General Certificate of Education
June 2009
Advanced Level Examination

MATHEMATICS **MD02**
Unit Decision 2

Monday 15 June 2009 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables
- an insert for use in Questions 1, 5 and 6 (enclosed).

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil or coloured pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MD02.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Fill in the boxes at the top of the insert.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Answer **all** questions.

1 [**Figure 1**, printed on the insert, is provided for use in this question.]

A decorating project is to be undertaken. The table shows the activities involved.

Activity	**Immediate Predecessors**	**Duration (days)**
A	–	5
B	–	3
C	–	2
D	A, B	4
E	B, C	1
F	D	2
G	E	9
H	F, G	1
I	H	6
J	H	5
K	I, J	2

(a) Complete an activity network for the project on **Figure 1**. *(3 marks)*

(b) On **Figure 1**, indicate:

(i) the earliest start time for each activity; *(2 marks)*

(ii) the latest finish time for each activity. *(2 marks)*

(c) State the minimum completion time for the decorating project and identify the critical path. *(2 marks)*

(d) Activity F takes 4 days longer than first expected.

(i) Determine the new earliest start time for activities H and I. *(2 marks)*

(ii) State the minimum delay in completing the project. *(1 mark)*

2 Two people, Rowena and Colin, play a zero-sum game.

The game is represented by the following pay-off matrix for Rowena.

Colin

	Strategy	$\mathbf{C_1}$	$\mathbf{C_2}$	$\mathbf{C_3}$
	$\mathbf{R_1}$	-4	5	4
Rowena	$\mathbf{R_2}$	2	-3	-1
	$\mathbf{R_3}$	-5	4	3

(a) Explain what is meant by the term 'zero-sum game'. *(1 mark)*

(b) Determine the play-safe strategy for Colin, giving a reason for your answer. *(2 marks)*

(c) Explain why Rowena should never play strategy R_3. *(1 mark)*

(d) Find the optimal mixed strategy for Rowena. *(7 marks)*

Turn over for the next question

3 Five lecturers were given the following scores when matched against criteria for teaching five courses in a college.

	Course 1	**Course 2**	**Course 3**	**Course 4**	**Course 5**
Ron	13	13	9	10	13
Sam	13	14	12	17	15
Tom	16	10	8	14	14
Una	11	14	12	16	10
Viv	12	14	14	13	15

Each lecturer is to be allocated to exactly one of the courses so as to maximise the total score of the five lecturers.

(a) Explain why the Hungarian algorithm may be used if each number, x, in the table is replaced by $17 - x$. *(2 marks)*

(b) Form a new table by subtracting each number in the table above from 17. Hence show that, by reducing **rows first** and then columns, the resulting table of values is as below.

0	0	3	3	0
4	3	4	0	2
0	6	7	2	2
5	2	3	0	6
3	1	0	2	0

(3 marks)

(c) Show that the zeros in the table in part (b) can be covered with two horizontal and two vertical lines. Hence use the Hungarian algorithm to reduce the table to a form where five lines are needed to cover the zeros. *(3 marks)*

(d) Hence find the possible allocations of courses to the five lecturers so that the total score is maximised. *(4 marks)*

(e) State the value of the maximum total score. *(1 mark)*

4 A linear programming problem involving variables x, y and z is to be solved. The objective function to be maximised is $P = 4x + y + kz$, where k is a constant. The initial Simplex tableau is given below.

P	***x***	***y***	***z***	***s***	***t***	***value***
1	-4	-1	$-k$	0	0	0
0	1	2	3	1	0	7
0	2	1	4	0	1	10

(a) In addition to $x \geqslant 0$, $y \geqslant 0$ and $z \geqslant 0$, write down **two** inequalities involving x, y and z for this problem. *(1 mark)*

(b) (i) The first pivot is chosen from the ***x*-column**. Identify the pivot and perform **one** iteration of the Simplex method. *(4 marks)*

(ii) Given that the optimal value of P has not been reached after this first iteration, find the possible values of k. *(2 marks)*

(c) Given that $k = 10$:

