General Certificate of Education

Mathematics 6360

MFP2 Further Pure 2

Mark Scheme

2009 examination - June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Set and published by the Assessment and Qualifications Alliance.

Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MFP2

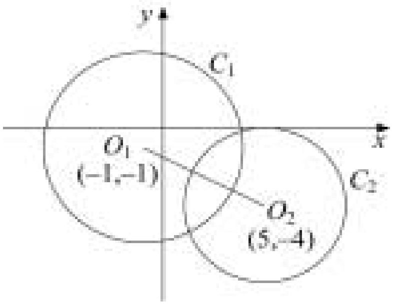
Q	Solution	Marks	Total	Comments
1(a)	$z^4 = 16\mathrm{e}^{\frac{4\pi\mathrm{i}}{12}}$	M1		Allow M1 if $z^4 = 2\mathrm{e}^{\frac{4\pi\mathrm{i}}{12}}$
	$= 16\left(\cos\frac{\pi}{3} + \mathrm{i}\sin\frac{\pi}{3}\right)$	A1		OE could be $2a\mathrm{e}^{\frac{\pi\mathrm{i}}{3}}$ or $2a\left(\cos\frac{\pi}{3} + \mathrm{i}\sin\frac{\pi}{3}\right)$
	$= 8 + 8\sqrt{3}\mathrm{i}$; $a = 8$	A1F	3	ft errors in 2^4
(b)	For other roots, $r = 2$	B1		for realising roots are of form $2\times\mathrm{e}^{i\theta}$
	$\theta = \frac{\pi}{12} + \frac{2k\pi}{4}$	M1A1		M1 for strictly correct θ i.e must be $\left(\text{their } \frac{\pi}{3} + 2k\pi\right)\times\frac{1}{4}$
	Roots are $2\mathrm{e}^{\frac{7\pi\mathrm{i}}{12}}, 2\mathrm{e}^{\frac{-5\pi\mathrm{i}}{12}}, 2\mathrm{e}^{\frac{-11\pi\mathrm{i}}{12}}$	A2,1,0 F	5	ft error in $\frac{\pi}{12}$ or r $\left[\text{accept } 2\mathrm{e}^{\left(\frac{\pi}{12}+\frac{2k\pi}{4}\right)i} \quad k = -1, -2, 1\right]$
	Total		**8**	
2(a)	$A = \frac{1}{2}, B = -\frac{1}{2}$	B1, B1F	2	For either A or B For the other
(b)	Method of differences clearly shown	M1		
	Sum $= \frac{1}{2}\left(1 - \frac{1}{2n+1}\right)$	A1		
	$= \frac{n}{2n+1}$	A1	3	AG
(c)	$\frac{1}{2(2n+1)} < 0.001$ or $\frac{n}{2n+1} > 0.499$	M1		Condone use of equals sign
	$1 < 0.004n + 0.002$ or $n > 0.998n + 0.499$			
	$n > \frac{0.998}{0.004}$ or $0.004n > 0.998$	A1		OE
	$n = 250$	A1F	3	ft if say 0.4999 used If method of trial and improvement used, award full marks for a completely correct solution showing working
	Total		**8**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$2+3\mathrm{i}$	B1	1	
(b)(i)	$\alpha\beta = 13$	B1	1	
(ii)	$\alpha\beta+\beta\gamma+\gamma\alpha = 25$	M1		M1A0 for -25 (no ft)
	$\gamma(\alpha+\beta) = 12$	A1F		
	$\gamma = 3$	A1F	3	ft error in $\alpha\beta$
(iii)	$p = -\sum\alpha = -7$	M1 A1F		M1 for a correct method for either p or q
	$q = -\alpha\beta\gamma = -39$	A1F	3	ft from previous errors p and q must be real for sign errors in p and q allow M1 but A0
	Alternative for (b)(ii) and (iii):			
(ii)	Attempt at $(z-2+3\mathrm{i})(z-2-3\mathrm{i})$	(M1)		
	$z^2-4z+13$	(A1)		
	cubic is $(z^2-4z+13)(z-3) \therefore \gamma = 3$	(A1)	(3)	
(iii)	Multiply out or pick out coefficients	(M1)		
	$p=-7,\ q=-39$	(A1, A1)	(3)	
	Total		**8**	
4(a)	Sketch, approximately correct shape	B1		B0 if curve touches asymptotes
	Asymptotes at $y = \pm 1$	B1	2	lines of answer booklet could be used for asymptotes be strict with sketch
(b)	Use of $u = \dfrac{\sinh x}{\cosh x}$	M1		
	$= \dfrac{\mathrm{e}^x - \mathrm{e}^{-x}}{\mathrm{e}^x + \mathrm{e}^{-x}}$ or $\dfrac{\mathrm{e}^{2x}-1}{\mathrm{e}^{2x}+1}$	A1		
	$u\left(\mathrm{e}^x + \mathrm{e}^{-x}\right) = \mathrm{e}^x - \mathrm{e}^{-x}$	M1		M1 for multiplying up
	$\mathrm{e}^{-x}(1+u) = \mathrm{e}^x(1-u)$	A1		A1 for factorizing out e's or M1 for attempt at $1+u$ and $1-u$ in terms of e^x
	$\mathrm{e}^{2x} = \dfrac{1+u}{1-u}$	m1		
	$x = \dfrac{1}{2}\ln\left(\dfrac{1+u}{1-u}\right)$	A1	6	AG

