf{M} is $\mathbf{M} = \mathbf{U} \mathbf{D} \mathbf{U}^{-1}$.

- (a) (i) Write down suitable matrices \mathbf{D} and \mathbf{U} , and find \mathbf{U}^{-1} . *(4 marks)*
- (ii) Hence determine the matrix \mathbf{M} . *(3 marks)*
- (b) Given that n is a positive integer, use the result $\mathbf{M}^n = \mathbf{U} \mathbf{D}^n \mathbf{U}^{-1}$ to show that:
- (i) when n is even, $\mathbf{M}^n = 3^n \mathbf{I}$;
- (ii) when n is odd, $\mathbf{M}^n = 3^{n-1} \mathbf{M}$. *(6 marks)*

8 (a) Matrix $\mathbf{M} = \begin{bmatrix} a & b & c \\ c & a & b \\ b & c & a \end{bmatrix}$. Without attempting to factorise, expand fully $\det \mathbf{M}$. (2 marks)

(b) Matrix $\mathbf{N} = \begin{bmatrix} d & e & f \\ f & d & e \\ e & f & d \end{bmatrix}$. Find the product matrix \mathbf{MN} . (3 marks)

(c) Prove that the product

$$(a^3 + b^3 + c^3 - 3abc)(d^3 + e^3 + f^3 - 3def)$$

can be written in the form $x^3 + y^3 + z^3 - 3xyz$, stating clearly each of x , y and z in terms of a , b , c , d , e and f . (2 marks)

END OF QUESTIONS

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