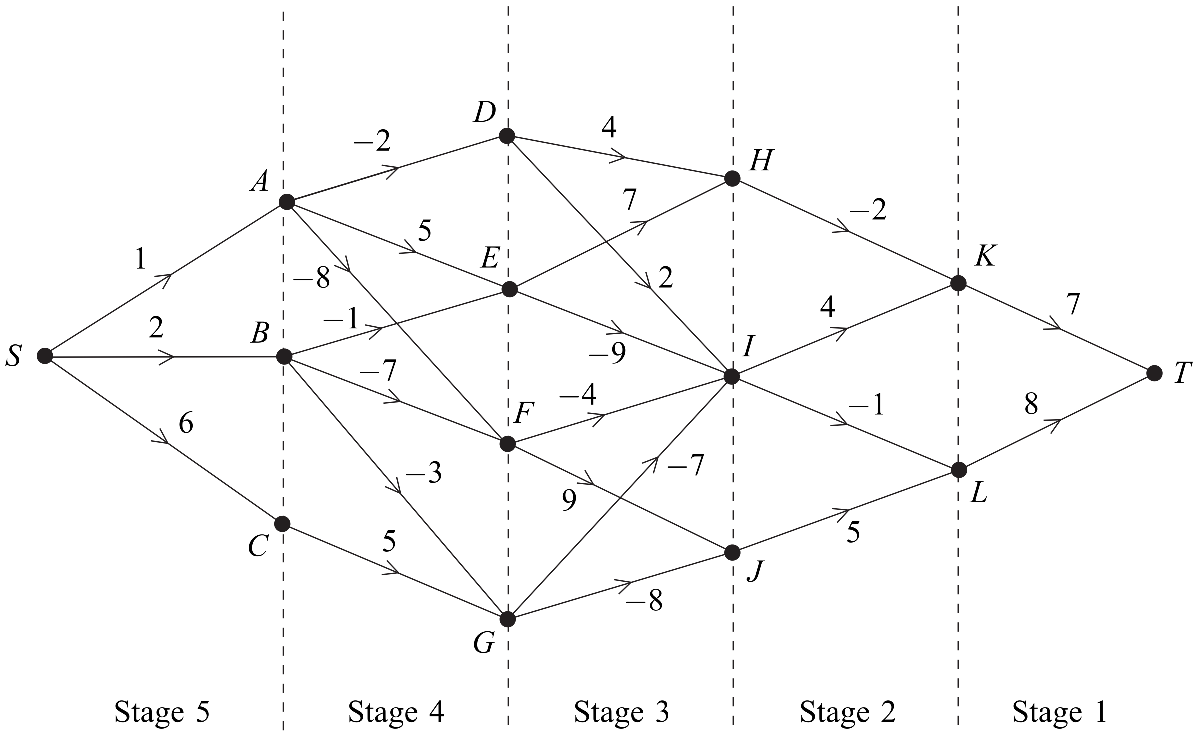
(i) perform one further iteration of the Simplex method; *(4 marks)*

(ii) interpret the final tableau. *(3 marks)*

Turn over for the next question

5 [**Figure 2**, printed on the insert, is provided for use in this question.]

A company has a number of stores. The following network shows the possible actions and profits over the next five years. The number on each edge is the expected profit, in millions of pounds. A negative number indicates a loss due to investment in new stores.