Q	Solution	Marks	Total	Comments
4(c)(i)	Use of $\tanh^2 x = 1 - \operatorname{sech}^2 x$	M1		
	Printed answer	A1	2	
(ii)	$(3\tanh x - 1)(\tanh x - 2) = 0$	M1		Attempt to factorise
	$\tanh x \neq 2$	E1		Accept tanh $x \neq 2$ written down but not ignored or just crossed out
	$\tanh x = \frac{1}{3}$	A1		
	$x = \frac{1}{2}\ln 2$	M1 A1F	5	ft
	Total		**15**	
5(a)	$(\cos\theta + \mathrm{i}\sin\theta)^{k+1} =$			
	$(\cos k\theta + \mathrm{i}\sin k\theta)(\cos\theta + \mathrm{i}\sin\theta)$	M1		
	Multiply out	A1		Any form
	$= \cos(k+1)\theta + \mathrm{i}\sin(k+1)\theta$	A1		Clearly shown
	True for $n = 1$ shown	B1		
	$P(k) \Rightarrow P(k+1)$ and $P(1)$ true	E1	5	provided previous 4 marks earned
(b)	$\frac{1}{z^n} = \frac{1}{\cos n\theta + \mathrm{i}\sin n\theta} = \cos n\theta - \mathrm{i}\sin n\theta$	M1A1		or $z^{-n} = \cos(-n\theta) + \mathrm{i}\sin(-n\theta)$ SC $(\cos\theta + \mathrm{i}\sin\theta)^{-n}$ quoted as $\cos n\theta - \mathrm{i}\sin n\theta$ earns M1A1 only
	$z^n + \frac{1}{z^n} = 2\cos n\theta$	A1	3	AG
(c)	$z + \frac{1}{z} = \sqrt{2}$			
	$2\cos\theta = \sqrt{2}$	M1		
	$\theta = \frac{\pi}{4}$	A1		
	$z^{10} + \frac{1}{z^{10}} = 2\cos\left(\frac{10\pi}{4}\right)$	M1		M0 for merely writing $z^{10} + \frac{1}{z^{10}} = 2\cos 10\theta$
	$= 0$	A1F	4	
	Total		**12**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
6(a)	Centre $-1-\mathrm{i}$ or $(-1,-1)$	B1		
	Radius 5	M1 A1F		ft incorrect centre if used
	$\lvert z+1+\mathrm{i}\rvert = 5$ or $\lvert z-(-1-i)\rvert = 5$	A1F	4	ft $\lvert z+1+\mathrm{i}\rvert = 10$ earns M0B1
(b)				
	C_1 correct centre, correct radius	B1F		ft errors in (a) but fit circles need to intersect and C_1 enclose $(0,0)$
	C_2 correct centre, correct radius	B1		
	Touching x-axis	B1F	3	error in plotting centre
(c)	$O_1O_2 = 3\sqrt{5}$	M1A1		allow if circles misplaced but O_1O_2 is still $3\sqrt{5}$
	Correct length identified	m1		
	Length is $9+3\sqrt{5}$	M1 A1F	5	ft if r is taken as 10
	Total		**12**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
7(a)(i)	$\frac{ds}{dx} = \sqrt{1+\left(\frac{dy}{dx}\right)^2} = \sqrt{1+\left(\frac{s}{2}\right)^2}$	M1A1		Allow M1 for $s = \int \sqrt{1+\left(\frac{s}{2}\right)^2}\,dx$ then A1 for $\frac{dy}{dx}$
	$= \frac{1}{2}\sqrt{4+s^2}$	A1	3	AG
(ii)	$\int \frac{ds}{\sqrt{4+s^2}} = \int \frac{1}{2}\,dx$	M1		For separation of variables; allow without integral sign
	$\sinh^{-1}\frac{s}{2} = \frac{1}{2}x + C$	A1		Allow if C is missing
	$C = 0$	A1		
	$s = 2\sinh\frac{1}{2}x$	A1	4	AG if C not mentioned allow $\frac{3}{4}$ **SC** incomplete proof of (a)(ii), differentiating $s = 2\sinh\frac{x}{2}$ to arrive at $\frac{ds}{dx} = \frac{1}{2}\sqrt{4+s^2}$ allow M1A1 only $\left(2/4\right)$
(iii)	$\frac{dy}{dx} = \sinh\frac{1}{2}x$	M1		
	$y = 2\cosh\frac{1}{2}x + C$	A1		Allow if C is missing
	$C = 0$	A1	3	Must be shown to be zero and CAO
(b)	$y^2 = 4\left(1+\sinh^2\frac{x}{2}\right)$	M1		Use of $\cosh^2 = 1+\sinh^2$
	$= 4+s^2$	A1	2	AG
	Total		**12**	
	TOTAL		**75**	