General Certificate of Education
June 2009
Advanced Level Examination

MATHEMATICS **MFP3**
Unit Further Pure 3

Thursday 11 June 2009 9.00 am to 10.30 am

For this paper you must have:
- a 12-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP3.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The function $y(x)$ satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x, y)$$

where $$\mathrm{f}(x, y) = \sqrt{x^2 + y + 1}$$

and $$y(3) = 2$$

(a) Use the Euler formula

$$y_{r+1} = y_r + h\,\mathrm{f}(x_r, y_r)$$

with $h = 0.1$, to obtain an approximation to $y(3.1)$, giving your answer to four decimal places. *(3 marks)*

(b) Use the formula

$$y_{r+1} = y_{r-1} + 2h\,\mathrm{f}(x_r, y_r)$$

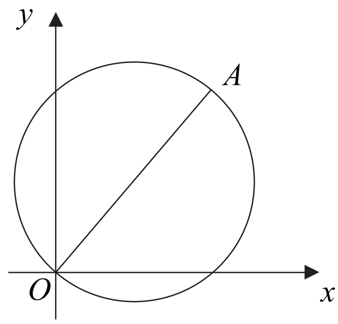
with your answer to part (a), to obtain an approximation to $y(3.2)$, giving your answer to three decimal places. *(3 marks)*

2 By using an integrating factor, find the solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} - y\tan x = 2\sin x$$

given that $y = 2$ when $x = 0$. *(9 marks)*

3 The diagram shows a sketch of a circle which passes through the origin O.



The equation of the circle is $(x-3)^2+(y-4)^2=25$ and OA is a diameter.

(a) Find the cartesian coordinates of the point A. *(2 marks)*

(b) Using O as the pole and the positive x-axis as the initial line, the polar coordinates of A are (k, α).

(i) Find the value of k and the value of $\tan\alpha$. *(2 marks)*

(ii) Find the polar equation of the circle $(x-3)^2+(y-4)^2=25$, giving your answer in the form $r = p\cos\theta + q\sin\theta$. *(4 marks)*

4 Evaluate the improper integral

$$\int_1^{\infty}\left(\frac{1}{x}-\frac{4}{4x+1}\right)\mathrm{d}x$$

showing the limiting process used and giving your answer in the form $\ln k$, where k is a constant to be found. *(5 marks)*

5 It is given that y satisfies the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+2\frac{\mathrm{d}y}{\mathrm{d}x}+5y=8\sin x+4\cos x$$

(a) Find the value of the constant k for which $y = k\sin x$ is a particular integral of the given differential equation. *(3 marks)*

(b) Solve the differential equation, expressing y in terms of x, given that $y = 1$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = 4$ when $x = 0$. *(8 marks)*

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6 The function f is defined by

$$f(x) = (9 + \tan x)^{\frac{1}{2}}$$

(a) (i) Find $f''(x)$. *(4 marks)*

(ii) By using Maclaurin's theorem, show that, for small values of x,

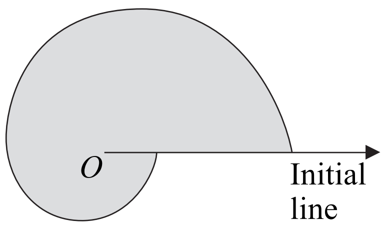
$$(9 + \tan x)^{\frac{1}{2}} \approx 3 + \frac{x}{6} - \frac{x^2}{216}$$ *(3 marks)*

(b) Find

$$\lim_{x \to 0} \left[\frac{f(x) - 3}{\sin 3x} \right]$$ *(3 marks)*

7 The diagram shows the curve C_1 with polar equation

$$r = 1 + 6e^{-\frac{\theta}{\pi}}, \quad 0 \leqslant \theta \leqslant 2\pi$$



(a) Find, in terms of π and e, the area of the shaded region bounded by C_1 and the initial line. *(5 marks)*

(b) The polar equation of a curve C_2 is

$$r = e^{\frac{\theta}{\pi}}, \quad 0 \leqslant \theta \leqslant 2\pi$$

Sketch the curve C_2 and state the polar coordinates of the end-points of this curve. *(4 marks)*

(c) The curves C_1 and C_2 intersect at the point P. Find the polar coordinates of P. *(5 marks)*

8 (a) Given that $x = t^2$, where $t \geqslant 0$, and that y is a function of x, show that:

(i) $2\sqrt{x}\,\dfrac{dy}{dx} = \dfrac{dy}{dt}$; *(3 marks)*

(ii) $4x\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} = \dfrac{d^2y}{dt^2}$. *(3 marks)*

(b) Hence show that the substitution $x = t^2$, where $t \geqslant 0$, transforms the differential equation

$$4x\frac{d^2y}{dx^2} + 2(1 + 2\sqrt{x})\frac{dy}{dx} - 3y = 0$$

into

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} - 3y = 0$$ *(2 marks)*

(c) Hence find the general solution of the differential equation

$$4x\frac{d^2y}{dx^2} + 2(1 + 2\sqrt{x})\frac{dy}{dx} - 3y = 0$$

giving your answer in the form $y = \mathrm{g}(x)$. *(4 marks)*

END OF QUESTIONS

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