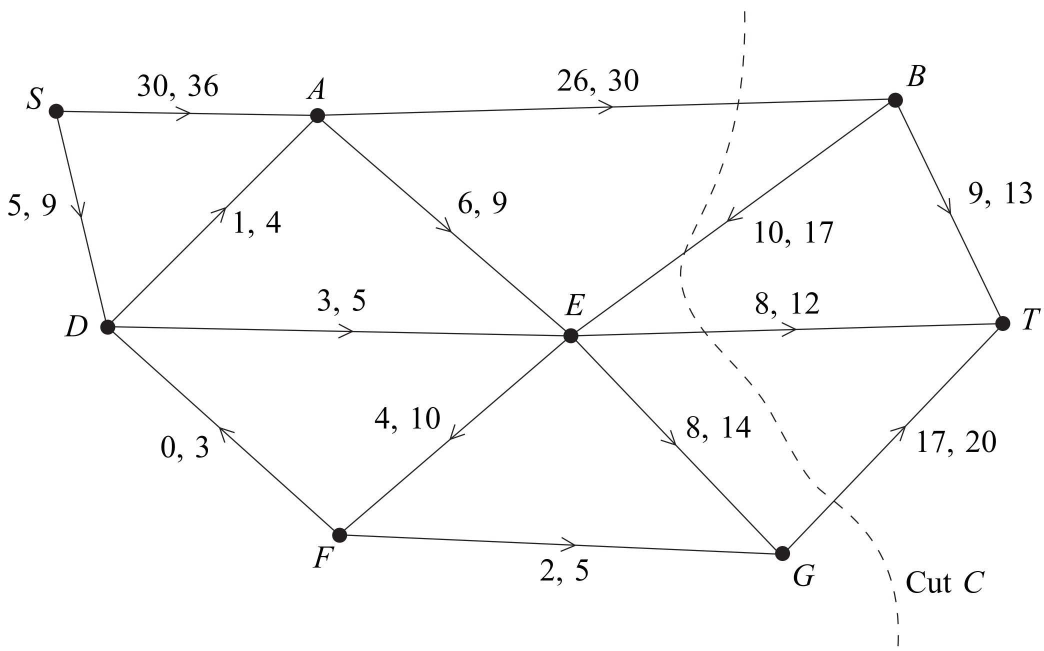


(a) **Working backwards from *T***, use dynamic programming to maximise the expected profits over the five years. You may wish to complete the table on **Figure 2** as your solution. *(7 marks)*

(b) State the maximum expected profit and the sequence of vertices from S to T in order to achieve this. *(2 marks)*

6 [**Figures 3, 4 and 5**, printed on the insert, are provided for use in this question.]

The network shows a system of pipes with the lower and upper capacities for each pipe in litres per second.



(a) Find the value of the cut C. *(2 marks)*

(b) **Figure 3**, on the insert, shows a partially completed diagram for a feasible flow of 40 litres per second from S to T. Indicate, on **Figure 3**, the flows along the edges AE, EF and FG. *(3 marks)*

(c) (i) Taking your answer from part (b) as an initial flow, indicate potential increases and decreases of the flow along each edge on **Figure 4**. *(3 marks)*

(ii) Use flow augmentation on **Figure 4** to find the maximum flow from S to T. You should indicate any flow augmenting paths in the table and modify the potential increases and decreases of the flow on the network. *(4 marks)*

(d) Illustrate the maximum flow on **Figure 5**. *(2 marks)*

(e) Find a cut with value equal to that of the maximum flow. *(2 marks)*

END OF QUESTIONS

There are no questions printed on this page

Surname		Other Names					
Centre Number						Candidate Number	
Candidate Signature							

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Insert

Insert for use in **Questions 1, 5 and 6**.

Fill in the boxes at the top of this page.

Fasten this insert securely to your answer book.

Turn over for Figure 1

Turn over ▶

Figure 1 (for use in Question 1)

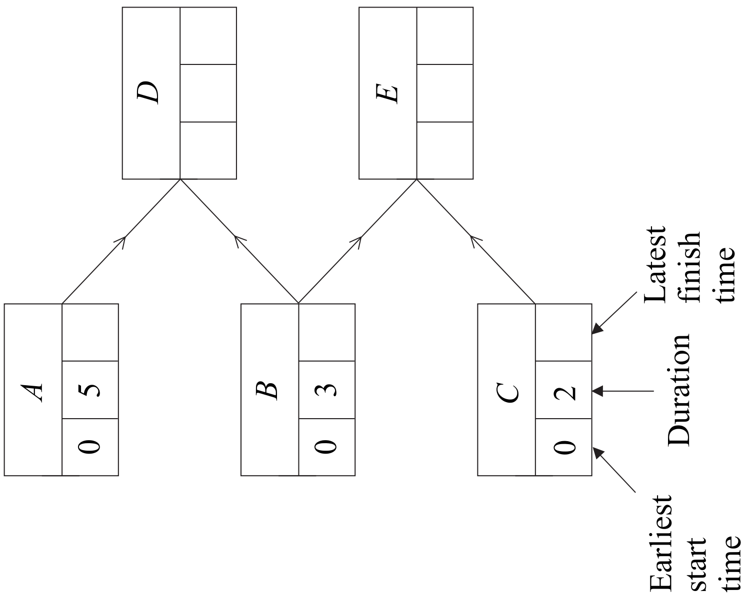


Figure 2 (for use in Question 5)

Stage	State	From	Value	
1	K	T	7	
	L	T	8	
2	H	K	$-2 + 7 = 5$	
	I	K		
		L		
	J	L		
3	D	H		
		I		

Turn over ▶

Figure 3 (for use in Question 6)

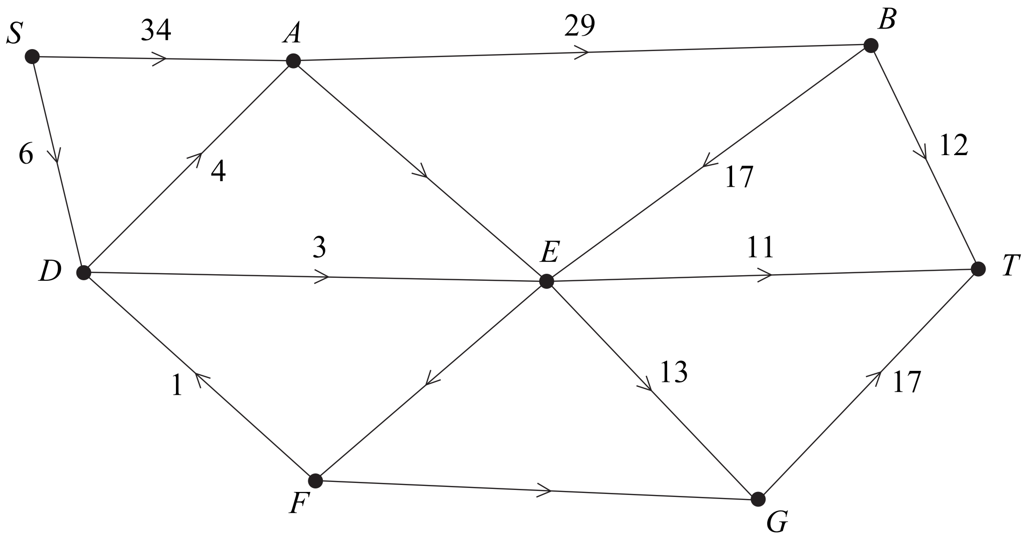
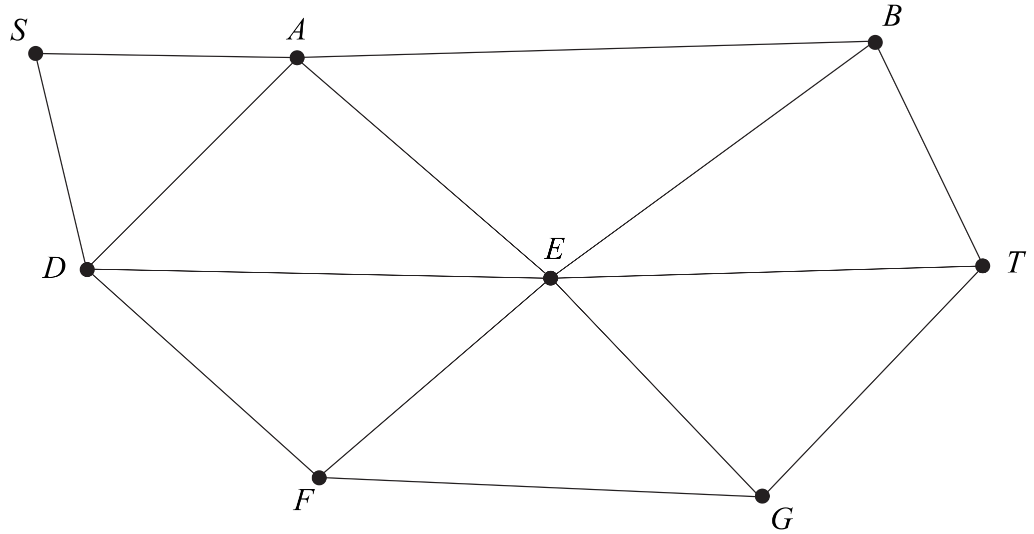


Figure 4 (for use in Question 6)



Path	Extra Flow

Figure 5 (for use in Question 6)